# Proceedings of the

# 1990 SEM Spring Conference on Experimental Mechanics

June 4 - 6, 1990

Albuquerque, New Mexico





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#### EXPERIMENTAL ANALYSIS OF STRESS INTENSITY FACTORS

#### FOR A VERTICAL SURFACE-BREAKING CRACK

#### LOCATED NEAR A CONCENTRATED FORCE

Yung-Mien Chen and James W. Dally Mechanical Engineering Department University of Maryland College Park, MD 20742

#### ABSTRACT

Two-dimensional photoelasticity is used to analyze the stress intensity factors for vertical surface-breaking cracks that are relevant to the process of wear particle generation. The specimen represents a half-plane with a crack intersecting the boundary at a right angle. A concentrated load is applied to the boundary of the half-plane in close proximity to the crack intersection. Three series of photoelastic experiments are conducted to determine the mixed mode stress intensity factors  $K_{I}$  and  $K_{II}$ . The mixed mode stress intensity factors are combined to give an effective stress intensity factor which controls the stability of vertical surface-breaking crack. The results indicate that the magnitude of  $K_{II}$  is much smaller than that of  $K_{I}$  for short cracks. However, as the crack becomes deeper the magnitude of  $K_{II}$  approaches that of  $K_{I}$ .

#### INTRODUCTION

During the past decade, significant progress has been made in developing models to describe the mechanisms and processes of wear. In order to extend these ideas and develop qualitative models for predicting wear rates, a better understanding of the details of the fracture process that precedes wear particle generation is needed. Fracture mechanics combined with two-dimensional photoelasticity is used to examine a vertical surface-breaking crack. The crack is subjected to a stress state produced by a concentrated force applied to the boundary of the half-plane near the crack intersection.

The wear particle generation process consists of the nucleation of microcracks, the propagation of these microcracks, and the subsequent formation of loose wear debris [1]. Metallurgical investigations [2] reveal that micropits are formed from small scratches that are oriented in the rolling direction. As tensile stresses concentrate at these micropits, a surface crack is nucleated and then extended under the action of cyclic sliding or rolling loads. Keer, et al. [3] have investigated the surface-breaking crack in an elastic deforming field under Hertzian loading with a numerical method based on integral equations of dislocation density functions. The results show that as the loading moves away from the crack, the absolute values of K<sub>I</sub> and K<sub>II</sub> tend to increase, reach a maximum, and then diminish. They indicate that there may be a tendency for the crack to propagate in a direction opposite to the direction of friction. Tunca and Laufer [4] have investigated the surface-breaking crack in an elastoplastic field using finite elements. The results for surface-breaking cracks in either an elastic or elastic-plastic zone show that crack growth is possible only by the shear mode in the compressive region. In the tensile region the cracks grow due to a combination of the opening and shear modes.

Crack extension, the final phase of the wear particle generation is considered in this paper. An intersecting crack is introduced in a half-plane oriented vertical to the boundary, as shown in Fig. 1. A concentrated load with both normal and tangential components is then applied to the boundary at a location close to the crack tip. The resulting stress field produces mixed mode stress intensity factors.

#### EXPERIMENTAL METHOD

Photoelastic experiments were conducted to investigate the stress distribution in the region near the tip of the vertical crack. The photoelastic models, illustrated in Fig. 1, which represent the half-plane in a state of plane stress, were fabricated from a polycarbonate sheet 6.3 mm thick. A crack intersecting the boundary of the half-plane at a right angle was saw-cut into each specimen. A concentrated force was applied to the boundary of the model with an aluminum wedge. The contact point on the wedge, shown in Fig. 2, was finished with a radius  $r=0.1\ mm$ . The applied force was inclined relative to the boundary of the half-plane to produce tangential and normal force components  $F_t$  and  $F_n$  by rotating the model relative to a vertical line of load application. The magnitude of the applied force was adjusted to generate a sufficient number of fringes required to determine the mixed mode stress intensity factors. The region of interest, 25 x 40 mm in size, shown as the cross-hatched area in Fig. 2, is small compared to the size of the specimen. The characteristic model dimensions a and b and the loads  $F_n$  and  $F_t$  are listed in Table 1 for all 15 of the photoelastic models studied.

Dark and light field isochromatic fringe patterns were recorded for each model. Typical fringe patterns showing enlargements of the region of interest are presented in Fig. 3. The fringe order N and the

corresponding positions (x and y) for 400 to 500 data points were recorded for each negative pair using a digitizing tablet. The data (N,x,y) were stored on a floppy disk for digital processing.

