Modern Physics

An Introductory Text

Jeremy I. Pfeffer
Shlomo Nir
Hebrew University of Jerusalem, Israel

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Preface

This book originated from the need for a suitable student text for the course *An Introduction to Modern Physics* first introduced by the authors at the Faculty of Agricultural, Food and Environmental Quality Sciences of the Hebrew University of Jerusalem. The primary goal of this course is to produce graduates who, whatever their field of specialisation, are 'modern physics literate'. It is open to all students who have successfully completed their first-year physics and mathematics studies.

The course sets out to recount, in terms amenable to non-physics majors, the development of the three seminal ideas—Special and General Relativity, Quantum Theory and the Nuclear Atom—out of which modern physics grew in the first half of the 20th century. These topics constitute the principle subject-matter of this book.

However, in addition to a final examination on these subjects, the students participating in the course are also required to submit a term-paper on any topic of their choice, providing it falls within the scope of modern physics or involves one or other of the industrial, technical or research applications derived from it. Accordingly, the scope of the book was widened beyond the narrow limits of the course material and chapters or sections devoted to such topics were added. Among those given a more detailed treatment are:

* Magnetism as a Relativistic Effect;
* The Interaction of Radiation with Matter—Spectroscopy;
* Fluorescence in Biological Systems and Membrane Research;
* Nuclear Structure and Elementary Particles—the Standard Model;
* The Design of Nuclear Weapons: fission and fusion weapons;
* Nuclear Reactors: the events at Chernobyl are described in detail;
* The Design and Use of Lasers;
* The Mössbauer Effect;
* Nuclear Magnetic Resonance and Magnetic Resonance Imaging;
* The Conduction of Electricity in Solids and Semiconductor Devices;

In each case, the underlying theory is presented together with a general description of the practical aspects of the application. For the sake of completeness we have also added:

* Quantum Electrodynamics—QED;
* Invariance, Symmetry and Conservation Laws.

In general, the presentation of the material emphasises the physical aspects of the phenomena. Problem solving is not a major or primary objective. Thus, the text presumes upon the spirit of the physical method recommended by J.J.Thomson:*

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* Sir Joseph J. Thomson 1856-1940, English physicist who was awarded the Nobel prize in 1906 for his research on the conduction of electricity through gases at low pressures.

The physical method has all the advantages in vividness which arise from the use of concrete quantities instead of abstract symbols ... we shall be acting in accordance with Bacon's dictum that the best results are obtained when a research begins with Physics and ends with Mathematics ...
The use of a physical theory will help to correct the tendency - which I think all will admit is by no means uncommon - to look on analytical processes as the modern equivalent of the Philosopher's Machine in the Grand Academy of Lagado, and to regard as the normal process of investigation in this subject the manipulation of a large number of symbols in the hope that every now and then some valuable result may happen to drop out.†

The impression that this is what doing physics is all about lies at the heart of the antipathy exhibited by so many students towards the subject. This text seeks to avoid this.

Notwithstanding, this is not a popular science book; it does not avoid the 'hard bits'. Studying physics requires a mental effort and there is no reason to hide this fact. Mathematics is incorporated into the main body of the text, but only to the extent required for descriptive or illustrative purposes. Most of the mathematical proofs have been separated from the main body of the text so as not to interrupt its flow; they appear in the worked examples and appendices and can be skipped at a first-reading. Questions, exercises and problems for student assignments will be found at the end of each part of the book; answers to these are to be found at the end of the book.

The techniques by which trigonometric functions, phasors (rotating vectors) and complex numbers are used in the mathematical description of wave motion are demonstrated in a supplementary section. A comprehensive index is also included.

In addition to its suitability as a student text for courses similar to that for which it was originally written, we would also recommend this book as a first reader and source text for students majoring in the physical sciences and engineering.

