JOHN W. BREWER

CONTROL SYSTEMS

Analysis, Design, and Simulation

CONTROL SYSTEMS
Analysis, Design, and Simulation

JOHN W. BREWER
Associate Professor
Department of Mechanical Engineering
University of California, Davis

Prentice-Hall, Inc., Englewood Cliffs, New Jersey

Library of Congress Cataloging in Publication Data

BREWER, JOHN W.
Control systems: analysis, design, and simulation.

Includes bibliographies.
1. Automatic control. 2. Control theory.
3. System analysis. I. Title.
TJ213.B64 629.8 73-19697
ISBN 0-13-171850-9

Dedicated to

M'lou,

Corrie,

Jeff

10 9 8 7 6 5 4 3 2 1

Printed in the United States of America

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA, PTY. LTD., *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LTD., *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*

Preface

It is the author's intention to integrate, into a form suitable for mechanical engineering undergraduates, an introductory treatment of

1. Physical system analysis,
2. Control system design, and
3. Sensitivity analysis.

It is the author's opinion that closed loop *insensitivity* to parameter change is the most important topic of feedback design theory.

A secondary objective is to prepare the student for modern control theory. State space concepts are introduced, and nonlinear regulators are discussed. The discussion of frequency domain centers on the Nyquist diagram because it is felt that the beginning student will, in this way, be prepared for advanced topics, such as Popov stability, if he thoroughly understands vector-loci techniques.

The outline for this text is prepared with the view that computers will play a major role in all aspects of modern engineering. For this reason, detailed instructions for construction of root locus diagrams, Bode diagrams, and Nyquist diagrams for complicated systems have been deleted. In place of such contruction rules, the author supplies discussion which will, it is hoped, increase the student's ability to extract all important information from these fundamental diagrams. Even at this date the construction of these diagrams is easily programmed on a computer (see Appendix C).

The author relied heavily on the contributions of Y. Takahashi, H.

Paynter, D. Karnopp, and R. Rosenberg in the theory of physical system analysis; R. Tomovic and I. Horowitz in sensitivity analysis; and W. Loscutoff in state space analysis. It is hoped that the text represents a true integration of the theory contributed by these researchers into a form suitable for the beginning student of systems theory.

There are several unique features in this text. The treatment of right half-plane zeros in the chapters on root locus and frequency domain techniques points up some interesting facts. State variable analysis is motivated with an introduction to simulation and synthesis techniques. Another unique feature is the use of frequency domain methods to show, for one simple example, the accuracy of various orders of truncated state variable models of a distributed parameter model. I think the users of this text will discover other innovations.

The text contains sufficient material for a one-year course on introductory systems theory even if the sections and chapters marked with an asterisk are omitted. The sections marked with an asterisk were included in order to provide the student with material that can aid his study of advanced topics.

A number of students made valuable corrections to and comments on the text. Ching Yung and John Seevers deserve special mention. Jeff Young bravely read the entire final draft and kindly concealed a good part of his amusement. Brian Crews assisted with figures.

H. Brandt and W. Giedt, Chairmen of the Department of Mechanical Engineering, supplied the encouragement to write this book. Acknowledgment is also made to typists Lois Wilson, Theda Strack, Winnie Letson, and Renee Dominguez; may their neurons unboggle and may they recover from their harrowing experience. A special thanks to Ruth Moorhead, who typed the final draft in such an excellent manner.

J.W.B.

Contents

*Chapters and sections marked with an asterisk contain reference material and may be omitted during first reading of the text.

