BARGER AND OLSSON

CLASSICAL MECHANICS: A MODERN PERSPECTIVE

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A Modern Perspective

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Preface

The study of classical mechanics offers an unequaled opportunity for physical insights into events of everyday life. For this reason it is desirable that a textbook for an intermediate-level course in mechanics be suitable for use both by physics majors and by students from other disciplines. This textbook attempts to meet this basic goal by presentation of topics of widespread popular interest. By repeated application of the principles of mechanics to such diverse topics as sports, seagulls, boomerangs, satellites, and tides, we try to develop physical intuition as well as proficiency with mathematical methods.

This text was designed for an intensive one-semester course in theoretical mechanics at the junior-senior level. A knowledge of general physics, integral calculus, and differential equations is assumed. The problems at the end of each chapter are designed to illustrate the methods developed in the text and to further stimulate the student's interest in mechanics. Since a mastery of problem-solving techniques is an essential requirement for a mechanics course, we have included a number of easy problems to permit the student to get a wide range of practice.

A major departure in our book from the conventional approach to the subject is the introduction of the Lagrange formulation of the equations of motion at an early stage (end of Chapter 2). In the conventional organization, Lagrange's equations are presented near the end of a one-semester course, and the student rarely develops a reasonable familiarity with lagrangian methods. Since our organization encourages the student to solve problems in later chapters by direct application of Newton's laws and Lagrange's equations, he can achieve a mastery of both techniques.

In our experience, about 85 percent of this text can be covered in a 15-week semester with three lecture hours per week. A majority of our students also attended an optional session each week for discussion of solutions to assigned homework problems. In the choice of material for lectures, any of the following sections can be selectively omitted without loss of continuity in the text: 2-9, 2-10, 2-12, 4-3, 4-6, 5-11, 6-4, 6-11, 6-12, 6-13. The last three sections of Chapter 7 can be covered or omitted, as time permits. Since the number of different topics in mechanics which can be discussed in the course of a semester is necessarily limited, we do not include chapters on strength of materials, continuous media, or relativity.

The first chapter contains novel one-dimensional applications involving frictional, gravitational, and harmonic forces in the sports of drag racing, sky diving, and archery. The simple harmonic oscillator with damping and driving forces is given appropriate attention. Chapters 2, 3, and 4 are organized around the fundamental conservation laws of energy, momentum, and angular momentum. As an application of energy conservation in Chapter 2, we calculate the minimum velocity needed to escape the earth's gravitational attraction. In Chapter 3 the Apollo moon rocket is used as a concrete example in a section dealing with variable mass. Collisions of billiard balls are discussed in center-of-mass and laboratory coordinate systems to develop familiarity with momentum-conservation methods. The concept of a differential cross section is introduced in a calculation of the likelihood that BBs ricochet off a cylindrical pipe in a given direction. In Chapter 4 the trajectories for planetary motion are derived by two alternative methods. The orbit period for a proposed NASA weather satellite is determined from Kepler's law. The forthcoming Grand Tours of our solar system on gravity-assistance trajectories are discussed as examples of the central-force problem. The differential cross section for Rutherford scattering is derived in the concluding section of Chapter 4.

In the treatment of rigid-body motion in Chapter 5, the return of a boomerang is explained in terms of the gyroscope effect. "Draw" and "follow" shots in billiards and the action of "superballs" are presented as intriguing examples of rigid-body rotations. Chapter 6 is concerned with applications of the law of motion in moving coordinate systems. The relevance of the centrifugal and Coriolis forces in a variety of physical situations is indicated. Several sections are devoted to the motion of spinning tops, concluding with an analysis of the flipping motion of the amazing tippie-top.

Chapter 7 begins with a proof that the net gravitational attraction of a point mass on a spherically symmetrical body acts as though the mass of the body were concentrated at its center. We then proceed to calculate the tides on earth due to the moon and sun. As a useful application of lagrangian methods in gravitation, we discuss the technique for automatic attitude stabilization of a satellite orbiting the earth. In somewhat more advanced sections of Chapter 7, we calculate the gravity field and shape of the oblate earth.

It is a pleasure to acknowledge helpful conversations with numerous colleagues and friends. Suggestions by Professors L. Durand, III, C. Goebel, and R. March were of particular value in the development and refinement of the text. A critical and thorough review of the manuscript by Professor C. Goebel, for which we are especially grateful, led to substantial improvements in various sections. We benefited by the able assistance of Mr. Kevin Geer as teaching assistant in charge of problem-solving sessions. Many thanks go to Mrs. Laurel Hermanson for typing the several drafts of the manuscript. One of us (V.B.) is grateful for the kind hospitality extended by Professor San Fu Tuan and other members of the Physics Department at the University of Hawaii, where part of the manuscript was prepared.

