Pattern Formation and Dynamics in Nonequilibrium Systems

Michael Cross and Henry Greenside

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MICHAEL CROSS
California Institute of Technology

HENRY GREENSIDE
Duke University

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PATTERN FORMATION AND DYNAMICS IN NONEQUILIBRIUM SYSTEMS

Many exciting frontiers of science and engineering require understanding of the spatiotemporal properties of sustained nonequilibrium systems such as fluids, plasmas, reacting and diffusing chemicals, crystals solidifying from a melt, heart muscle, and networks of excitable neurons in brains.

This introductory textbook for graduate students in biology, chemistry, engineering, mathematics, and physics provides a systematic account of the basic science common to these diverse areas. This book provides a careful pedagogical motivation of key concepts, discusses why diverse nonequilibrium systems often show similar patterns and dynamics, and gives a balanced discussion of the role of experiments, simulation, and analytics. It contains numerous illustrative worked examples, and over 150 exercises.

This book will also interest scientists who want to learn about the experiments, simulations, and theory that explain how complex patterns form in sustained nonequilibrium systems.

MICHAEL CROSS is a Professor of Theoretical Physics at the California Institute of Technology, USA. His research interests are in nonequilibrium and nonlinear physics including pattern formation, chaos theory, nanomechanical systems, and condensed matter physics, particularly the theory of liquid and solid helium.

HENRY GREENSIDE is a Professor in the Department of Physics at Duke University, USA. He has carried out research in condensed matter physics, plasma physics, nonequilibrium pattern formation, and theoretical neurobiology. He is also involved with outreach programs to stimulate interest in science and physics at junior high school and high school levels.

"This book by Cross and Greenside presents a comprehensive introduction to an important area of natural science, and assembles in one volume the essential conceptual, theoretical, and experimental tools a serious student will need to obtain a modern understanding of pattern formation outside of equilibrium. The masterful 50-page Introduction lays out the essential questions and provides motivation to the reader to explore the subsequent chapters, beginning with simple ideas and growing progressively in mathematical sophistication and physical depth. Careful attention is paid to the relationship between the theoretical methods and controlled laboratory experiments or numerical simulations. I can highly recommend this book to any student or researcher interested in a deepened understanding of nonequilibrium spatiotemporal patterns."

Pierre Hohenberg, New York University

"This book gives an excellent didactic introduction to pattern formation in spatially extended systems. It can serve both as the basis for an advanced undergraduate or graduate course as well as a reference. It is one of those books that will never outlive its usefulness. It is a must for anyone interested in nonlinear, nonequilibrium physics."

Eberhard Bodenschatz, MPI for Dynamics and Self-Organization, University of Goettingen, Cornell University

"This book fills a long-standing need, and is certain to be an instant classic. The physics of pattern forming systems is diverse but the theoretical core of the subject, along with many of the most important applications, can be learned from this splendid book. It is bound to be a key text for courses, as well as a much cited reference."

Stephen Morris, University of Toronto

To our families

Katy, Colin and Lynn
Peyton, Arthur and Noel

Preface

This book is an introduction to the patterns and dynamics of sustained nonequilibrium systems at a level appropriate for graduate students in biology, chemistry, engineering, mathematics, physics, and other fields. Our intent is for the book to serve as a second course that continues from a first introductory course in nonlinear dynamics. While a first exposure to nonlinear dynamics traditionally emphasizes how systems evolve in time, this book addresses new questions about the spatiotemporal structure of nonequilibrium systems. Students and researchers who succeed in understanding most of the material presented here will have a good understanding of many recent achievements and will be prepared to carry out original research on related topics.

We can suggest three reasons why nonequilibrium systems are worthy of study. First, observation tells us that most of the Universe consists of nonequilibrium systems and that these systems possess an extraordinarily rich and visually fascinating variety of spatiotemporal structure. So one answer is sheer basic curiosity: where does this rich structure come from and can we understand it? Experiments and simulations further tell us that many of these systems – whether they be fluids, granular media, reacting chemicals, lasers, plasmas, or biological tissues – often have *similar* dynamical properties. This then is the central scientific puzzle and challenge: to identify and to explain the similarities of different nonequilibrium systems, to discover unifying themes, and, if possible, to develop a quantitative understanding of experiments and simulations.