#### DATA ANALYSIS

The stresses in the region of interest are produced by the interaction of two singularities. One singularity of order  $r^{-1}$  is due to the concentrated load and the other of order  $r^{-1/2}$  is due to the crack tip. The analysis of the stresses depends on the crack length, and the approach used for both the shallow and deep cracks is described below:

#### Shallow crack : a/b < 1

For a shallow crack, the two singularities strongly interact with each other, as indicated in the fringe pattern presented in Fig. 3a. The stresses in the region located between the two singularities are described by superimposing the Boussinesq equation [5] and two modes of the generalized Westergaard equations [6-8]. The Boussinesq equation accounts for the stresses due to the concentrated load and the generalized Westergaard equations gives the mixed mode stresses produced by the presence of the crack tip. The Boussinesq [5] representation of the stress components relative to the co-ordinate system defined in Fig. 4 is:

$$\sigma_{\xi} = -\frac{2 F}{\pi \xi} \cos(\alpha + \eta), \quad \sigma_{\eta} = 0 \quad \text{and} \quad \tau_{\xi \eta} = 0$$
 (1)

where F is the load per unit thickness. Note, in Fig. 4 that the line LL perpendicular to the loading force separates the radial stress field into two different regions with  $\sigma_{\xi}$  tensile in the upper left and compressive in the lower right. The tensile region is of primary importance since positive values of the opening mode stress-intensity factor  $K_{I}$  that markedly affect the extension of the crack occur in the tensile region.

Sanford [7] has shown the generalized Westergaard equation for the opening mode and Chona [8] has extended the development to cover the in-plane shearing mode. An expression for the stresses in the region, between the two singularities, obtained by superimposing the stress components from the Boussinesq equations and the two sets of modified Westergaard equations, is given by:

$$\sigma_{xx} = -\frac{2 F}{\pi \xi} \cos(\eta + \alpha) \cos^2 \eta + \sum_{i=0}^{\infty} A_i r^{i-1/2} [\cos(i-1/2)\theta - (i-1/2) \sin\theta \sin(i-3/2)\theta]$$

$$+ \sum_{j=0}^{\infty} B_j r^j [-j \sin\theta \sin(j-1)\theta + 2 \cos(j\theta)]$$

$$+ \sum_{m=0}^{\infty} C_m r^{m-1/2} [(m-1/2) \sin\theta \cos(m-3/2)\theta + 2 \sin(m-1/2)\theta]$$

$$+ \sum_{n=1}^{\infty} D_n r^n [\sin(n\theta) + n \sin\theta \cos(n-1)\theta]$$
(2)

$$\begin{split} \sigma_{yy} &= -\frac{2 \ F}{\pi \ \xi} \ \text{Cos}(\eta + \alpha) \ \text{Sin}^2 \eta \ + \sum_{i=0}^{\infty} \ A_i \ r^{i-1/2} \ [ \ \text{Cos}(i-1/2)\theta \ + (i-1/2) \ \text{Sin}\theta \ \text{Sin}(i-3/2)\theta ] \\ &+ \sum_{j=0}^{\infty} \ B_j \ r^j \ [ \ j \ \text{Sin}\theta \ \text{Sin}(j-1)\theta \ ] \\ &+ \sum_{m=0}^{\infty} \ C_m \ r^{m-1/2} \ [ -(m-1/2) \ \text{Sin}\theta \ \text{Cos}(m-3/2)\theta \ ] \ + \sum_{n=1}^{\infty} \ D_n \ r^n \ [ \ \text{Sin}(n\theta) \ - \ n \ \text{Sin}\theta \ \text{Cos}(n-1)\theta \ ] \\ &\tau_{xy} &= \frac{F}{\pi \xi} \ \text{Cos}(\eta + \alpha) \ \text{Sin}(\ 2\eta \ ) \ + \sum_{i=0}^{\infty} \ A_i \ r^{i-1/2} \ [ -(i-1/2) \ \text{Sin}\theta \ \text{Cos}(\ i-3/2)\theta \ ] \\ &+ \sum_{j=0}^{\infty} \ B_j \ r^j \ [ -j \ \text{Sin}\theta \ \text{Cos}(j-1)\theta \ - \ \text{Sin}(j\theta) \ ] \\ &+ \sum_{m=0}^{\infty} \ C_m \ r^{m-1/2} \ [ -(m-1/2) \ \text{Sin}\theta \ \text{Sin}(m-3/2)\theta \ + \ \text{Cos}(m-1/2)\theta \ ] + \sum_{n=1}^{\infty} \ D_n \ r^n \ [ \ -n \ \text{Sin}\theta \ \text{Sin}(n-1)\theta \ ] \end{aligned} \tag{4}$$

where the stress intensity factors are given by

$$K_{I} = \sqrt{2\pi} A_{0} \quad \text{and} \quad K_{II} = \sqrt{2\pi} C_{0}$$
 (5)

Obviously, this solution violates the boundary conditions of the body described in Fig. 1. The Boussinesq equation satisfies the boundary of the half-plane, but violates the stress-free condition on the crack faces. Also, the generalized Westergaard equations satisfy the stress free condition of the crack faces but violate the stress-free conditions on the boundary of the half-plane. Analysis of results described later

indicate that these boundary violations do not produce large deviations in the stresses in the near field.

