

BENJAMIN C. KUO

DIGITAL  
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# DIGITAL CONTROL SYSTEMS

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# PREFACE

[Prefis] 序言  
vi 序言 摘要 vii

This book represents a fairly extensive revision of the 1977 edition of **DIGITAL CONTROL SYSTEMS** published by the SRL Publishing Company. Since 1963, the author has written several books on the subject of digital and sampled-data control systems. The first text, **ANALYSIS AND SYNTHESIS OF SAMPLED-DATA CONTROL SYSTEMS**, was published in 1963 by Prentice-Hall, Inc. This earlier book features the classical analysis and design of sampled-data control systems, and has a chapter on nonlinear systems. In general, the book treats a wide range of sampled-data control systems problems in a comprehensive manner. The second book, **DISCRETE-DATA CONTROL SYSTEMS**, was published in 1970 by Prentice-Hall, Inc. The book represents a completely new approach from the 1963 edition by concentrating on the state variable approach and modern control theory. **DIGITAL CONTROL SYSTEMS** is a complete revision and expansion of **DISCRETE-DATA CONTROL SYSTEMS**.

In the 1960's, at the height of the expansion of the aerospace industries, development of sampled-data control systems theory enjoyed the most prolific period. A large number of papers and books were published during this period on discrete-data and sampled-data control systems. In recent years although research and development activities in the aerospace sector have stabilized, the advances made in microcomputers and microprocessors and the process control industry have infused new life to the growth of digital control systems. However, the popularity of microprocessor control has put new emphasis on digital

control systems theory. Because microprocessors are slow digital machines, and are usually equipped with a small wordlength, it is necessary to place importance on these constraints and the effects of time delays and amplitude quantization when designing a digital control system. Many practicing engineers apply microprocessors in control systems simply by implementing an analog control algorithm and using a very small sampling period, in order to keep the overall system stable. While this scheme may work in many non-critical situations, it would not be an acceptable solution for more critical design cases.

For those who are familiar with the first edition, **DIGITAL CONTROL SYSTEMS**, the following major revisions are made in the current edition:

1. The conventional analysis and design methods are expanded. This step was taken because a majority of the systems designed in the industry still rely on the use of the conventional design techniques. This happens to be true for both analog and digital control systems.
2. The design of digital control systems is greatly expanded. Chapter 10 presents a variety of the design techniques encompassing the conventional as well as the modern control theory.
3. A chapter on microprocessor control is introduced. It is difficult to draw a line on how much and what portion of the microprocessor applications should be included in a text such as this. Most electrical engineering curricula now have courses on the application of microprocessors. The author believes that the learning of the programming aspects should not belong to this book. However, it is impossible that every reader of this text is proficient in microcomputer programming. It is quite possible that the control system engineer is assigned with the task of designing the digital control system, and a programmer or software engineer would probably be more proficient in programming the digital controller. However, it is important that the control system designer understands how a microcomputer works, and all the programming ramifications, so that a realistic digital controller can be designed subject to all the physical constraints of the microcomputer. With this in mind, Chapter 14 introduces the basic components and the programming logic of a microprocessor without dwelling into specific details, but places emphasis on pointing out all the effects due to time delays and quantization due to a finite wordlength.
4. In order to conserve space and make room for all the expanded material, the chapters on statistical design have been eliminated. Since this new edition is now intended to be used at the senior or first-year graduate level, the omission of the material on statistical design may not be a serious handicap to the majority of the users.
5. The home problems have been expanded. Many practical analysis and design problems are introduced in the problem sections.

As a college text, the material presented should be more than adequate for a one-semester course. This will give the instructor some flexibility in selecting the subjects he would like to cover. All the material has been class-tested by the author teaching at the University of Illinois at Urbana-Champaign. The book is also prepared with the needs of the practicing engineers in mind; it should be suitable as a reference and for self-study purposes.

