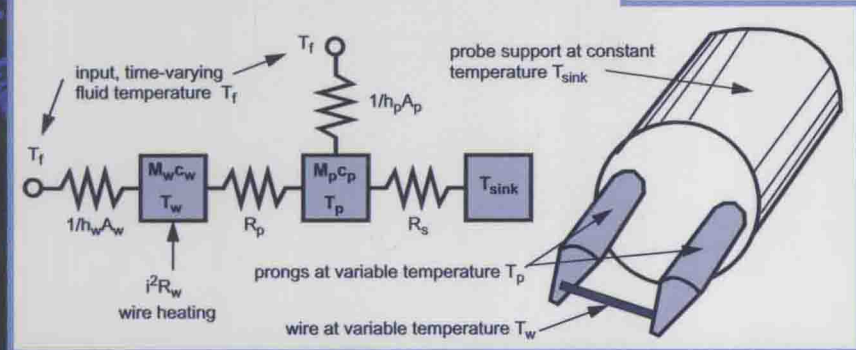
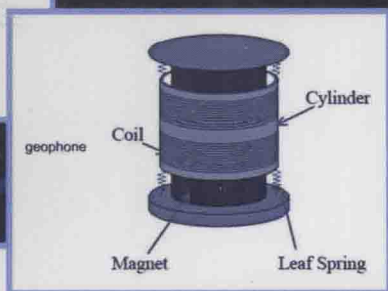
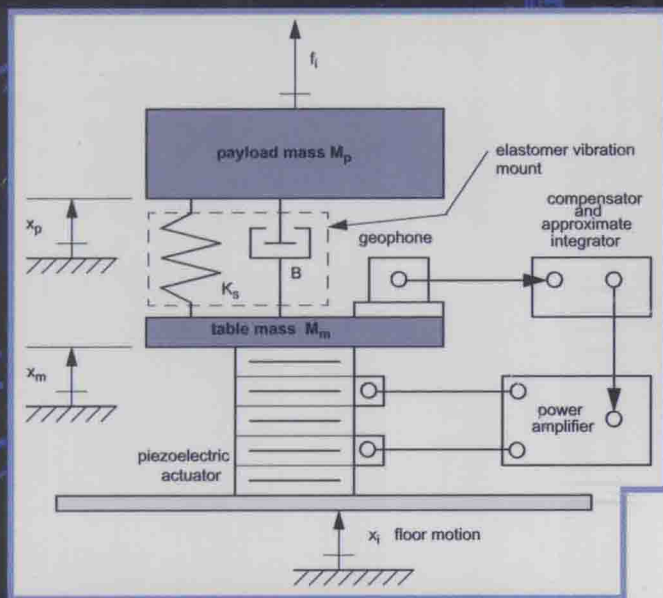


INSTRUMENTATION DESIGN STUDIES



Ernest O. Doebelin



CRC Press
Taylor & Francis Group

INSTRUMENTATION DESIGN STUDIES

Ernest O. Doebelin



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **Informa** business

MATLAB® and Simulink® are trademarks of the MathWorks, Inc. and are used with permission. The MathWorks does not warrant the accuracy of the text or exercises in this book. This book's use or discussion of MATLAB® and Simulink® software or related products does not constitute endorsement or sponsorship by the MathWorks of a particular pedagogical approach or particular use of the MATLAB® and Simulink® software.

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

© 2010 by Taylor and Francis Group, LLC
CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works

Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number: 978-1-4398-1948-7 (Hardback)

This book contains information obtained from authentic and highly regarded sources. Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access www.copyright.com (<http://www.copyright.com/>) or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

Library of Congress Cataloging-in-Publication Data

Doebelin, Ernest O.
Instrumentation design studies / Ernest Doebelin.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-4398-1948-7
1. Engineering instruments--Design and construction. 2. Systems engineering. 3.
Automatic control. I. Title.