The authors acknowledge their deep indebtedness to the many excellent standard reference works to which they had recourse during the writing of this book. They also wish to thank Dr. German Kälberman who read and commented on parts of the text and Daniel Pfeffer who prepared the computer simulations and provided the technical advice without which this camera-ready manuscript could never have been completed.

Jeremy I. Pfeffer
Shlomo Nir

Rehovot 2000

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† *Notes on Recent Researches in Electricity and Magnetism*, J.J.Thomson, Oxford at the Clarendon Press (1893).

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Part One The Birth of a New Physics

Classical physics has its origins in the 17th century, with Galileo's[1] experiments on falling bodies, Kepler's[2] calculations of the planetary orbits and Newton's[3] postulates and mathematical laws of motion. During the following two centuries, the Newtonian system of mechanics, together with its theory of gravitation, dominated scientific thinking; its achievements were unprecedented. Perhaps its most impressive feat was predicting the existence of the planet Neptune. Calculations based on Newtonian mechanics indicated that the slight perturbations that had been observed in the orbit of the planet Uranus could be accounted for by the presence of an additional planet in the solar system, one previously unknown. In 1846, this new planet, Neptune, was found at the exact position in space indicated by the calculations.[4]

By the middle of the 19th century, the ancient mystery of the nature of heat had been solved by the kinetic theory of matter and the science of thermodynamics. The latter had also provided an understanding for the 'arrow of time' inherent in the workings of Nature.

Classical physics reached its zenith in the second half of the 19th century with the publication of Maxwell's[5] theory of electromagnetism and the discovery of the electromagnetic waves whose existence it had predicted. This theory summarised and unified everything that was known at the time about electrical and magnetic phenomena and provided the first comprehensive conceptual basis for the science of optics.

However, towards the end of the 19th century, it became clear that there were important physical phenomena for which classical physics had no satisfactory explanation. The electron, X-rays and radioactivity, all of which were discovered within a few years of each other in the last decade of the century, were beyond the competence of classical physics to explain. Moreover, there were even instances where the hypotheses and laws of classical physics were found to be totally incompatible with the results of experiments in more conventional fields, such as studies of the speed of light relative to different observers and the emission and absorption of heat radiation. It became apparent that despite its great achievements, some of the most fundamental

----

1 Galileo Galilei 1564-1642, the Italian physicist and astronomer who first asserted that "the book of nature is written in the language of mathematics".

2 Johannes Kepler 1571-1630, German astronomer.

3 Sir Isaac Newton 1642-1727, English mathematician, astronomer and physicist.

4 In a similar technique that is currently being used in the search for planetary systems other than the solar system, the 'wobbling' observed in the motion of certain stars is attributed to the presence of large but as yet unseen orbiting bodies (planets).

5 James Clerk Maxwell 1831-1879, Scottish mathematician and physicist.

principles underlying classical physics were incorrect. Clearly, fresh ideas and theories were needed.

At the beginning of the 20th century, three new hypotheses were put forward that changed the face of physics. They were:

1. The theory of relativity;
2. Quantum theory;
3. The nuclear model of the atom.

This triad laid the foundations of modern physics. They and the effects derived from them are the subject of this book.

The new theories of modern physics not only resolved the problems left unanswered by classical physics but extended the reach of the physical sciences into previously unknown fields. In general, the familiar and well established laws of classical physics remain valid for dealing with phenomena that occur on a 'normal' scale, in the human sense of the term. However, when dealing with phenomena occurring on the cosmic scale on the one hand, and on the atomic scale on the other, only the more comprehensive laws of modern physics can be employed.

Our study of 20th century physics opens with the discovery of the electron, the first elementary particle to be identified. This is followed by a short review of the classical theory of electromagnetic waves. These two topics provide the basis for the comprehensive examination of the theory of special relativity that follows. Part One of this book concludes with a brief survey of the theory of general relativity.

