NONLINEAR
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Kuppalapalle Vajravelu
Robert A. Van Gorder

Nonlinear Flow Phenomena and Homotopy Analysis

Fluid Flow and Heat Transfer

流动非线性及其同伦分析

流体力学和传热

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NONLINEAR PHYSICAL SCIENCE

非线性物理科学

NONLINEAR PHYSICAL SCIENCE

Nonlinear Physical Science focuses on recent advances of fundamental theories and principles, analytical and symbolic approaches, as well as computational techniques in nonlinear physical science and nonlinear mathematics with engineering applications.

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Foreword

"The essence of mathematics lies entirely in its freedom"

by Georg Cantor (1845—1918).

Solving nonlinear problems is inherently difficult. Perturbation techniques are mostly used to gain analytic approximations of nonlinear equations. Unfortunately, perturbation methods depend too heavily on small physical parameters, and perturbative results are valid only in cases of weak nonlinearity. Although some non-perturbation techniques were developed to overcome the restrictions of perturbation methods, neither of them can guarantee the convergence of approximation series.

The homotopy analysis method (HAM) is a promising analytic method for highly nonlinear equations, which has been successfully applied in science, applied mathematics, finance, and engineering. Based on the concept of homotopy in topology, the homotopy analysis method is being developed to solve nonlinear problems independent of any small physical parameters. Especially, the HAM introduces a totally new concept "convergence-control" by means of the "convergence-control parameter" that provides a simple way to guarantee the convergence of approximation series. In fact, it is the "convergence-control parameter" that makes the HAM different from all of other analytic methods. As a result, unlike other analytic methods, the HAM is valid for highly nonlinear problems.

Thanks to contributions of many researchers in dozens of countries, the HAM has been developed and modified greatly in theory, and widely applied in many fields. Written by two outstanding scholars, the book "Nonlinear Flow Phenomena and Homotopy Analysis" describes some theoretical developments and new attempts of the HAM, together with typical applications in flow and heat transfer of fluids. Especially, the authors discuss the choice of initial guess, auxiliary linear operator, auxiliary function, convergence-control parameter and so on, from the viewpoints of applied mathematician. The selected examples are fundamental in nature and are important for new users to understand and use the HAM. Obviously, this book is of benefit to the advancement and wide application of the HAM.

Shijun Liao

May 26th 2012

Preface

Over the last decade, the homotopy analysis method has come into prevalence as it allows one to construct reasonably accurate approximations to nonlinear differential equations. A wide variety of mathematical problems, appearing in areas as diverse as fluid mechanics, chemical engineering, biology, finance, theoretical physics and aerospace engineering, have been solved by means of homotopy analysis. Though, the homotopy analysis method is frequently employed in the literature, there are still a number of questions which remain open regarding the method, due in part to the generality of the method.

In the present monograph, we highlight some of the key points which need to be understood by those working in applied mathematics, physics and applied science and engineering in order to apply the homotopy analysis method. That is, the book helps the reader to develop the toolset needed to apply the method, without sifting through the endless literature on the area. Issues of the optimal selection of the auxiliary linear operator, auxiliary function, convergence control parameter, and initial approximations, are discussed heuristically. Furthermore, advanced and less frequently seen methods, such as multiple homotopies and nonlinear auxiliary operations, are discussed. As mentioned above, there are very many applications of the homotopy analysis method in the literature. In selecting applications and specific problems to work through, we have restricted our attention to the fluid phenomena of fluid flow and heat transfer as such problems introduce a wide variety of mathematical problems yet allow for a sufficiently narrow focus. Hence, in order to illustrate various properties and tools useful when applying the homotopy analysis method, we have selected recent research in the area of fluid flow and heat transfer.

We appreciate the support and motivation of prof. A.C.-J. Luo. We also acknowledge the role of Higher Education Press (China) and Springer for making this book a reality. K. V. Prasad helped in typing some of the examples.