TA165.D547 2010
620'.0042--dc22

2009028747

Visit the Taylor & Francis Web site at
<http://www.taylorandfrancis.com>

and the CRC Press Web site at
<http://www.crcpress.com>

INSTRUMENTATION DESIGN STUDIES

Preface

All my earlier books (*Dynamic Analysis and Feedback Control* [1962]; *Measurement Systems*, five editions [1966–2004]; *System Dynamics* [1972]; *System Modeling and Response* [1980]; *Control System Principles and Design* [1985]; *Engineering Experimentation* [1995]; and *System Dynamics: Modeling, Analysis, Simulation, Design* [1998]) were designed as engineering textbooks to be used as aids in teaching undergraduate and graduate courses in the areas of system dynamics, measurement, and control. They were thus organized to progress in a carefully designed sequence of chapters, which led the student from simple basic concepts toward progressively more comprehensive and practical views of the field under study. As is usual in textbooks, each chapter included homework problems designed to stimulate students' personal understanding of important concepts. While these books were originally intended for teaching purposes in engineering schools, their judicious blending of useful theory with practical hardware and design considerations made them appealing also to engineering practitioners who wanted to update their education in specific areas.

This book is still devoted to the same general areas (system dynamics, measurement, and control) but departs from the textbook format to address the needs of practicing engineers working in those fields, which are sometimes collected under the heading "instrumentation." As is common with this type of book, homework problems are not included. While all the chapters certainly have a common interest in the overall field, each is largely self-contained in addressing an important subarea of the subject. As such, they are readily accessible to readers with a specific interest in improving their expertise in the chosen topic. While the book is not designed for a specific academic course, it could be profitably used as additional enrichment reading for any number of specific courses, or possibly for a single seminar-type experience.

Central to all the chapter treatments is the close integration and widespread use of appropriate software, such as MATLAB®/Simulink® (dynamic system simulation), Minitab® (statistical tools), and Mathcad (general engineering computation). To facilitate readers' comprehension of software applications, detailed appendices in the form of sharply focused and user-friendly *mini-manuals* are provided for MATLAB/Simulink and Minitab. (Most Mathcad applications are sufficiently self-explanatory and user-friendly that additional explanation is not warranted.) While engineering software packages provide extensive printed manuals and/or online help, in my experience these aids are too voluminous and unfocused to allow efficient use for specific application areas, such as instrumentation. These appendix manuals are

specifically addressed to the text's application areas, and thus can be used by the reader in an efficient and time-saving manner.

This new book is largely based on a series of homework projects that I developed over many years for an advanced measurement course/lab populated by a mix of engineering seniors and graduate students. This experience was valuable in showing me the best ways to present the material, which was continuously revised over the years. The homework project manual included extensive notes that led the student through the particular topic and required certain calculations and explanations at each of the steps in the development. In adapting this manual to the needs of this book, I replaced the homework sections with a complete presentation and explanation of the solutions required of the students. I also adapted the format to meet the needs of the new audience, and augmented the technical material with any new developments that I was familiar with. I hope this book will be a useful and interesting learning tool for engineers in the instrumentation field.

Ernest O. Doebelin

MATLAB® is a registered trademark of The MathWorks, Inc. For product information, please contact:

The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098 USA
Tel: 508 647 7000
Fax: 508-647-7001
E-mail: info@mathworks.com
Web: www.mathworks.com

Author

Ernest O. Doebelin was born in Germany in 1930, but left for the United States in 1933. His elementary and secondary schooling were in the public schools of Cleveland, Ohio, and North Ridgeville, Ohio. He received his BSc in mechanical engineering (1952) from the Case Institute of Technology, Cleveland, Ohio, and his MSc and PhD (in 1954 and 1958, respectively) in mechanical engineering from the Ohio State University, Columbus. While working on his PhD, he was a full-time instructor, and under the guidance of the department chairman, S.M. Marco, taught many of the undergraduate mechanical engineering courses. This experience contributed to his lifelong interest in the entire mechanical engineering curriculum, and gave his subsequent teaching and writing in more restricted areas a “generalist” flavor.

