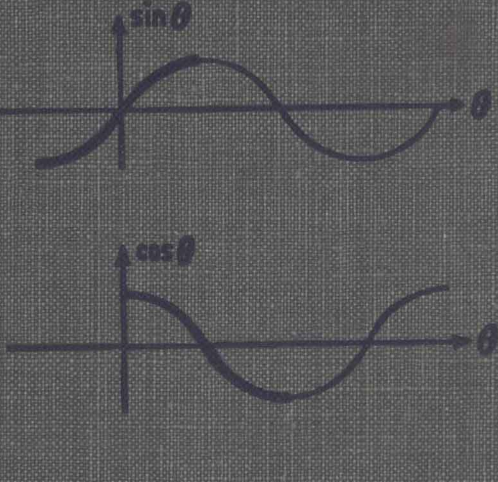
PLANE TRIGONOMETRY

WITH TABLES



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PLANE TRIGONOMETRY

BY

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PLANE TRIGONOMETRY

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PREFACE

The principal objective of the author in writing this trigonometry textbook has been teachability. The book should prove especially beneficial to students who have a weak mathematical background and to those who have not yet acquired the habit of orderly and independent thinking. Some of the features of the text are:

1. Many of the exercises include true-false questions to test the student's ability to avoid pitfalls and to detect camouflaged truths. An effort has been made to thwart the development of such false notions as "In logarithms, division is replaced by subtraction" and "To find the square root of a number, divide its logarithm by 2." The duty of the instructor is, not only to teach correct methods, but also to convince the student of the error in the false methods.

2. The memory work has been reduced to a bare minimum. All unnecessary formulas and concepts have been deliberately omitted. Some of these are cologarithms, confusing reduction formulas, the "fourth" property of logarithms, and formulas for cot $(A + B)$ and cot $(A - B)$.

3. The new charactcristic rule for logarithms has been proved by classroom experiment to be effective, especially in finding a number from its logarithm. Instructors who prefer the old rule will find it listed as an alternative.

4. Definite instructions are given for proving identities and solving trigonometric equations. The subject of identities is approached gradually with practice in algebraic operations with the trigonometric functions.

5. A careful explanation of approximations and significant figures is given early in the text. The principle of accuracy in figures is adhered to tenaciously throughout

the book. The answers to computation problems are given with no more accuracy than is justified by the given data.

6. All problem sets are carefully graded and contain an abundance of simple problems that involve nothing more than the principles being discussed. The first half of each set of computation problems requires no interpolation. This enables the student to concentrate on the new concepts without being burdened with confusing interpolations.

7. Miscellaneous points include (*a*) interesting applied problems, (*b*) problems that are encountered in the calculus, (*c*) a careful explanation of the concept of infinity, (*d*) memory schemes, (*e*) the uses of the sine and cosine curves, and (*f*) a note to the student.

For the convenience of teachers, brief outlines for two-semester-hour and three-semester-hour courses are given.

TWO-SEMESTER-HOUR COURSE OF 30 LESSONS
(Allowing 3 hours for quizzes)

Chapter	1	2	3	4	5	6	7	8	9	10	11	12
Number of Lessons	3	4	$2\frac{1}{2}$	1	$1\frac{1}{2}$	1	3	1	4	$1\frac{1}{2}$	$3\frac{1}{2}$	1

THREE-SEMESTER-HOUR COURSE OF 45 LESSONS
(Allowing 5 hours for examinations)

Chapter	1	2	3	4	5	6	7	8	9	10	11	12
Number of Lessons	4	4	4	2	2	2	5	2	6	2	5	2

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E. RICHARD HEINEMAN.

LUBBOCK, TEXAS,
August, 1942.

NOTE TO THE STUDENT

A mastery of the subject of trigonometry requires (1) a certain amount of memory work and (2) a great deal of practice and drill in order to acquire experience and skill in the application of the memory work. Your instructor is a "trouble-shooter" who attempts to prevent you from going astray, supplies missing links in your mathematical background, and tries to indicate the "common sense" approach to the problem. The memory work in any course is one thing that the student can and should perform by himself. The least you can do for your instructor and yourself is *to commit to memory each definition and theorem as soon as you contact it.* This can be accomplished most rapidly, not by reading, but by writing the definition or theorem until you can reproduce it without the aid of the text.