V. Barger
M. Olsson

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CHAPTER 1

The Beginnings

Classical mechanics is one of the most satisfying subjects of study in all of science. In order to understand and appreciate how both everyday and esoteric things in our world work, some knowledge of the principles of mechanics is essential. In this age everyman needs to know mechanics to fulfill and enrich his daily existence.

The formulation of classical mechanics represents a giant milestone in man's intellectual and technological history, as the first mathematical abstraction of physical theory from empirical observation. This crowning achievement is rightly accorded to Isaac Newton (1642–1720), who modestly acknowledged that if he had seen further than others, "it is by standing upon the shoulders of Giants." However, the great Laplace characterized Newton's work as the supreme exhibition of individual intellectual effort in the history of the human race.

Newton translated his interpretation of various physical observations into a compact mathematical theory. Three centuries of experience indicate that all mechanical behavior in the everyday domain can be understood from Newton's theory. His simple hypotheses are now elevated to the exalted status of laws, and these are our point of embarkation into the subject.

1-1 NEWTONIAN THEORY

The newtonian theory of mechanics is customarily stated in three laws. According to the first law, a particle continues in uniform motion unless a force acts on it. The first law is a fundamental observation that physics is simpler when viewed from a certain kind of coordinate system, called an *inertial frame*. One cannot define an inertial frame except by saying that it is a frame in which Newton's laws hold. However, once one finds (or imagines) one such frame, all other inertial frames are moving in straight lines at constant velocity (i.e., nonaccelerating) with respect to it. A coordinate system fixed on the surface of the earth is not an inertial frame because of the accelerations due to the rotation of the earth, and its motion around the sun. Nevertheless, for many purposes it is an adequate approximation to regard a coordinate frame fixed on the earth's surface as an inertial frame. Indeed, Newton himself discovered nature's true laws while riding on the earth!

The meat of Newton's theory is contained in the second law, which states that *the time rate of change of momentum of a particle is equal to the force acting on the particle*,

$$F = \frac{dp}{dt} \tag{1-1}$$

where the momentum p is given by the product of (mass) $\times$ (velocity) for the particle.

$$p = mv \tag{1-2}$$

The second law provides a definition of force. The physics content of the second law depends on empirical forms for the forces as functions of positions and velocities. The force in Eq. (1-1) can be a function of x, v, and t, and so

$$F(x, v, t) = \frac{dp}{dt} = m\frac{d^2x}{dt^2}$$

is a differential equation. While Newton's laws promise to apply to any situation in which one can specify the force at all times, very few interesting physical problems lead to force laws amenable to simple mathematical solution. To approximate the true force law by a sufficiently accurate approximate form is one of the arts that will be taught in this book. However, in this modern age of digital computers, one can handle incredibly complicated force laws by the brute-force method of numerical integration.

In the special case $F = 0$, integration of Eq. (1-1) gives $p =$ constant in accordance with the first law. A more familiar expression of the second law in Eq. (1-1) is

$$F = ma \tag{1-3}$$

where $a = dv/dt$ is the acceleration.

The third law states that if particle A experiences a force due to particle B, then B feels simultaneously a force of equal magnitude but in the opposite direction. This law is extremely useful, especially in the treatment of rigid-body motion, but its range of applicability is not as universal as the first two laws. The third law breaks down when the interaction between the particles is electromagnetic.

It is a remarkable fact that macroscopic phenomena can be explained by such a simple set of mathematical laws. As we shall see, the mathematical solutions to some problems can be complex; nevertheless, the physical basis is just Eq. (1-1). Of course, there is still a great deal of physics to put into Eq. (1-1), namely, the laws of force for specific kinds of interactions.

1-2 INTERACTIONS

The gravitational and electromagnetic forces determine our whole condition of life. Newton deduced the following force law for gravitation by studying data phenomenologically fitted by Kepler on the motion of planets and satellites in our solar system.

$$F = -\frac{GM_1M_2}{r^2} \tag{1-4}$$

The force between masses M_1 and M_2 is proportional to the masses and inversely proportional to the square of the distance between them. The negative sign in Eq. (1-4) denotes an attractive force between the masses. Newton proposed that this gravitational law was universal, the same force applying on the earth as between celestial bodies. The universality of the gravitational law can be verified, and the proportionality constant G determined, by delicate experimental measurements of the force between masses in the laboratory. The value of G is

$$G = 6.67 \times 10^{-11} m^3/(\text{kg})(\text{s}^2) \tag{1-5}$$

The dominant gravitational force on an object located on the surface of the earth is due to the attraction from the earth. The gravitational attraction on a point mass from a spherically symmetric body acts as if all the mass of the body were concentrated at its center, as Newton rigorously proved from his invention of calculus. We will give a proof of this assertion in Chap. 7. For an object of mass m on the surface of the earth, the force law of Eq. (1-4) becomes

$$F = -m\frac{M_e G}{R_e^2} = -mg \tag{1-6}$$

where g is the gravitational acceleration,

$$g = 9.8 \text{ m/s}^2$$

The values of mass and radius of the earth in Eq. (1-6) are

$$R_e = 6{,}371 \text{ km}$$
$$M_e = 5.97 \times 10^{24} \text{ kg}$$

Since the earth's radius is large, the gravitational force on an object anywhere between the surface of the earth and the top of the atmosphere ($\approx$200 km up) is given with reasonable accuracy by Eq. (1-6). Consequently, in many applications on earth, we can neglect the variation of the gravitational force with position.