As an assistant professor, he was assigned to develop the curricular area of *instrumentation and control*, which in those early years consisted of only a single course. Over the years, he developed and taught eight courses in the areas of *system dynamics, measurement, and control*, ranging from the sophomore to the PhD level. Seven of these courses had laboratories which he designed, developed, and taught. Textbooks for all these courses were written along with comprehensive lab manuals and software mini-manuals. In a career that was focused on teaching, Prof. Doebelin was fortunate to win many awards. These included several departmental, college, and alumni recognitions, and the university-wide distinguished teaching award (five selectees yearly from the entire university faculty). The American Society for Engineering Education (ASEE) also presented him with the Excellence in Laboratory Instruction Award. After retirement in 1990, he continued to teach in lectures and labs, but for one quarter a year. He also worked on a volunteer basis at Otterbein College, Westerville, Ohio, a local liberal arts school, developing and teaching a course on *understanding technology*, as an effort to address the nationwide problem of technology illiteracy within the general population. As a further hobby after retirement, he has become a politics/economics junkie, focusing particularly on alternative views of globalization.

Contents

Preface.....	xi
Author.....	xiii
1. Introduction to Statistical Design of Experiments: Experimental Modeling of a Cooling System for Electronic Equipment	1
1.1 Introduction	1
1.2 Basic Concepts	2
1.3 Mathematical Formulation	3
1.4 Full-Factorial and Fractional-Factorial Experiments	4
1.5 Run-Sequence Randomization.....	5
1.6 Validation Experiments.....	6
1.7 Example Experiment: Modeling an Electronics Cooling Process	6
1.8 Using Minitab® to Design the Experiment and Then Analyze the Results	8
1.9 Multiple Regression: A General Tool for Analyzing Experiment Data and Formulating Models	15
1.10 Stepwise Regression	17
2. Vibration Isolation for Sensitive Instruments and Machines	31
2.1 Introduction	31
2.2 Passive Spring/Mass Isolators	32
2.3 Passive Air-Spring Systems	41
2.4 Active Air-Spring Systems.....	60
2.5 Low-Frequency Isolation Using Negative-Spring-Constant Devices.....	69
2.6 Active Electromechanical Vibration Isolation	80
2.7 Tuned Vibration Absorbers and Input-Shaping Methods.....	99
3. Design of a Vibrating-Cylinder, High-Accuracy Pressure Transducer	111
3.1 Introduction	111
3.2 Basic Concept.....	112
3.3 Cylinder Natural Frequency Calculations	118
3.4 Use of an Unstable Feedback System to Maintain Continuous Oscillation	120
3.5 Nyquist and Root-Locus Studies of System Operation	129

3.6	Simulation of the Complete System	131
3.7	Ultraprecision Calibration/Measurement Using a 15-Term Calibration Equation, Built-In Temperature Compensation, and Microprocessor Data Reduction.....	136
4.	A Fast ("Cold-Wire") Resistance Thermometer for Temperature Measurements in Fluids.....	147
4.1	Introduction	147
4.2	Circuitry and Wire Details	148
4.3	Estimating the Self-Heating Error	152
4.4	Estimating the Sensitivity to Desired and Spurious Inputs	158
4.5	Dynamic Response to Fluid Temperature Fluctuations.....	159
4.6	Use of Current Inputs for Dynamic Calibration.....	163
4.7	Electronic Considerations	165
4.8	Effect of Conduction Heat Transfer at the Wire Ends	167
5.	Piezoelectric Actuation for Nanometer Motion Control	177
5.1	Introduction	177
5.2	Mechanical Considerations	180
5.3	Actuators, Sensors, and Mounting Considerations	186
5.4	Control System Design.....	198
6.	Preliminary Design of a Viscosimeter.....	229
6.1	Introduction	229
6.2	Definition of Viscosity	229
6.3	Rotational Viscosimeters.....	231
6.4	Measurement of Torque	233
6.5	Dynamic Measurements.....	236
6.6	Velocity Servos to Drive the Outer Cylinder	242
6.7	Calibration.....	253
6.8	Corrections to the Simplified Theory.....	258
6.9	Non-Newtonian Fluids	260
6.10	The Concept of the Representative Radius	262
6.11	The Concept of Effective Length	263
6.12	Cylinder Design according to German Standards.....	265
6.13	Designing a Set of Cylinders.....	268
6.14	Temperature Effect on Viscosity	269
6.15	Temperature Control Methods.....	273
6.16	Uncertainty Analysis.....	288
6.17	Encoder Angular Position and Speed Measurement.....	298
6.18	Practical Significance of the Shear Rate	300
6.19	Fitting a Power-Law Model for a Non-Newtonian Fluid	304
	Bibliography.....	310