In working the problems, do not continually refer back to the illustrative examples. Study the examples so thoroughly (by writing them) that you can reproduce them with your text closed. Only after the examples are entirely clear and have been completely mastered should you attempt the unsolved problems. These problems should be worked *without referring to the text.*

GREEK ALPHABET

Alpha	Α, α	Nu	Ν, ν
Beta	Β, β	Xi	Ξ, ξ
Gamma	Γ, γ	Omicron	Ο, ο
Delta	Δ, δ	Pi	Π, π
Epsilon	Ε, ε	Rho	Ρ, ρ
Zeta	Ζ, ζ	Sigma	Σ, σ, ς
Eta	Η, η	Tau	Τ, τ
Theta	Θ, θ	Upsilon	Υ, υ
Iota	Ι, ι	Phi	Φ, φ
Kappa	Κ, κ	Chi	Χ, χ
Lambda	Λ, λ	Psi	Ψ, ψ
Mu	Μ, μ	Omega	Ω, ω

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PLANE TRIGONOMETRY

CHAPTER 1

THE TRIGONOMETRIC FUNCTIONS

1. Trigonometry. Trigonometry is that branch of mathematics which deals primarily with six ratios called the trigonometric functions. These ratios are important for two reasons. First, they are the basis of a theory which is used in other branches of mathematics as well as in physics and engineering. Second, they are used in solving triangles. From geometry we recall that two sides and the included angle of a triangle suffice to fix its size and shape. It will be shown later that the length of the third side and the size of the remaining angles can be computed by means of trigonometry.

2. Directed segments. A directed line is a line upon which one direction is considered positive; the other, negative. Thus in Fig. 1 the arrowhead indicates that all

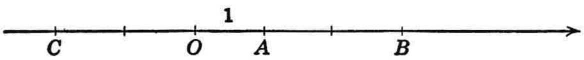

FIG. 1.

segments measured from left to right are positive. Hence if $OA = 1$ unit of length, then $OB = 3$, and $BC = -5$. Observe that since the line is directed, CB is not equal to BC. However, $BC = -CB$; or $CB = -BC$. Also note that $OB + BC + CO = 0$.

3. The rectangular coordinate system. A rectangular (or Cartesian) coordinate system consists of two perpendicular *directed* lines. It is conventional to draw and direct these lines as in Fig. 2. The **x-axis** and the **y-axis** are called the **coordinate axes;** their intersection O is called the **origin.** The position of any point in the plane is fixed by its distances from the axes.

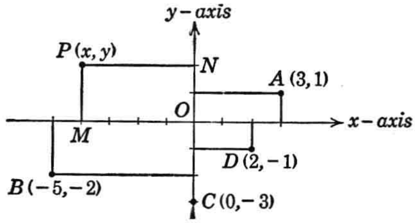

FIG. 2.

*The x-coordinate** (or x) *of point P is the directed segment NP* (or *OM*) *measured from the y-axis to point P. The y-coordinate** (or y) *of point P is the directed segment MP, measured from the x-axis to point P.* It is necessary to remember that each coordinate is measured *from axis to point.* Thus the x of P is NP (not PN); the y of P is MP (not PM). The point P, with x-coordinate x and y-coordinate y, is denoted by $P(x, y)$. It follows that the x of any point to the right of the y-axis is positive; to the left, negative.

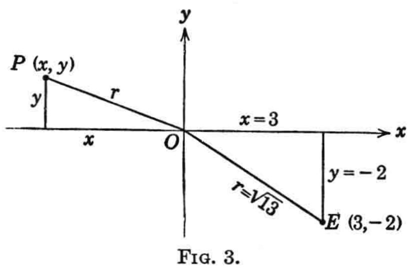

FIG. 3.

Also the y of any point above the x-axis is positive; below, negative.

To plot a point means to locate and indicate its position on a coordinate system. Several points are plotted in Fig. 2.