Chapter 1.1

The Electron

When first investigated in the 18th and 19th centuries, electrical and magnetic phenomena were generally construed in terms of *æthereal fluids*, as were those associated with heat and light. These fluids were thought to comprise minute mutually repelling particles. Thus, heat was either thought to be vibrations in the fluid *caloric* or an accumulation of this fluid in the interstices of materials. Light was either a flux of particles emitted at high speed from luminous bodies or the vibrations of a ubiquitous fluid æther. Electric fluids—*electricity*—flowed readily through metals and other conductors but did not penetrate insulators such as paper and glass. Opinion was divided as to whether there was just one electric fluid or two—a *positive* fluid and a *negative* fluid.

The possibility that electricity might not be a continuous fluid was first raised in the middle of the 19th century following Faraday's[6] quantitative researches on electrolysis. These showed the existence of a systematic relationship between the amount of electricity passed through an electrolytic cell and the quantity of material that undergoes chemical reaction (electrolysis) in the cell. Thus, the passage of a certain amount of electricity—96,500 coulombs in modern terms—always liberates a gram-equivalent of substance from the electrolyte, whether it is the metal released at the negative cathode or the non-metal at the positive anode. Putting aside his misgivings about atomism, Faraday recognised that this suggested electricity might be *atomic* in nature and that a natural indivisible unit of electricity exists. In 1891 Stoney[7] suggested that this natural unit of electricity be called an *electron*.

On this view, every ion carries an integer multiple of this natural unit. For example, a silver ion, Ag^+, carries a single natural unit of positive charge; a typical copper ion, Cu^{++}, carries two such units. Given that a gram-equivalent of monovalent ions comprises a mole, the magnitude of the natural unit of electricity, e, can be calculated by dividing the 96,500 coulombs by Avogadro's number, $6.02 \cdot 10^{23}$:

$$e = \frac{96{,}500}{6.02 \cdot 10^{23}} = 1.6 \cdot 10^{-19}\,\mathrm{C} \tag{1.1}$$

The term *electron* is now used to designate the elementary particle that carries the natural unit of negative charge which was first identified towards the end of the 19th century in experiments on the conduction of electricity through gases at very low pressures. At atmospheric pressure, gases do not usually conduct electricity. However, at reduced pressures of 0.5mmHg to 10mmHg and with applied potentials of several

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6 Michael Faraday 1791-1867, English chemist and physicist.

7 George Stoney 1826-1911, Irish physicist.

thousand volts, they can be made to pass a current. These greatly reduced gas pressures were first achieved at the end of the 19th century following the advances made at that time in vacuum pump technology. The gases were contained in narrow glass tubes, called *discharge tubes*, into which suitable electrodes had been inserted. The passage of the current is accompanied by the appearance of striking colours in the tubes (Fig 1.1).

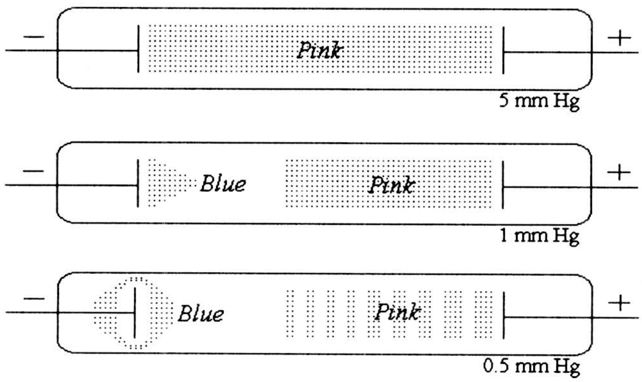

Fig 1.1 The conduction of electricity through air at low pressures in a discharge tube. The colours result from the excitation and ionisation of the atoms of the gas in the discharge tube. The bands and the coloured and dark regions arise from the variations in the electric field strength throughout the tube.

At still lower pressures, ~0.02mmHg, the colours disappear but the glass tube itself begins to glow with a green hue. An object placed in front of the cathode (the negative electrode) casts a shadow on the opposite wall of the discharge tube (Fig 1.2). Certain minerals, when placed in front of the cathode, fluoresce with brilliant colours. It appears that something is being emitted from the cathode; this emanation was given the name *cathode rays*.