7. Infrasonic and Ultrasonic Microphones	311
7.1 Introduction.....	311
7.2 Infrasonic Microphones.....	312
7.3 Diaphragm Compliance Calculation.....	314
7.4 Microphone Transfer Function	316
7.5 System Simulation.....	317
7.6 Adjusting Diaphragm Compliance to Include Air-Spring Effect	320
7.7 Calibration.....	328
7.8 Wind Noise Filtering with Pipes and Spatial Arrays	331
7.9 Ultrasonic Microphones.....	342
7.10 Ultrasonic Acoustics Pertinent to Leak Detection	351
8. Some Basic Statistical Tools for Experiment Planning	355
8.1 Introduction.....	355
8.2 Checking Data for Conformance to Some Theoretical Distribution.....	357
8.3 Confidence Intervals for the Average (Mean) Value	366
8.4 Comparing Two Mean Values: Overlap Plots and Confidence Intervals.....	371
8.5 Confidence Intervals for the Standard Deviation	375
8.6 Specifying the Accuracy Needed in Individual Measurements to Achieve a Desired Accuracy in a Result Computed from Those Measurements	379
9. Multiaxial Force/Torque Measurement: Thrust Stands for Jet Engines and Rocket Engines	387
9.1 Introduction.....	387
9.2 Dynamics of Thrust Stand Force/Torque Measurement	389
9.3 Characteristics of Elastic Elements in Three Dimensions	392
9.4 Dynamic Response Equations of the Thrust Stand	397
9.5 Matrix Methods for Finding Natural Frequencies and Mode Shapes	401
9.6 Simulink® Simulation for Getting the Time Response to Initial Conditions and/or Driving Forces/Moments	408
9.7 Frequency Response of the Thrust Stand.....	416
9.8 Matrix Frequency Response Methods	421
9.9 Simulation of the Asymmetric System: Use of Simulink Subsystem Module	427
9.10 Static Calibration of Thrust Stands.....	432
9.11 Damping of Thrust Stands	442
9.12 Flexure Design.....	455

10. Shock Calibrator for Accelerometers.....	467
10.1 Introduction.....	467
10.2 Description of the Calibrator.....	468
10.3 Review of Basic Impact Calculations	471
10.4 Simulation of the Coefficient of Restitution Drop-Test Experiment.....	474
10.5 Some Analytical Solutions.....	486
10.6 Simulation of the Pneumatic Shock Calibrator Apparatus.....	491
10.7 Concluding Remarks.....	505
11. Shock Testing and the Shock Response Spectrum.....	507
11.1 Introduction.....	507
11.2 Analysis and Simulation of Response to Shock Inputs.....	509
11.3 Shock Response Spectrum.....	516
11.4 Practical Shock Testing and Analysis	520
11.5 Pyrotechnic Shock.....	533
11.6 Vibration Shakers as Shock Pulse Sources	539
11.7 Design of a Shock Isolator.....	545
11.8 Relation of SRS to Actual Mechanical Damage.....	552
11.9 Measurement System and Data Acquisition/Processing Considerations.....	558
Appendix A: Basic MATLAB®/Simulink® Techniques for Dynamic Systems.....	561
A.1 Basic Simulink Techniques.....	561
A.1.1 A Little History	561
A.1.2 Basic Concepts.....	562
A.1.3 Graphing Methods.....	564
A.1.4 The Simulink Functional Blocks.....	566
A.1.5 Running a Simulation	567
A.1.6 Configuring the <i>To Workspace</i> Blocks.....	570
A.1.7 “Wiring” the Block Diagram (Connecting and Manipulating the Blocks).....	571
A.1.8 Setting Numerical Values of System Parameters: MATLAB Scripts/Files.....	572
A.1.9 Making Tables of Numerical Values of Variables	574
A.1.10 “Detailed” versus “Compact” Simulation Formats (Basic Equations versus Transfer Functions)	575
A.1.11 Simulating Sets of Simultaneous Equations	575
A.1.12 Frequency-Selective Filters.....	583
A.1.13 Working with Random Signals.....	591
A.1.14 Generating Random Signals, with Prescribed PSD, for Vibration Testing.....	604
A.1.15 Example Applications of Some Functional Blocks.....	608