8 Simulation and Synthesis of Dynamic Systems: State Space Analysis* 207

9 Error Analysis of Simulation Studies* 257

Introduction: Feedback Control and Dynamic Systems Analysis

1

1.1 Feedback Control

To illustrate the concept of feedback control, the applied mathematician Norbert Wiener described a human being grasping an object.[13] The process is illustrated in Figure 1.1. The *desired hand position* is the position of the object to be grasped. The brain compares this position with the actual hand position, which is measured and *fed back* to the brain by the eye. The difference between the actual and desired position of the hand is the *error signal*, which, if not zero, causes another part of the brain (the *compensator*) to send appropriate signals to muscle cells. This *prime mover* converts available energy (stored chemical energy in this case) into useful work and moves the hand or *controlled object*. If all the elements of the feedback system are working properly, the hand will be moved in a manner so as to continually reduce the magnitude of the error signal until the hand is close enough to the object to grasp it.

The description of many other physiological functions in such a manner is quite popular, and, as a matter of fact, many different types of pathologies may be thought of as malfunctions of one of the elements of some physiological feedback system.[6] Wiener identified similar types of feedback loops in social organizations.[13] Allen describes the operation of feedback in economic systems.[1]

It is important to gain some insight into the necessity for feedback itself and for the compensator element within a feedback loop. In concise terms,

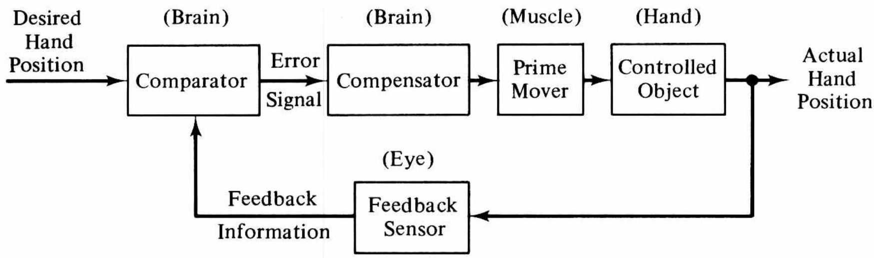

Figure 1.1. Elements of a feedback system; Wiener's classical example.

the feedback loop corrects for inaccuracies in the operation of individual loop elements and for the occurrence of disturbing signals. The eye, after all, provides only an estimate of the three-dimensional position, and the muscle contractions do not respond perfectly accurately and consistently to stimuli from the brain. However, the overall feedback system accuracy is much higher than the accuracy of either of these components because the compensator will always react to nonzero error and eventually force the hand to the correct position. Examples of disturbing signals are unforeseen forces applied to the hand from an external source (e.g., the result of the operation of a physiological feedback system of a coed) and physiological signals which affect the operation of the muscle. The effect of disturbance is an increase in the magnitude of the error signal, which, once again, is corrected by the operation of the feedback system.

A third function provided by the feedback system is compensation for *parameter variation.* Muscle fatigue could cause a great deal of change in the response of the prime mover in the feedback loop illustrated in Figure 1.1. A remarkable fact about feedback systems is that the variation in performance of the system is often far less than the associated variation of performance of any of the individual elements within the loop. Thus, one would expect that a great deal of muscle fatigue (parameter change) would occur before the performance of the feedback system, described by Wiener, is affected. It is little wonder that evolutionary processes provided the human body with many feedback loops.

The final concept to be gleaned from this simple example is the role of the compensator in the smooth operation of the feedback system. The hand, of course, has inertia and, once set in motion by the muscle prime mover, will continue to move until the muscles exert a braking or stopping force. If the braking force is exerted too late, the hand will overshoot its target. The resulting error will activate the feedback mechanism, which will tend to bring the hand back to the correct position, but a better-quality performance would be one in which the hand is brought smoothly to the desired position with little or no overshoot. No overshoot could be accomplished if the braking action were anticipatory, i.e., if the braking were applied slightly early. It is the function of the compensator to operate upon the feedback information and provide the anticipatory control actions.

The main objective of this text is to present the application of the feedback principles, described above, to the design of automatically controlled engineering systems. An example of such a system is the first radar-controlled antiaircraft gun, which Wiener helped develop during World War II. The development of this instrument is a significant event in the history of automatic control because, for the first time, mathematical techniques which had previously been used to design electrical feedback amplifiers were abstracted and applied to the design of a mechanical system. Abstract mathematical techniques will be strongly emphasized in this text.