It is assumed that the reader has a knowledge on the basic principles of feedback control systems. Background on the basics of matrix algebra, Laplace transform is essential. A prior knowledge of the state-variable analysis and microprocessor programming would be helpful.

Chapter 1 gives a general introduction to the subject of digital control systems. Some typical examples of digital control systems are presented.

Chapter 2 covers signal conversion, signal processing, and the sampling operation. It gives the motivation to the mathematical and analytical treatment of digital data and signals in control systems. Chapter 3 gives a careful treatment of the z-transform theory. Modified z-transform, signal flow graph applications, and nonuniform and multirate digital control systems are included in the chapter. Chapter 4 gives a comprehensive treatment of state variable technique applied to digital systems. Time-invariant as well as time-varying systems are included. Chapter 5 covers the subject on stability. Various methods of testing the stability of linear digital control systems are given. The second method of Liapunov and its use in optimal system design is also covered. The subject on digital simulation is given in Chapter 6. Numerical integration and the z-form methods are discussed. Also included in this chapter is the subject on digital redesign, which is on how to approximate a continuous-data system by an equivalent digital system. Chapter 7 is on time-domain analysis of digital control systems. Emphasis is placed on the prediction of time response from pole-zero locations, root locus method, steady-state and transient analysis. Chapter 8 is on frequency-domain analysis of digital control systems. The conventional methods such as the Nyquist criterion, Bode plot, Nichols' chart, gain margin and phase margin are all extended to digital control systems. Chapter 9 is devoted to the subject on controllability, observability and time-optimal control. Chapter 10 covers a wide range of design techniques, from conventional to state-feedback and output-feedback controls. Chapters 11 through 13 are devoted to optimal control of digital control systems. The subjects include maximum principle, optimal linear regulator design, dynamic programming, sampling-period sensitivity, and observer design. The last chapter, Chapter 14, is on microprocessor control.

The book contains a large number of illustrative examples, and many are derived from the practical experiences of the author, although most problems are simplified in order to be suitable for textbook presentation. Many practical digital control systems problems are given in the problems section at the end

of each chapter. A solution manual to the problems is available, and qualified users should make inquiry to the publisher.

The author is grateful to many of his graduate students whose assistance in the form of thesis work and classroom discussions has contributed in many ways to the preparation of this book. He wishes to acknowledge his daughter Tina for doing most of the illustrations in the book, and his other two daughters, Lori and Linda, for proofreading the typed manuscript. Special thanks go to Jane Carlton, his former secretary, for typing the complete manuscript in a professional and efficient manner. Finally, the author wishes to pay tribute to his wife, Margaret, for her contribution in many ways to this and many other projects throughout the years. Without her moral and physical assistance, these projects could never have been accomplished.

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Champaign, Illinois

# CONTENTS

## 1 INTRODUCTION

- 1.1 Introduction 1
- 1.2 Advantages of Sampling in Control Systems 5
- 1.3 Examples of Discrete-Data and Digital Control Systems 5

## 2 SIGNAL CONVERSION AND PROCESSING

11

- 2.1 Introduction 11
- 2.2 Digital Signals and Coding 12
- 2.3 Data Conversions and Quantization 19
- 2.4 Sample-and-Hold Devices 23
- 2.5 Digital-to-Analog (D/A) Conversion 28
- 2.6 Analog-to-Digital (A/D) Conversion 33
- 2.7 Mathematical Treatment of the Sampling Process 39
- 2.8 The Sampling Theorem 59
- 2.9 Some  $s$ -Plane Properties of  $F^*(s)$  60
- 2.10 Reconstruction of Sampled Signals 62
- 2.11 The Zero-Order Hold 66