- A.2 Basic Frequency-Domain Techniques in MATLAB 622
 - A.2.1 Introduction 622
 - A.2.2 Choosing the Frequency Values to Be Used in a Calculation 623
 - A.2.3 Units for Amplitude Ratios and Phase Angles..... 623
 - A.2.4 Computing and Graphing the Frequency-Response Curves..... 624
 - A.2.5 Fitting Analytical Transfer Functions to Experimental Frequency-Response Data 628
 - A.2.6 Using *sys* and *tf* Statements to Form Transfer Functions..... 631
 - A.2.7 Matrix Frequency Response Calculation 633
 - A.2.8 Finding the Frequency Spectra of Time-Varying Signals 635
 - A.2.8.1 Frequency Spectrum of Periodic Signals 636
 - A.2.8.2 Frequency Spectrum of Transient Signals..... 639
 - A.2.8.3 Frequency Spectrum of Random Signals 645
 - A.2.9 Overlap Processing 649
 - A.2.10 Experimental Modeling of Systems Using Frequency-Response Testing..... 651
- Appendix B: Basic Statistical Calculations Using Minitab 661**
 - B.1 Introduction 661
 - B.2 Getting Data into Minitab Using the Worksheet 664
 - B.3 Manipulating Data in the Worksheet..... 666
 - B.4 Graphing Tools for General Graphing 668
 - B.5 Checking Physical Data for Conformance to Some Theoretical Distribution..... 671
 - B.6 Computing Mean and Standard Deviation (and Their Confidence Intervals) for a Sample of Data..... 673
 - B.7 Transforming a Non-Gaussian Distribution to a Gaussian One 674
 - B.8 Comparing Means and Standard Deviations 679
 - B.9 Multiple Regression 682
 - B.9.1 Curve Fitting..... 683
 - B.9.2 Model Building 689
 - B.9.2.1 Best Subsets Regression..... 693
 - B.9.2.2 Validation Experiments..... 696
 - B.9.2.3 Analyze Factorial Design 697
 - B.9.2.4 Nonnumerical Factors 697
 - B.9.2.5 3-Level Experiments (See Also Chapter 1) 698
- Index 699**

1

Introduction to Statistical Design of Experiments: Experimental Modeling of a Cooling System for Electronic Equipment

1.1 Introduction

The *statistical design of experiments* (DOE) is the subject of entire large books and academic courses. Its various techniques are widely practiced in industry and have achieved many successful practical applications. Many engineers have little or no familiarity with this important approach and the purpose of this chapter is essentially to raise your consciousness of this topic. The development of true expertise must of course depend on further study and practical experience. Hopefully this introduction will at least make you aware of the general approach so that you will consider it when facing new experimental projects as they arise. Widely available statistical software (such as the Minitab whose use is explained in Appendix B) makes the application of the methods much easier and quicker than was the case in earlier years. Because I was convinced of the importance of making these methods accessible to all undergraduate mechanical engineers, I included two chapters on these topics in a textbook published in 1995.* Chapter 2 of that book introduces general basic concepts to readers with no background in statistics while chapter 4 develops the methods of DOE. My idea was that the existing books and courses required so much time and effort that most engineers and students would not make this investment, so I tried in these two chapters to simplify and streamline the material by extracting what I thought were the essential ideas and methods. If in the future you want to go beyond what is presented in this short chapter, you might start with these two chapters since they will “get you going” in the shortest time. Of course, if you find that you use these methods regularly, you might go to the more detailed texts (or short courses offered by many companies and software suppliers) for deeper background. Chapters 2 and 4 of my 1995 textbook provide links to such resources.

* E.O. Doebelin, *Engineering Experimentation: Planning, Execution, Reporting*, McGraw-Hill, New York, 1995.

1.2 Basic Concepts

We now present, in a severely condensed (but hopefully still useful) form, the basic concepts of DOE. The problems dealt with can be described as follows. We have, say, some manufacturing process that produces a product or material that has one or more attributes associated with quality and/or cost. This quality parameter depends on several process variables that we are able to set within a certain range of values. The process is sufficiently complex that physical/mathematical modeling to reveal the relation of the quality parameter to the set of process variables has proven not possible or insufficiently accurate. We therefore propose to *run an experiment* in which we “exercise” the process by setting the process variables at several combinations of values and then measure the value of the quality parameter that results. We then analyze this data to develop a *mathematical model*, which predicts the effects of the process variables on the quality parameter. Many times such modeling allows us to find which variables are the most significant, and also the *optimum combination* of process variable settings; that is, one that maximizes quality or minimizes cost.