In the radar-controlled weapon, the aircraft angular position is the desired angular position of the weapon and is measured by radar. The actual weapon position can be measured with a potentiometer and an electrical error signal is generated. The compensator is an electrical network which must be designed to provide the proper anticipation. The prime mover supplies the energy to drive the system and, in this case, is a hydraulic motor which is activated by an electrical signal from the compensator. The controlled object is the weapon, which has inertias and frictions which must be overcome by the prime mover in order to reduce the error. An important type of parameter variation is the extreme variation in the frictional properties of the grease-lubricated connections which is induced by temperature variations and/or contaminants. The feedback systems developed for such weapons are insensitive to such parameter variations.

1.2 Early Examples of Mechanical Feedback Systems[5]

While World War II marks the beginning of the development of mathematical automatic feedback control theory by Wiener and Bode and others, the actual use of feedback systems greatly precedes this event. Since several of the early feedback devices are still used today in some form or another, it is instructive to study the early applications of feedback.

The first example is the control of the volume flow rate of a fluid, which was accomplished by the Alexandrian Greek Ktesibios in the third century B.C.[5] The device is illustrated in Figure 1.2. The variable to be controlled is the flow rate at the outlet. The main purpose of this simple feedback system is to correct for large fluctuations or disturbances in the supply pressure. To understand the operation of the system, assume that some large, unforeseen increase occurs in the supply pressure. The inlet flow rate will increase, causing an error signal, i.e., a difference between inlet flow rate and outlet flow

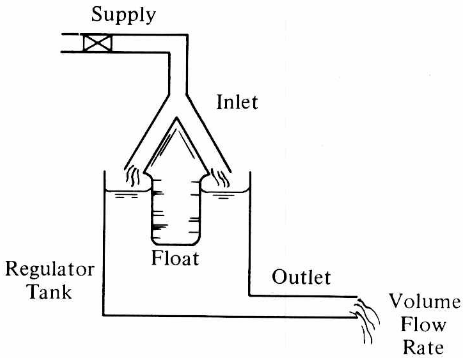

Figure 1.2. Flow control system of Ktesibios (third century B.C.).

rate. This error signal will ultimately cause the float to rise and restrict the inlet flow area. The system will then come to equilibrium as the inlet flow rate is decreased until it equals the outlet flow rate. Note that the system comes to equilibrium with a slightly increased head in the regulator tank and, since outlet flow rate is proportional to this head, with a slightly increased outlet flow rate. The fact that makes the system effective is that the inlet flow rate is strongly dependent on inlet flow area and, hence, on the vertical position of the float. Thus, negligibly small changes in outlet flow rate (and regulator tank head) are associated with large supply pressure disturbances. The main application of Ktesibios' invention was in the development of water clocks. Notice that no compensator or prime mover is included in Ktesibios' feedback loop.

The development of feedback sensors (see Figure 1.1) seems to have been the crucial step in the synthesis of early engineering feedback systems. Mayr describes many of these sensors.[5] For instance, the invention of the thermostat by the seventeenth-century Dutchman Cornelis Drebbel (working in England) led to the development of feedback temperature control systems. An improved eighteenth-century version of the thermostat is illustrated in Figure 1.3. The tube is immersed in the medium whose temperature is to be controlled. Because of differences between the coefficients of expansion of iron and lead, temperature changes in the medium induce corresponding changes in the displacement of the upper edge of the tube, x. The displacement, x, serves as *feedback information* (see Figure 1.1) in temperature feedback loops. The displacement, x, is input to a linkage which regulates the angular position of a damper, which in turn regulates the flow of gas to a combustion chamber (the prime mover). Large variations in the quantity and quality of fuel in the combustion chamber motivated the synthesis of these early temperature control systems.

Eighteenth- and nineteenth-century British millwrights were very inventive and provided many developments in feedback devices and synthesized several interesting feedback loops.[5] It is instructive to speculate on the reason