**3 THE Z-TRANSFORM**

78

- 3.1 Definition of the z-Transform 78
- 3.2 Evaluation of z-Transforms 81
- 3.3 Mapping Between the s-Plane and the z-Plane 84
- 3.4 The Inverse z-Transform 88
- 3.5 Theorems of the z-Transform 95
- 3.6 Limitations of the z-Transform Method 103
- 3.7 The Pulse Transfer Function
- 3.8 Pulse Transfer Function of the Zero-Order Hold and the  
Relation Between  $G(s)$  and  $G(z)$  108
- 3.9 Responses Between Sampling Instants 111
- 3.10 Signal-Flow-Graph Method Applied to Digital Systems 121
- 3.11 Multirate Digital Systems 137

**4 THE STATE VARIABLE TECHNIQUE**

163

- 4.1 Introduction 163
- 4.2 State Equations and State Transition Equations of  
Continuous-Data Systems 164
- 4.3 State Equations of Digital Systems with Sample-and-Hold 170
- 4.4 State Equations of Digital Systems with All-Digital Elements 172
- 4.5 State Transition Equations of Digital Systems 172
- 4.6 State Transition Equation of Digital Time-Invariant Systems 176
- 4.7 Digital Simulation and Approximation 177
- 4.8 Solution of the Time-Invariant Discrete State Equation by  
the z-Transformation 178
- 4.9 Relation Between State Equation and Transfer Function 181
- 4.10 Characteristic Equation, Eigenvalues and Eigenvectors 185
- 4.11 Diagonalization of the A Matrix 192
- 4.12 Jordan Canonical Form 195
- 4.13 Methods of Computing the State Transition Matrix 198
- 4.14 Digital Adjoint Systems 206
- 4.15 Relationship Between State Equations and High-Order  
Difference Equations 208
- 4.16 Transformation to Phase-Variable Canonical Form 210
- 4.17 The State Diagram 218
- 4.18 Decomposition of Digital Systems 225

4.19	State Diagrams of Sampled-Data Control Systems	231
4.20	Response of Sampled-Data Systems Between Sampling Instants Using the State Approach	237
4.21	Analysis of Discrete-Time Systems with Multirate, Skip-Rate, and Nonsynchronous Samplings	239
4.22	State-Plane Analysis	244
<b>5</b>	<b>STABILITY OF DIGITAL CONTROL SYSTEMS</b>	<b>267</b>
5.1	Introduction	267
5.2	Definitions of Stability	270
5.3	Stability Tests of Digital Systems	277
5.4	The Second Method of Liapunov	287
<b>6</b>	<b>DIGITAL SIMULATION AND DIGITAL REDESIGN</b>	<b>303</b>
6.1	Introduction	303
6.2	Digital Simulation - Digital Model with Sample-and-Hold	303
6.3	Digital Simulation - Numerical Integration	308
6.4	Digital Simulation by the z-Forms	315
6.5	Digital Redesign	321
<b>7</b>	<b>TIME-DOMAIN ANALYSIS</b>	<b>344</b>
7.1	Introduction	344
7.2	Comparison of Time Responses of Continuous-Data and Digital Control Systems	346
7.3	Correlation Between Time Response and Root Locations in the s-Plane and the z-Plane	353
7.4	Effect of the Pole-Zero Configurations in the z-Plane Upon the Maximum Overshoot and Peak Time of Transient Response of Sampled-Data Systems	356
7.5	Root Loci for Digital Control Systems	366
7.6	Steady-State Error Analysis of Digital Control Systems	373