#### Deep crack : a/b > 1

As the crack is extended, the interaction of the two singularities weakens, as indicated in Fig. 3b. Indeed, when the crack is sufficiently deep, the loading force becomes remote, and the solution for the stress field around the crack tip is essentially governed by two modes of the generalized Westergaard equations. In this case  $\sigma_{xx}$ ,  $\sigma_{yy}$  and  $\tau_{xy}$  are determined from eq (2), (3) and (4), by simply removing the first (Boussinesq) term.

#### NUMERICAL PROCEDURE

From the stress-optic law [9], the fringe order N is related to the stresses by:

$$N = (h/f_{\sigma})(\sigma_1 - \sigma_2) = (2h/f_{\sigma})\tau_{max}$$
(6)

where h is the model thickness and f is the material fringe value. Recall that:

$$\tau_{\text{max}}^2 = D^2 + T^2 = [Nf_{\text{c}}/(2h)]^2$$
 (7)

where

$$D = (\sigma_{yy} - \sigma_{xx})/2 \qquad \text{and} \qquad T = \tau_{xy}$$
 (8)

Combine eqs (5) to (6), and define a function G as:

$$G_{k}(A_{1},B_{1},C_{m},D_{n}) = (D_{k}^{2} + T_{k}^{2}) - [N_{k}f_{0}/(2 h)]^{2} \equiv 0$$
 (9)

where the subscript k identifies the point selected at position  $(r_k, \theta_k)$  with the fringe order  $N_k$ . If an exact set of coefficients are used,  $G_k \equiv 0$  and the value of  $(D_k^2 + T_k^2)$  coincides with the experimentally determined quantity  $[N_k f_{\infty}/(2h)]^2$ .

The purpose of the analysis is to determine the coefficients  $A_i, B_j, C_m, D_n$  with a numerical procedure that insures accuracy in extracting  $K_I$  and  $K_{II}$  from the isochromatic fringe data  $(N_k, r_k, \theta_k)$ . Since the eq (9) is nonlinear in terms of the unknown coefficients, a numerical approach based on a combination of the Newton Raphson and least squares method [10] is required. In applying this procedure one begins with a truncated series which includes  $A_0, B_0, C_0$ . The best estimates of these three coefficients are obtained and used as first estimates for an expanded series including additional terms. The expansion of the series is continued in steps of two additional terms until divergence occurs and a solution is not possible. In this study, the series was truncated at fifteen terms to avoid divergence for shallow cracks. For deep cracks, with only the Westergaard equations, the series was extended to nineteen terms before divergence occurred.

#### VERIFICATION OF NUMERICAL PROCEDURE

The results for  $K_I$  and  $K_{II}$  from this numerical procedure are verified by statistics and by input-output comparisons. The input-output comparisons are made between the original experimental data and fringe patterns which are reconstructed by using the coefficients. The coefficients  $A_I, B_J$ ,  $C_I$  and  $D_I$  are substituted into eq (6) to determine  $N(r,\theta)$ . Integer order values of N are then plotted to produce reconstructed (theoretical) fringe patterns. The reconstructed fringe patterns are compared with the original data to determine the adequacy of the analysis in determining the coefficients. Typical reconstructed patterns for the shallow and deep cracks are presented in Fig. 5a and 5b, respectively. The dots represent the reconstructed fringes, and the numbers represent the original data points. The relatively close match of the data points with the reconstructed fringes over the region of interest demonstrates that the numerical approach can be used to determine  $K_I$  and  $K_{II}$  for both shallow and deep cracks with reasonable accuracy.

In the data extraction for each model, 400 to 500 data points were selected from the fringe patterns. For the shallow cracks, the data points were distributed over a circular region near the crack tip with 3 < r < 15 mm, giving some preference for points with smaller r. For the deeper cracks, most of the data points were distributed over a fan-like region in front of the crack tip with 3 < r < 10 mm. In addition, a small number of points were collected from the region between the crack tip and the loading point. Twenty subsets of data, 70 points each, were sampled at random from the complete data set and each sub set was analyzed to obtain the best-fit coefficients. The results of these 20 analyses for each experiment including the mean values, standard deviation and coefficient of variation for  $K_{\rm I}$  and  $K_{\rm II}$  are presented in Table 2, 3 and 4.

These results provide a statistical assessment of the adequacy of the numerical process. The numerical results for the shallow crack, presented in Table 2, show the effect of oblique loads with constant model dimensions for a and b. The coefficient of variation ranged from 4.1 to 9.1 per cent for  $K_{I}$  (slightly higher than those in deep crack case), and from 12.9 to 25.8 per cent for  $K_{II}$  (much higher than those in deep crack case). In fact the standard deviations are nearly the same for both shallow and deep crack cases. However, the mean values of  $K_{II}$  are much smaller for the shallow crack case and this elevates