The static Coulomb force between two charges e_1 and e_2 is similar in form to the gravitational-force law of Eq. (1-4).

$$F = \frac{e_1 e_2}{r^2} \tag{1-7}$$

This force is attractive if the charges are opposite in sign and repulsive if the charges are of the same sign.

Another force with a wide range of application is the spring force or Hooke's law, which is expressed as

$$F = -kx \tag{1-8}$$

Here k is a spring constant which is dependent on the properties of the spring and x is the extension of the spring from its relaxed position. This particular force law is a very good approximation in many physical situations, such as the stretching or bending of materials which are initially in equilibrium.

Frictional forces play a crucial role in damping or retarding motion initiated by other forces. The static frictional force between two solid surfaces is

$$|f| \leq \mu_s N \tag{1-9}$$

The force f acts to prevent sliding motion. N is the perpendicular force (normal force) holding the surfaces together, and μ_s is a material-dependent coefficient. Equation (1-9) is an *approximate* formula for frictional forces which has been deduced from empirical observations. The frictional force which retards the motion of sliding objects is given by

$$f = \mu_k N$$

It is observed that this force is nearly independent of the velocity of the motion, for velocities which are neither too small or too large. For a given set of surfaces, the coefficient of kinetic friction μ_k is less than the coefficient of static friction μ_s.

Frictional laws to describe the motion of a solid through a fluid or a gas are often complicated by such effects as turbulence. However, for sufficiently small velocities, the approximate form

$$f = -bv \tag{1-10}$$

where b is a constant, holds. At higher, but still subsonic velocities, the frictional-force law

$$f = -cv^2 \tag{1-11}$$

is approximately true. The drag force on a propeller airplane is remarkably well represented by a constant times the square of the velocity.

Externally imposed forces can take on a variety of forms. Of those depending explicitly on time, sinusoidal oscillating applied forces like

$$F = F_0 \cos \omega t \tag{1-12}$$

are frequently encountered in physical situations.

In a general case the forces can be position-, velocity-, and time-dependent.

$$F = F(x, v, t) \tag{1-13}$$

Among the most interesting and easily solved examples are those in which the forces depend on only one of the above three variables, as illustrated by the examples in the following three sections.

1-3 THE DRAG RACER: FRICTIONAL FORCE

A number of interesting engineering-type problems can be solved from straightforward application of Newton's laws. As an illustration, suppose we want to design a drag racer which will achieve maximum possible acceleration when starting from rest. We assume that the engine of the racer can apply an arbitrary torque to the rear wheels, and our problem is to determine the optimal weight distribution of the racer. The external forces on the racer which must be taken into account are (1) gravity, (2) the normal forces supporting the racer at the wheels, and (3) the frictional forces which oppose the rotation of the powered rear wheels. A sketch indicating the various external forces is given in Fig. 1-1. The gravity force Mg acts as if all the weight were concentrated at the center of mass. This is a familiar

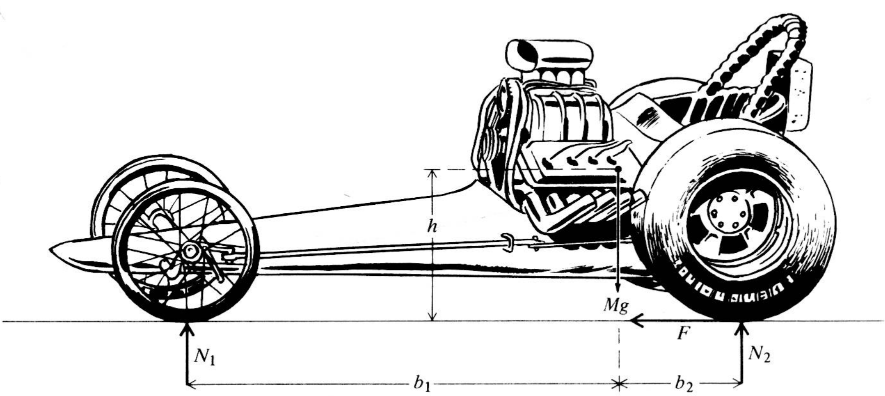

FIGURE 1-1 Forces acting on a drag racer.