<b>8</b>	<b>FREQUENCY-DOMAIN ANALYSIS</b>	<b>393</b>
3.1	Introduction	393
3.2	The Nyquist Plot	395
3.3	The Bode Diagram	405
3.4	Gain Margin and Phase Margin	407
3.5	The Gain-Phase Diagram and the Nichols Chart	409
3.6	Bandwidth Considerations	411
3.7	Computer Programs for Frequency-Domain Plots	414
<b>9</b>	<b>CONTROLLABILITY AND OBSERVABILITY</b>	<b>421</b>
9.1	Introduction	421
9.2	Definitions of Controllability	423
9.3	Theorems on Controllability (Time-Varying Systems)	424
9.4	Theorems on Controllability (Time-Invariant Systems)	426
9.5	Definitions of Observability	430
9.6	Theorem on Observability (Time-Varying Systems)	430
9.7	Theorems on Observability (Time-Invariant Systems)	431
9.8	Dual Relation Between Observability and Controllability	434
9.9	Relationship Between Controllability, Observability and Transfer Functions	436
9.10	Controllability and Observability Versus Sampling Period	438
9.11	Illustrative Examples	439
9.12	Invariant Theorem on Controllability	444
<b>10</b>	<b>DESIGN OF DIGITAL CONTROL SYSTEMS</b>	<b>452</b>
10.1	Introduction	452
10.2	Cascade Compensation by Continuous-Data Controllers	455
10.3	Feedback Compensation with Continuous-Data Controllers	468
10.4	The Digital Controller	472
10.5	Design of Digital Control Systems with Digital Controllers Through the Bilinear Transformation	489
10.6	Design in the $z$ -Plane Using the Root Locus Diagram	496
10.7	The Digital PID Controller	509
10.8	Design of Digital Control Systems with Deadbeat Response	514
10.9	Pole-Placement Design by State Feedback (Single-Input)	528

10.10 Pole-Placement Design by State Feedback (Multiple-Input)	537
10.11 Pole-Placement Design by Incomplete State Feedback or Output Feedback	540
10.12 Design of Digital Control Systems with State Feedback and Dynamic Output Feedback	549
10.13 Realization of State Feedback by Dynamic Controllers	555
<b>11 DESIGN BY MAXIMUM PRINCIPLE</b>	<b>579</b>
11.1 The Discrete Euler-Lagrange Equation	579
11.2 The Discrete Maximum (Minimum) Principle	585
11.3 Time-Optimal Control with Energy Constraint	591
<b>12 OPTIMAL LINEAR DIGITAL REGULATOR DESIGN</b>	<b>601</b>
12.1 Introduction	601
12.2 Linear Digital Regulator Design (Finite-Time Problem)	603
12.3 Linear Digital Regulator Design (Infinite-Time Problem)	609
12.4 Principle of Optimality and Dynamic Programming	613
12.5 Solution of the Discrete-Riccati Equation	617
12.6 Sampling Period Sensitivity	628
<b>13 DIGITAL STATE OBSERVER</b>	<b>646</b>
13.1 Introduction	646
13.2 Design of the Full-Order State Observer	648
13.3 Design of the Reduced-Order State Observer	660
<b>14 MICROPROCESSOR CONTROL</b>	<b>670</b>
14.1 Introduction	670
14.2 Basic Computer Architecture	671
14.3 Machine-Language Programming	680
14.4 Microprocessor Control of Control Systems	689
14.5 General Discussions on the Effects of Some Limitations of Microprocessor-Based Control Systems	694
14.6 Effects of Finite Wordlength on Controllability and Closed-Loop Pole Placement	694

14.7 Time Delays in Microprocessor-Based Control Systems 699

14.8 Effects of Quantization - Least-Upper Bound on

Quantization Error 705

APPENDIX A

722

INDEX

725

# INTRODUCTION

## 1.1 INTRODUCTION

In recent years significant progress has been made in discrete-data and digital control systems. These systems have gained popularity and importance in all industries due in part to the advances made in digital computers, and more recently in microcomputers, as well as the advantages found in working with digital signals.

Discrete-data and digital control systems differ from the continuous-data or analog systems in that the signals in one or more parts of these systems are in the form of either a pulse train or a numerical code. The terms, sampled-data systems, discrete-data systems, discrete-data, discrete-time systems, and digital systems have been used loosely in the control literature. Strictly, sampled-data refers to signals that are pulse-amplitude modulated, that is, trains of pulses with signal information carried by the amplitudes, whereas digital data usually refer to signals which are generated by digital computers or digital transducers and thus are in some kind of coded form. However, it will be shown later that a practical system such as an industrial process control usually is of such complexity that it contains analog and sampled as well as digital data. Therefore, in this text we shall use the term discrete-data systems in a broad sense to describe all systems in which some form of digital or sampled signals takes place.