The distance from the origin O to point P is called the **radius vector** (or r) of P. This distance r is not directed

* The x-coordinate and y-coordinate are also called the *abscissa* and *ordinate,* respectively.

and *is always positive* by agreement. Hence with each point of the plane we can associate three coordinates: x, y, and r. The radius vector r can be found by using the Pythagorean* relation $x^2 + y^2 = r^2$.

The coordinate axes divide the plane into four parts called **quadrants** as indicated in Fig. 4.

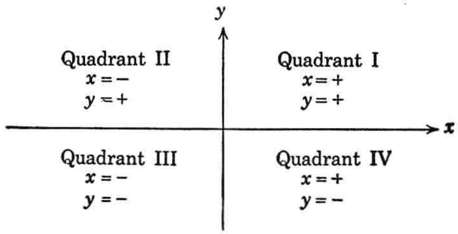

FIG. 4.

Examples. To find r for the point $(5, -12)$, use

$$r^2 = 5^2 + (-12)^2 = 169, \qquad r = 13.$$

If $x = 15$ and $r = 17$, we obtain y by using $x^2 + y^2 = r^2$. Hence $(15)^2 + y^2 = (17)^2$; $225 + y^2 = 289$; $y^2 = 64$; $y = \pm 8$. If the point is in quadrant I, $y = 8$; but if the point is in quadrant IV, $y = -8$. (Since x is positive the point cannot lie in either quadrant II or quadrant III.)

EXERCISE 1

1. Using coordinate paper, plot the points $(-3, 4)$, $(-3, -\sqrt{7})$, $(-2, 0)$, $(6, -2)$. Find the value of r for each point.

2. Plot on coordinate paper and find the value of r: $(5, 12)$, $(-1, 1)$, $(0, -4)$, $(\sqrt{5}, -\sqrt{2})$.

3. Use the Pythagorean theorem to find the missing coordinate and then plot the point:

(*a*) $x = 12$, $r = 13$, point is in Q I.†
(*b*) $y = -5$, $r = 6$, point is in Q III.
(*c*) $x = 0$, $r = 3$, y is negative.

4. Use the Pythagorean theorem to find the missing coordinate; plot the point:

(*a*) $y = -4$, $r = 5$, point is in Q IV.
(*b*) $x = -2\sqrt{5}$, $r = 6$, point is in Q II.
(*c*) $y = -4$, $r = 4$.

* *Pythagorean theorem: The square of the hypotenuse of a right triangle equals the sum of the squares of its legs.*

† Q I means quadrant I.

5. What is the x of all points on the y-axis? What is the y of all points on the x-axis?

6. In which quadrants is each of the following ratios positive? negative? (*a*) y/r. (*b*) x/r. (*c*) y/x.

4. Trigonometric angles. In geometry an angle was thought of as the "opening" between two lines. *A*

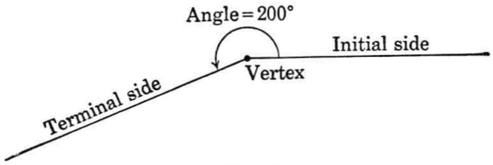

FIG. 5.

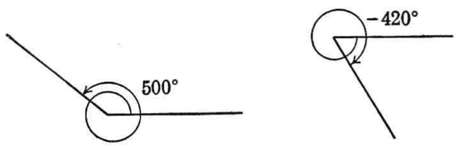

FIG. 6.

trigonometric angle is an amount of rotation required to move a line from one position to another. A *positive* angle is generated by *counterclockwise* rotation; a *negative* angle, by *clockwise* rotation. Figure 5 illustrates the terms used and shows an angle of 200°. Figure 6 shows angles of 500° and −420°. The −420° angle may be thought of as the amount of rotation effected by the minute hand of a clock between 12:15 and 1:25. *To draw a trigonometric angle, we need,* in addition to its sides, *a curved arrow extending from its initial side to its terminal side.*

5. Standard position of an angle. *An angle is said to be in* **standard position** *if its vertex is at the origin and its initial side coincides with the positive x-axis.* An angle is said to be in a certain quadrant if its terminal side lies in that quadrant when the angle is in standard position. For example, 600° is in the third quadrant; or, −70° is a fourth-quadrant angle.

