

Design and Analysis of Experiments

DOUGLAS C. MONTGOMERY

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Edition



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Design and Analysis of Experiments

Seventh Edition

International Student Version

DOUGLAS C. MONTGOMERY

Arizona State University



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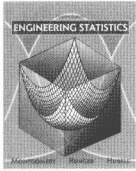
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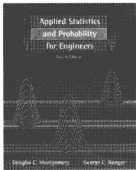
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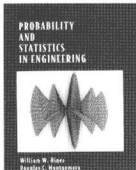
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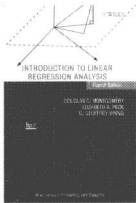


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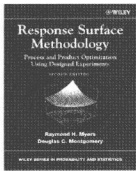
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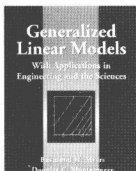


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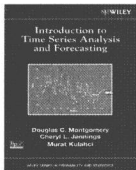


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An introductory text or reference on Generalized Linear Models (GLMs). The range of theoretical topics and applications appeals both to students and practicing professionals.



Introduction to Time Series Analysis and Forecasting

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Methods for modeling and analyzing time series data, to draw inferences about the data and generate forecasts useful to the decision maker. Minitab and SAS are used to illustrate how the methods are implemented in practice. For advanced undergrad/first-year graduate, with a prerequisite of basic statistical methods. Portions of the book require calculus and matrix algebra.

Design and Analysis of Experiments

International Student Version

Preface

Audience

This is an introductory textbook dealing with the design and analysis of experiments. It is based on college-level courses in design of experiments that I have taught over 30 years at Arizona State University, the University of Washington, and the Georgia Institute of Technology. It also reflects the methods that I have found useful in my own professional practice as an engineering and statistical consultant in the many areas of science and engineering, including the transactional research required for successful technology commercialization and product realization.

The book is intended for students who have completed a first course in statistical methods. This background course should include at least some techniques of descriptive statistics, the normal distribution, and an introduction to basic concepts of confidence intervals and hypothesis testing for means and variances. Chapters 10, 11, and 12 require some familiarity with matrix algebra.

Because the prerequisites are relatively modest, this book can be used in a second course on statistics focusing on statistical design of experiments for undergraduate students in engineering, the physical and chemical sciences, mathematics, and other fields of science. For many years I have taught a course from the book at the first-year graduate level in engineering. Students in this course come from all of the fields of engineering, materials science, physics, chemistry, mathematics, operations research, and statistics. I have also used this book as the basis of an industrial short course on design of experiments for practicing technical professionals with a wide variety of backgrounds. There are numerous examples illustrating all of the design and analysis techniques. These examples are based on real-world applications of experimental design and are drawn from many different fields of engineering and the sciences. This adds a strong applications flavor to an academic course for engineers and scientists and makes the book useful as a reference tool for experimenters in a variety of disciplines.

About the Book

The seventh edition is a major revision of the book. I have tried to maintain the balance between design and analysis topics of previous editions; however, there are many new topics and examples, and I have reorganized much of the material. There is much more emphasis on the computer in this edition.

Design-Expert, JMP, and Minitab Software

During the last few years a number of excellent software products to assist experimenters in both the design and analysis phases of this subject have appeared. I have included output from three of these products, Design-Expert, JMP, and Minitab at many points in the text. Minitab and JMP are widely available general-purpose statistical software packages that have good data analysis capabilities and that handle the analysis of experiments with both fixed and random factors (including the mixed model) quite nicely. Design-Expert is a package focused exclusively on experimental design. All three of these packages have many capabilities for construction and evaluation of designs and extensive analysis features. Student versions of Design-Expert and JMP are available as a packaging option with this book, and their use is highly recommended. I urge all instructors who use this book to incorporate computer software into your course. (In my course, I bring a laptop computer and a computer projector to every lecture, and every design or analysis topic discussed in class is illustrated with the computer.) To request this book with the student version of JMP or Design-Expert included, contact your local Wiley representative. You can find your local Wiley representative by going to www.wiley.com/college and clicking on the tab for “Who’s My Rep?”