Figure 1-1 shows the basic elements of a typical closed-loop control system with sampled data. The sampler simply represents a device or operation at the output of which the signal is of the form of a periodic or aperiodic pulse train, with no information transmitted between two consecutive pulses. Figure 1-2 illustrates a common way of how a sampler operates. A continuous input signal  $e(t)$  is sampled by a sampler, and the output of the sampler is a sequence of pulses. In the present case the sampler is assumed to have a uniform sampling rate. The magnitudes of the pulses at the sampling instants represent the values of the input signal  $e(t)$  at the corresponding instants. In general, the sampling schemes exist in many variations; some of these are the periodic, cyclic-rate, multirate, random, and pulsewidth modulated samplings. The most common variations found in practical systems are the single-rate and multirate samplings. These various types of sampling and their effects will be discussed in the subsequent chapters of this book.

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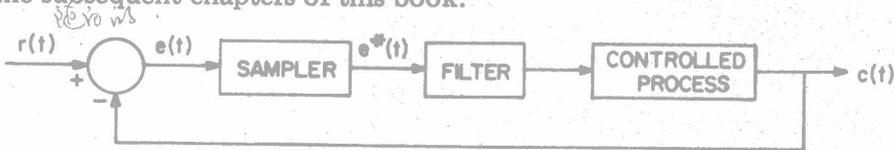


Fig. 1-1. Closed-loop sampled-data control system.

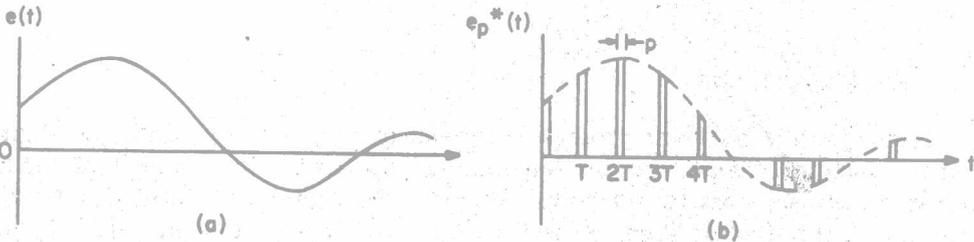


Fig. 1-2. (a) Continuous-data input to sampler.  
(b) Discrete-data output of sampler.

The filter located between the sampler and the controlled process is used for the purpose of smoothing, since most controlled processes are naturally designed and constructed to receive analog signals.

A digital control system is defined as one in which the signal at one or more points of the system is expressed in a numerical code for digital-computer or digital-transducer processing in the system. The block diagram of a typical digital control system is shown in Fig. 1-3. The appearance of digitally-coded signals, such as binary-coded, in certain parts of the system requires the use of digital-to-analog (D/A) and analog-to-digital (A/D) converters. Although there are basic differences between the structures and components of a sampled-data and a digital control system, we shall show that from an analytical standpoint

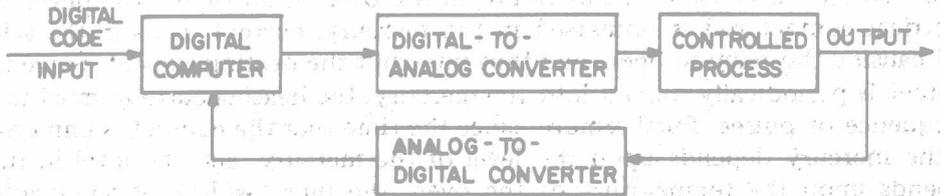


Fig. 1-3. A typical digital control system.

both types of systems are treated by the same analytical tools.