A second reason for studying nonequilibrium phenomena is their importance to technology. Although the many observed spatiotemporal patterns are often interesting in their own right, an understanding of such patterns – e.g. being able to predict when a pattern will go unstable or knowing how to select a pattern that maximizes some property like heat transport – is often important technologically. Representative examples are growing pure crystals, designing a high-power coherent laser, improving yield and selectivity in chemical synthesis, and inventing new electrical control techniques to prevent epilepsy or a heart attack. In these and other cases such as forecasting the weather or predicting earthquakes, improvements in the design, control, and prediction of nonequilibrium systems are often limited by our incomplete understanding of sustained nonequilibrium dynamics.

Finally, a third reason for learning the material in this book is to develop specific conceptual, mathematical, and numerical skills for understanding complex phenomena. Many nonequilibrium systems involve continuous media whose quantitative description is given in terms of nonlinear partial differential equations. The solutions of such equations can be difficult to understand (e.g. because they may evolve nonperiodically in time and be simultaneously disordered in space), and questions such as "Is the output from this computer simulation correct?," "Is this simulation producing the same results as my experimental data?" or "Is experimental noise relevant here?" may not be easily answered. As an example, one broadly useful mathematical technique that we discuss and use several times throughout the book is multiscale perturbation theory, which leads to so-called "amplitude equations" that provide a quantitatively useful reduction of complex dynamics. We also discuss the role of numerical simulation, which has some advantages and disadvantages compared to analytical theory and experimental investigation.

To help the reader master the various conceptual, mathematical, and numerical skills, the book has numerous worked examples that we call etudes. By analogy to a musical etude, which is a composition that helps a music student master a particular technique while also learning a piece of artistic value, our etudes are one- to two-page long worked examples that illustrate a particular idea and that also try to provide a non-trivial application of the idea.

Although this book is intended for an interdisciplinary audience, it is really a physics book in the following sense. Many of the nonequilibrium systems in the Universe, for example a germ or a star, are simply too complex to analyze directly and so are ill-suited for discovering fundamental properties upon which a general quantitative understanding can be developed. In much of this book, we follow a physics tradition of trying to identify and study simple idealized experimental systems that also have some of the interesting properties observed in more complex systems.

Thus instead of studying the exceedingly complex dynamics of the Earth's weather, which would require in turn understanding the effects of clouds, the solar wind, the coupling to oceans and ice caps, the topography of mountains and forest, and the effects of human industry, we instead focus our experimental and theoretical attention on enormously simplified laboratory systems. One example is Rayleigh–Bénard convection, which is a fluid experiment consisting of a thin horizontal layer of a pure fluid that is driven out of equilibrium by a vertical temperature difference that is constant in time and uniform in space. Another is a mixture of reacting and diffusing chemicals in a thin layer of gel, with reservoirs of chemicals to sustain the reaction. The bet is then that to understand aspects of what is going on in the weather or in an epileptic brain, it will be useful to explore some basic questions first for convection and other well-controlled laboratory systems. Similarly, as we discuss later in the book, there are conceptual, mathematical, and computational advantages if one studies simplified and reduced mathematical models such as the Swift–Hohenberg and complex Ginzburg–Landau equations when trying to understand the much more difficult partial differential equations that describe physical systems quantitatively. The experiments, models, simulations, and theory discussed in this book – especially the numerous comparisons of theory and simulation with experiment – will give the reader valuable insights and confidence about how to think about the more complex systems that are closer to their interests.

As background, readers of this book should know the equivalent of an introductory nonlinear dynamics course at the level of Strogatz's book [99]. Readers should feel comfortable with concepts such as phase space, dissipation, attractors (fixed points, limit cycles, tori, and strange attractors), basins of attractors, the basic bifurcations (super- and subcritical, saddle-node, pitchfork, transcritical, Hopf), linear stability analysis of fixed points, Lyapunov exponents, and fractal dimensions. A previous exposure to thermodynamics and to fluid dynamics at an undergraduate level will be helpful but is not essential and can be reviewed as needed. The reader will need to be competent in using multivariate calculus, linear algebra, and Fourier analysis at a junior undergraduate level. Several appendices in this book provide concise reviews of some of this prerequisite material, but only on those parts that are important for understanding the text.

There is too much material in this book for a single semester class so we give here some suggestions of what material could be covered, based on several scenarios of how the book might be used.