While the study of manufacturing processes is perhaps the application of most economic significance, DOE methods are directly applicable to other situations. For example, the NASA Johnson Space Center (Houston, Texas) ran experiments on the Space Shuttle’s life-support system, which removes water vapor and carbon dioxide from the cabin atmosphere. The rate of removal of carbon dioxide was the quality parameter and the process variables were: temperature of a bed of absorbent material, partial pressure of water vapor in the inlet stream, partial pressure of carbon dioxide in the input stream, and total gas-flow rate. Physical/mathematical modeling of this system had not provided a good understanding of process behavior or reliable predictions of the effects of the process variables on CO₂ removal rate, so an experimental approach was undertaken. (More details of this application, including a complete set of real-world data and its analysis to provide a useful model are given at the end of this chapter.)

Finally, DOE methods are used for *computer experiments*, where the data are generated, not by a physical experiment but rather by a computer simulation.* For example, a finite element analysis (FEA) study of a machine part might be interested in the effects of various dimensions and material properties on the stress, deflection, or natural frequency. One can, of course, run such a simulation over and over with various combinations of input parameters in an attempt to find parameter values which minimize stress or deflection, or maximize natural frequency. Since each such run may be quite expensive, and the search for the optimum lacks much guidance as to “which way

* A. Rizzo, Quality engineering with FEA and DOE, *Mechanical Engineering*, May 1994, pp. 76–78.

to go," this approach requires many runs and thus may be quite inefficient. DOE methods allow us to choose a relatively small number of parameter combinations to run, formulate a model relating our quality parameter to dimensions and material properties, and then use this model to predict the optimum combination.

1.3 Mathematical Formulation

With the above background, we can see that all these applications can be thought of mathematically as a problem of finding a functional relation between a set of process variables and some quality parameter. In DOE parlance, the process variables are called *factors* and the quality parameter is called the *response* (y). Mathematically

$$y = b_0 + b_1 f_1(x_1) + b_2 f_2(x_2) + b_3 f_3(x_3) + \dots \quad (1.1)$$

Here the f_i 's are functions which can involve any of the process variables in any way. If we call the process variables x_a, x_b, x_c, \dots then, for example, f_1 might be $\sqrt{x_a(x_b/x_c)}$. The standard methods of DOE require that the functional relation in Equation 1.1 be linear in the coefficients b , but the $f(x)$ functions can take *any* form. While the restrictions put on Equation 1.1 prevent the use of certain kinds of functions, this form is sufficiently versatile to meet most, but not all practical needs. The advantage realized by the restrictions is that the solution for the unknown b values is readily accomplished by routine computerized methods of linear algebra. An experiment consists of choosing the functional forms of the $f(x)$'s, running the experiment to get a numerical value for y (the dependent variable) that results from each set of x 's, and then analyzing these data to find the numerical values of the b 's. We have to use at least as many sets of x 's as there are b 's in our model if we want to get a solution (n linear equations in n unknowns). Usually we use *more* sets of x 's, which makes the equation set *overdetermined* and requires use of least-squares solution methods, but these fortunately are also part of standard linear algebra. Each set of x 's and the associated response y constitutes one run of our experiment.

While Equation 1.1 allows a very large variety of functions to be used, many useful applications employ a much more restricted class of functions. A major class of such applications is the so-called *screening experiment*. Here, we have a situation where we have identified, by using our familiarity with the physical process, a number (sometimes as large as 10 or 15) of process variables (*factors*) which might influence the quality parameter of interest. We want to run a frugal experiment that will narrow this rather long list down to a few factors that really matter, which we will then study in more detail. Such experiments

often use only two values of each process variable, a high value and a low value. Since we generally know the allowable ranges of the process variables, we can choose these high and low values numerically for each process variable. (An approach to *multivariable* experimentation much used in science and engineering is to hold all variables except one at *constant* values and then change this one variable over some range, thus isolating the effect of this variable. Doing this in turn for each of the variables, we hope to discover useful relations. While such an approach is common and can lead to useful results, the whole premise of DOE is that a more efficient method lets all the variables change *simultaneously*.) Thus the next step in the DOE procedure is to define the *combinations* of variable settings that will be used; each such combination is called a *run* of the experiment. For example, a run of a four-factor screening experiment might be to set all four factors to their individual high values. Another run might be to set factors 1 and 2 at their high values and factors 3 and 4 at their low values. While one might use “common sense” to define the set of runs, more systematic and efficient ways are available.