The use of sampled data in control systems can be traced back to at least seventy years ago. Some of these early applications of sampled data were for the purpose of improving the performance of the control system in one form or another. For instance, in the chopper-bar galvanometer described by Oldenbourg and Sartorius [1], (see Fig. 1-4) the sampling operation produces greater system sensitivity to a low-level input signal.

With reference to Fig. 1-4, a small signal is normally applied to the galvanometer coil. The chopper bar is lowered periodically, and the projected pointer of the galvanometer causes the load to be driven in proportion to the signal strength. The torque which is applied to the load shaft is thus determined by the chopper-bar drive rather than just by the torque developed in the galvanometer coil.

Another known example of an early application of the sampling concept in control systems is the constant-temperature oven devised by Gouy [2]. The system consists of the elements shown in Fig. 1-5. The purpose of this system

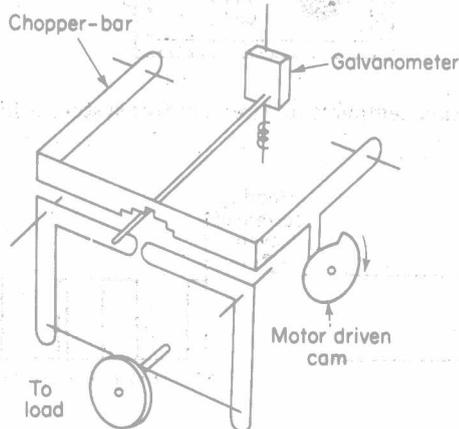


Fig. 1-4. Chopper-bar galvanometer.

is to maintain a constant temperature in the oven at all times. Whenever the electrical contact rod is immersed in the mercury, current flows in the relay coil causing the relay to open, and thus interrupt the heating current. Since the contact is periodically dipped into the mercury, the heating current consists of a sequence of pulses. Furthermore, since the time that the contact is immersed in the mercury depends upon the level of the mercury, and the level in turn depends upon the temperature of the oven, the pulse widths of the heating current are varied in proportion to the temperature of the oven, but the pulses all have equal amplitudes. A typical set of signals of the sampled-data temperature control system is shown in Fig. 1-6. In contrast to the conventional sampling scheme, such as the one described in Fig. 1-2, in which the amplitude of the signal is sampled and transmitted by the amplitude of a pulse, the sampled-data shown in Fig. 1-6 have a constant pulse amplitude, but with the signal information carried by the widths of the pulses. Therefore, we see that in practice there are many ways of sampling a signal, or perhaps more appropriately, of representing a signal in sampled form.

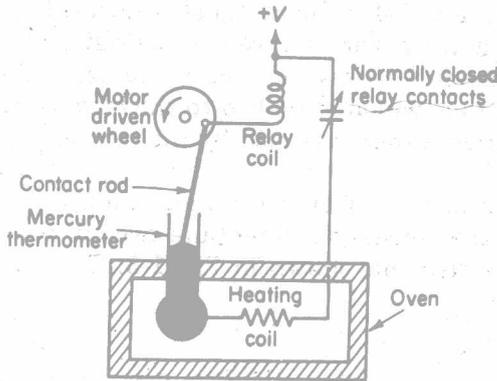


Fig. 1-5. A constant-temperature oven control system utilizing sampled-data.

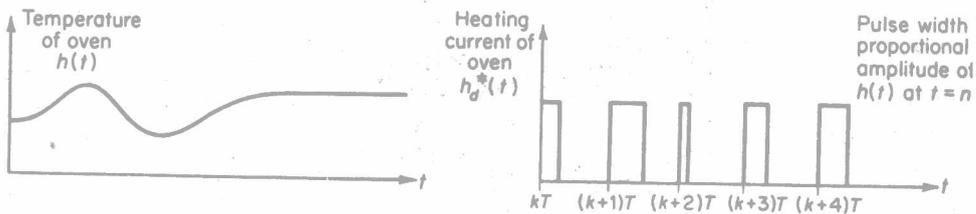


Fig. 1-6. Input and output of pulsewidth sampler.