Angles are said to be **coterminal** if their terminal sides coincide when the angles are in standard position. For example, 200°, 560°, −160° are coterminal angles. From a trigonometric viewpoint these angles are not equal; they are merely coterminal.

EXERCISE 2

Place each of the following angles in standard position; draw a curved arrow to indicate the rotation. Draw and find the size of two other angles, one positive and one negative, that are coterminal with the given angle.

1. 330°.	**2.** 135°.	**3.** 225°.	**4.** 120°.
5. 400°.	**6.** 540°.	**7.** 90°.	**8.** 315°.

Each of the following points is on the terminal side of a positive angle in standard position. Plot the point; draw the terminal side of the angle; indicate the angle by a curved arrow; use a protractor to find to the nearest degree the size of the angle.

9. (10, 2).	**10.** (8, −3).	**11.** (−5, 5).	**12.** (0, 6).
13. (−7, −1).	**14.** $(-5, 5\sqrt{3})$.	**15.** (−4, 0).	**16.** (−2, 5).

17. A wheel makes 1,000 revolutions per minute. Through how many degrees does it move in one second?

6. Definitions of the trigonometric functions of a general angle. The whole subject of trigonometry is based upon the six **trigonometric functions.** The names of these functions, with their abbreviations in parentheses, are: sine (**sin**), cosine (**cos**), tangent (**tan**), cotangent (**cot**), secant (**sec**), cosecant (**csc**). In a certain sense, the following definitions are the most important in this book.

A COMPLETE DEFINITION OF THE TRIGONOMETRIC FUNCTIONS OF ANY ANGLE θ

1. *Place the angle θ* in standard position.*
2. *Choose any point P on the terminal side of θ.*
3. *Drop a perpendicular from P to the x-axis, thus forming a triangle of reference for θ.*
4. *The point P has three coordinates x, y, r, in terms of which we define the following trigonometric functions:*

* See Greek alphabet opposite p. 1.

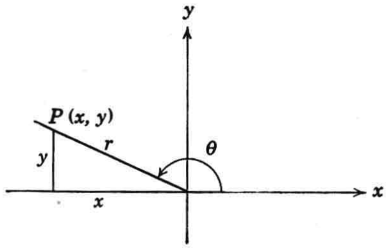

FIG. 7.

$$\sin\theta = \frac{y}{r},$$

$$\cos\theta = \frac{x}{r},$$

$$\tan\theta = \frac{y}{x},$$

$$\cot\theta = \frac{x}{y},$$

$$\sec\theta = \frac{r}{x},$$

$$\csc\theta = \frac{r}{y}.$$

A function of θ is a quantity whose value can be determined whenever the value of θ is given. For example, $3\theta^2 + 1$ is a function of θ. If θ has the value 5, then $3\theta^2 + 1$ has the value 76. If $\theta = -4$, $3\theta^2 + 1 = 49$. Likewise $\theta^3 + 7$ and 8θ are functions of θ. In fact, 9 is a function of θ. Why?

In order to prove that $\sin\theta$ is a function of θ, we shall show that the value of $\sin\theta$ is independent of the choice of point P on the terminal side of θ. Let $P'(x', y')$ be any other point on OP. Then, using the coordinates of P', we have $\sin\theta = y'/r'$. Since triangles $OP'M'$ and OPM are similar, it follows that

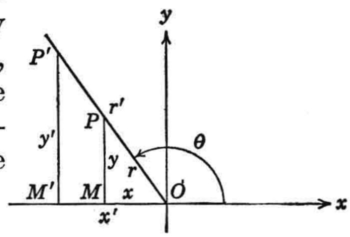

FIG. 8.

$$y'/r' = y/r,$$

and the value of $\sin\theta$ is the same whether it is obtained by using P or by using P'. If, however, the value of θ is changed, then the value of y/r is changed, and the value of $\sin\theta$ is changed. Since the value of $\sin\theta$ depends upon the value of θ, and is independent of the choice of P, we can say that $\sin\theta$ is a function of θ. *The values of the trigonometric functions* of θ depend solely upon the value of θ.*

* Three other functions sometimes used are versed sine, coversed sine, and