The first six chapters present the basic core material and should be covered in most classes for which this book is a main text. By the end of Chapter 6, most of the main ideas have been introduced, at least qualitatively. The successive chapters present more advanced material that can be discussed selectively. For example, those particularly interested in the systematic treatment of stationary patterns may choose to complete the semester by studying all or parts of Chapters 7 and 8, which provide quantitative discussions of two-dimensional patterns and localized structures, and Chapter 9 which is a more qualitative discussion of stationary patterns far from onset. For a less mathematical approach, it is possible to leave out the more technical Chapters 7 and 8 and move straight to Chapter 9 although we recommend including the first three subsections of Section 7.3 on the central question of the competition between stripes, two-dimensional lattices, and quasiperiodic patterns (these sections can be read independently of the remainder of the chapter). If the interest is more in dynamical phenomena, such as oscillations, propagating pulses, and waves (which may be the case if applying the ideas to signalling phenomena in biology is a goal), the class may choose to skip Chapters 7–9, pausing briefly to study Section 8.3 on fronts, and move immediately to Chapters 10 and 11 on oscillatory patterns and excitable media. Numerical simulations are vital to many aspects of the study of pattern formation, and to nonlinear dynamics in general, and so any of the above suggestions may include all or parts of Chapter 12.

In learning about nonequilibrium physics and in writing this book, the authors have benefited from discussions with many colleagues and students. We would like to thank Philip Bayly, Bob Behringer, Eshel Ben-Jacob, Eberhard Bodenschatz, Helmut Brand, Hugues Chaté, Peilong Chen, Elizabeth Cherry, Keng-Hwee Chiam, Bill Coughran Jr., Peter Daniels, David Egolf, Bogdan Epureanu, Paul Fischer, Jerry Gollub, Roman Grigoriev, James Gunton, Craig Henriquez, Alain Karma, Kihong Kim, Paul Kolodner, Lorenz Kramer, Andrew Krystal, Eugenia Kuo, Ming-Chih Lai, Herbert Levine, Ron Lifshitz, Manfred Lücke, Paul Manneville, Dan Meiron, Steve Morris, Alan Newell, Corey O'Hern, Mark Paul, Werner Pesch, Joel Reisman, Hermann Riecke, Sam Safran, Janet Scheel, Berk Sensoy, Boris Shraiman, Eric Siggia, Matt Strain, Cliff Surko, Harry Swinney, Shigeyuki Tajima, Gerry Tesauro, Yuhai Tu, Wim van Saarloos, and Scott Zoldi. We would like to express our appreciation to John Bechhoefer, Roman Grigoriev, Pierre Hohenberg, Steven Morris, and Wim Van Saarloos for helpful comments on early drafts of this book. And we would like especially to thank Guenter Ahlers and Pierre Hohenberg for many enjoyable and inspiring discussions over the years.

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Henry Greenside would like to thank his wife, Noel Greis, for her patience, support, and good cheer while he was working on this book.

Readers can find supplementary material on the book website at

`http://mcc.caltech.edu/pattern-formation-book/`,

or

`http://www.phy.duke.edu/~hsg/pattern-formation-book/`.

A list of errata will also be available on these websites.

We would be grateful to readers if they could forward to us comments they might have about the material and its presentation. These comments can be e-mailed to us at `mcc@caltech.edu` or `hsg@phy.duke.edu`.

Contents

1

Introduction

In this opening chapter, we give an informal and qualitative overview – a pep talk – to help you appreciate why sustained nonequilibrium systems are so interesting and worthy of study.

We begin in Section 1.1 by discussing the big picture of how the Universe is filled with nonequilibrium systems of many different kinds, a consequence of the fact that the Universe had a beginning and has not yet stopped evolving. A profound and important question is then to understand how the observed richness of structure in the Universe arises from the property of not being in thermodynamic equilibrium. In Section 1.2, a particularly well studied nonequilibrium system, Rayleigh–Bénard convection, is introduced to establish some vocabulary and insight regarding what is a nonequilibrium system. Next, in Section 1.3, we extend our discussion to representative examples of nonequilibrium patterns in nature and in the laboratory, to illustrate the great diversity of such patterns and to provide some concrete examples to think about. These examples serve to motivate some of the central questions that are discussed throughout the book, e.g. spatially dependent instabilities, wave number selection, pattern formation, and spatiotemporal chaos. The humble desktop-sized experiments discussed in this section, together with theory and simulations relating to them, can also be regarded as the real current battleground for understanding nonequilibrium systems since there is a chance to compare theory with experiment quantitatively.

Next, Section 1.4 discusses some of the ways that pattern-forming nonequilibrium systems differ from the low-dimensional dynamical systems that you may have seen in an introductory nonlinear dynamics course. Some guidelines are also given to determine qualitatively when low-dimensional nonlinear dynamics may not suffice to analyze a particular nonequilibrium system. In Section 1.5, a strategy is given and explained for exploring nonequilibrium systems. We explain why fluid dynamics experiments have some advantages over other possible experimental systems and why certain fluid experiments such as Rayleigh–Bénard convection are