1.4 Full-Factorial and Fractional-Factorial Experiments

If there are k factors and each is to be restricted to two values, it becomes clear that to explore all possible combinations will require an experiment of 2^k runs. Such an experiment is called a *full-factorial* type. When k gets large, a full-factorial experiment can be prohibitively expensive in time and money, so we sometimes use *fractional-factorial* experiments. These use a carefully chosen subset of the runs of the full factorial, reducing the amount of information we can glean, but also cutting the costs. From Equation 1.1, however, it is clear that to find, say, four b values, we must have at least four runs (four equations in four unknowns). Fractional-factorial experiments usually define their runs using an orthogonality principle. Our abbreviated presentation will not attempt to explain this concept, and fortunately, standard statistics software (such as Minitab) provides the desired run definitions. The most common screening experiment attempts to find only the so-called *main effects* of the factors. Then Equation 1.1 takes the simple form:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \cdots \quad (1.2)$$

where the x 's are now the factors (independent process variables) themselves. That is, we seek only the linear effects of the individual factors. This simple model has some theoretical foundation in that any smooth nonlinear function can be approximated (for small changes away from some chosen operating point) by linear terms. (For $y = f(x)$, the tangent line to the curve and for $z = f(x, y)$, the tangent plane to the surface give a geometrical interpretation.)

Sometimes, the model will benefit from the so-called *interaction terms* such as $b_1x_1x_2$ (two-factor interaction) or $b_1x_1x_2x_3$ (three-factor interaction), with interactions higher than two-factor being very rarely used. If Equation 1.2 were to be augmented with higher powers of factors, such as bx_1^2 , we would find that the analysis software would fail; to deal with such terms we would need a screening experiment which uses *three* settings (high, medium, low) for each factor, which expands the scope and cost of the experiment, but is sometimes necessary. The intuitive reason for this behavior is that *two points can only determine a straight line; it takes three to allow curvature*.

An important consideration in choosing between full-factorial and fractional-factorial experiment designs is the issue of *confounding*. In a full-factorial experiment we are able to distinguish both main effects and interactions. This capability is lost, to some extent, in fractional-factorial experiments; the main effects and some interactions are said to be *confounded*. The *degree* of confounding is given by the *resolution level* of the design; common designs being designated as Resolution III, Resolution IV, or Resolution V. Resolution III designs have the smallest number of runs, but can only isolate the *main effects* of the factors; interaction terms cannot be reliably identified. Resolution IV designs require more runs, but can find main effects and two-way interactions. Higher order (three-way, four-way, etc.) interactions are confounded and thus not identifiable. Resolution V designs can find main effects, two-way interactions and three-way interactions. Since three-way interactions are not common, most fractional-factorial designs use either Resolution 3 or 4. See the appendix material on Minitab for further details on this topic.

1.5 Run-Sequence Randomization

Another consideration is that of *run-sequence randomization*. If an experiment has, say, 8 runs, does it matter what *sequence* we use in actually performing these runs? There are a number of possible reasons for randomizing the sequence of the runs rather than blindly using a nominal sequence given by common sense, some book, or software. When we originally list the factors which we believe effect the process response variable, there are always some *other* factors that we consciously or unconsciously leave out of our list. Often these are subtle effects of the human operator who carries out the experiment. If operators are somewhat inexperienced, their task performance may improve, due to learning as they go from the first runs to the last; this can bias the results. On the other hand, operators may become fatigued as they work through the sequence causing poorer performance later in the sequence. If an apparatus is operated at several different power levels, as we change from one run to the next, we approach the new steady-state condition through some sort of transient. These transients usually approach the new