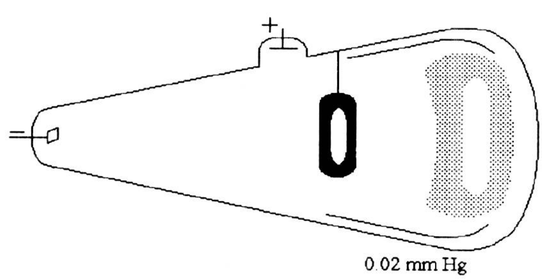

Fig 1.2 The shadow cast by an object in the path of the rays in a cathode ray tube.

In further experiments it was found that the cathode rays were deflected by a magnetic field as would a stream of negative charge (Fig 1.3a). Furthermore, a small paddle wheel positioned between the electrodes rotated under their impact; switching the polarity of the electrodes reversed the direction of the rotation (Fig 1.3b). These two phenomena suggested the cathode rays might be negatively charged particles. Nevertheless, many physicists at the time still considered them to be of an æthereal rather than a material nature.

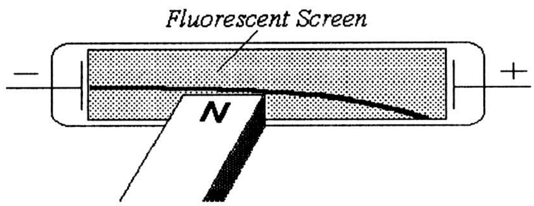

Fig 1.3a The deflection of cathode rays in a magnetic field. The direction of the deflection shows that it is negative charge that flows with the cathode rays.

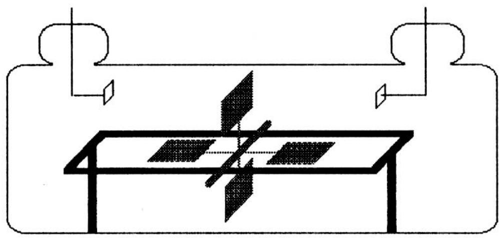

Fig 1.3b The paddle wheel rotates under the impact of the cathode rays.

Convinced that the cathode rays were in fact charged particles of matter, J.J.Thomson set out in 1897 to measure their velocity, v, and the ratio, q/m, between their charge, q, and their mass, m. In one of the experiments he conducted, a narrow collimated beam of cathode rays was aimed along the length of a very low pressure glass discharge tube. After emerging from the hole in the anode, the beam passed through a thin slit, between a pair of vertical coils and, finally, between the horizontal parallel plates of a condenser. The green spot that appeared on the glass at the far end of the tube indicated where the beam impinged upon it (Fig 1.4).

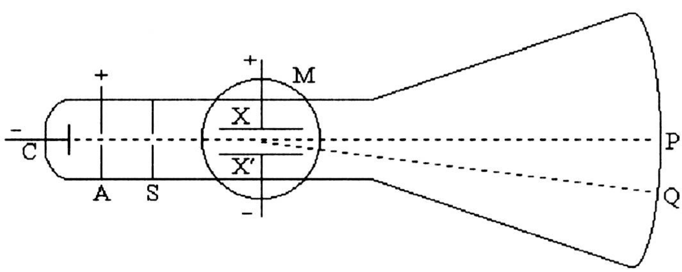

Fig 1.4 The type of very low pressure discharge tube used by Thomson for the determination of the ratio between the charge and the mass of the particles (electrons) in cathode rays. A collimated beam was produced by the arrangement of the cathode, C, the pierced anode, A, and the slit, S. The circle M represents the pair of coils, one on each side of the tube. Passing a current through these coils produces a uniform horizontal magnetic field in the gap between them. X and X′ are the parallel plates of the condenser. Connecting the plates to a source of a potential produces a uniform vertical electric field between them. An undeflected particle beam strikes the point P.