Orlando, Florida
2012

Kuppalapalle Vajravelu
Robert A. Van Gorder

Contents

Chapter 1
Introduction

The processes in the world we live in are, more often than not, governed by nonlinearity. Hence, in mathematics, and also in many other sciences in which quantitative models are useful, we often wish to obtain solutions for nonlinear equations. In the field of differential equations, many results pertaining to linear differential equations are well known and have been in existence for quite a while. However, in the area of nonlinear differential equations, there is little in the way of a unifying theory. In many cases, exact solutions for nonlinear differential equations are not to be found, and often we must resort to numerical schemes in order to gain an understanding of a solution to a particular nonlinear equation. When exact or analytical solutions are obtained, one often faces with difficulty of generalizing such results to other nonlinear differential equations.

Due to such difficulties, we frequently seek to obtain approximate solutions to a nonlinear problem, valid over some restricted region in the domain of the original problem. One technique which has shown great promise over the past few years is the homotopy analysis method [1–6]. By use of the method, numerous nonlinear differential equations have been studied in great detail (see, for instance, [7–44]). Like many other perturbation techniques, this method is very useful as it allows us to obtain approximate solutions to nonlinear differential equations. The homotopy analysis method is unique among other perturbation techniques as it allows us to effectively control the region of convergence and rate of convergence of a series solution to a nonlinear differential equation, via control of an initial approximation, an auxiliary linear operator, an auxiliary function and a convergence control parameter.

However, with such great freedom comes the dilemma of deciding just how to proceed. There have been a number of nonlinear differential equations to which the homotopy analysis method has been applied, however the selection of initial approximation, auxiliary linear operator, auxiliary function, and convergence control parameter varies greatly from equation to equation and author to author. That said, there are several underlying themes that become apparent when one examines the literature on the area. Building on such themes, we hope to add some structure and formality to the application of the homotopy analysis method. In particular, we discuss several features of the method and the choices one can make in the selection of the initial approximation, auxiliary linear operator, auxiliary function, and the convergence control parameter. As said in [1],

> "...it is necessary to propose some pure mathematical theorems to direct us to choose the initial approximation, the auxiliary linear operator, and the auxiliary function. These mathematical theorems should be valid in rather general cases without any prior knowledge so that we can apply them without any physical back grounds. Up to now, it is even an open question if such kinds of pure mathematical theorems exist or not."
>
> —— S. J. Liao [1]

We hope that this book helps in achieving this long range goal. We present a number of ways in which one may select the initial approximation, auxiliary linear operator, auxiliary function, and the convergence control parameter when attempting to solve a nonlinear differential equation by the homotopy analysis method. We also focus our attention on the properties of solutions resulting from such a choice of the initial approximation, auxiliary linear operator, auxiliary function, and the convergence control parameter. These choices play a large role in the computational efficiency. Further, we discuss the convergence properties of solutions obtained through the homotopy analysis method.

We primarily discuss nonlinear ordinary differential equations and associated nonlinear operators. However, such discussion is usually general enough to use for solving various nonlinear partial differential equations, as well. We discuss many cases in general while still maintaining applicability of the results to actually computing solutions via the homotopy analysis method. As frequent users of the method, we understand the importance of implementing the presented results.

We note that a good companion to this book will be Liao's original work [1], Beyond Perturbation: Introduction to the Homotopy Analysis Method, which gives some guidelines as to the selection of the auxiliary linear operator, initial guess, auxiliary function, and so on. Knowledge of the first half of this text, in addition to the present article, shall give anyone new to the homotopy analysis method a good idea of how to implement the method.

The outline of the book will be as follows. In Chapters 2 – 4, which comprise the first part of the book, we outline the general method of homotopy analysis. This first set of chapters serves as an outline to the method, which can be directly employed by researchers in engineering, applied physics, and other applied sciences. While the discussion is general, there is no knowledge of advanced mathematics required. We keep the discussion general here, so as to provide such a framework for researchers. In order to give the reader the best preparation for using the method, we realize that often the best way to convey information is through worked examples. Thus, at the end of Chapters 2 – 4 we provide multiple real-world examples of nonlinear equations that have been solved via homotopy analysis, in order to illustrate the theoretical material.

In the second part of the book, Chapters 5 – 6 shift the focus to concrete examples and consider problems in fluid mechanics and heat transfer governed by nonlinear differential equations in order to give real world examples of the application of homotopy analysis. These problems are pulled from the literature, to give a sampling of work in the field. Such specific examples will benefit the reader in seeing how the general methods of Chapters 2 – 4 may be applied to actual problems of physical relevance. We consider such problems in Chapters 5 and 6.

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