Empirical Model

I have continued to focus on the connection between the experiment and the model that the experimenter can develop from the results of the experiment. Engineers (and physical and chemical scientists to a large extent) learn about physical mechanisms and their underlying mechanistic models early in their academic training, and throughout much of their professional careers they are involved with manipulation of these models. Statistically designed experiments offer the engineer a valid basis for developing an *empirical* model of the system being investigated. This empirical model can then be manipulated (perhaps through a response surface or contour plot, or perhaps mathematically) just as any other engineering model. I have discovered through many years of teaching that this viewpoint is very effective in creating enthusiasm in the engineering community for statistically designed experiments. Therefore, the notion of an underlying empirical model for the experiment and response surfaces appears early in the book and receives much more emphasis.

Factorial Designs

I have expanded the material on factorial and fractional factorial designs (Chapters 5–9) in an effort to make the material flow more effectively from both the reader’s and the instructor’s viewpoint and to place more emphasis on the empirical model. There is new material on a number of important topics, including follow-up experimentation following a fractional factorial, non-regular and non-orthogonal designs, and small, efficient resolution IV and V designs.

Additional Important Topics

The chapter on response surfaces (Chapter 11) immediately follows the material on factorial and fractional factorial designs and regression modeling. I have expanded Chapter 12 on robust parameter design and process robustness experiments. Chapters 13 and 14 discuss experiments involving random effects and some applications of these concepts to nested and split-plot designs. Because there is expanding industrial interest in these designs, Chapters 13 and 14 have several new topics. Chapter 15 is an overview of important design and analysis topics: nonnormality of the response, the Box–Cox method for selecting the form of a transformation, and other alternatives; unbalanced factorial experiments; the analysis of covariance, including covariates in a factorial design, and repeated measures.

Experimental Design

Throughout the book I have stressed the importance of experimental design as a tool for engineers and scientists to use for product design and development as well as process development and improvement. The use of experimental design in developing products that are robust to environmental factors and other sources of variability is illustrated. I believe that the use of experimental design early in the product cycle can substantially reduce development lead time and cost, leading to processes and products that perform better in the field and have higher reliability than those developed using other approaches.

The book contains more material than can be covered comfortably in one course, and I hope that instructors will be able to either vary the content of each course offering or discuss some topics in greater depth, depending on class interest. There are problem sets at the end of each chapter. These problems vary in scope from computational exercises, designed to reinforce the fundamentals, to extensions or elaboration of basic principles.

Course Suggestions

My own course focuses extensively on factorial and fractional factorial designs. Consequently, I usually cover Chapter 1, Chapter 2 (very quickly), most of Chapter 3, Chapter 4 (excluding the material on incomplete blocks and only mentioning Latin squares briefly), and I discuss Chapters 5 through 8 on factorials and two-level factorial and fractional factorial designs in detail. To conclude the course, I introduce response surface methodology (Chapter 11) and give an overview of random effects models (Chapter 13) and nested and split-plot designs (Chapter 14). I always require the students to complete a term project that involves designing, conducting, and presenting the results of a statistically designed experiment. I require them to do this in teams because this is the way that much industrial experimentation is conducted. They must present the results of this project, both orally and in written form.

The Supplemental Text Material

For the seventh edition I have prepared supplemental text material for each chapter of the book. Often, this supplemental material elaborates on topics that could not be discussed in greater detail in the book. I have also presented some subjects that do not appear directly in the book, but an introduction to them could prove useful to some students and professional practitioners. Some of this material is at a higher mathematical level than the text. I realize that instructors use this book with a wide array of audiences, and some more advanced design courses could possibly benefit from including several of the supplemental text material topics. This material is in electronic form on the World Wide Website for this book, located at www.wiley.com/go/global/montgomery.

Website

Current supporting material for instructors and students is available at the website www.wiley.com/go/global/montgomery. The supplemental text material described above is available at the site, along with electronic versions of data sets used for examples and homework problems, a course syllabus, and some representative student term projects from the course at Arizona State University.

Student Companion Site

The student's section of the textbook website contains the following:

1. The supplemental text material described above
2. Data sets from the book examples and homework problems, in electronic form
3. Sample Student Projects

Instructor Companion Site

The instructor's section of the textbook website contains the following:

4. Solutions to the text problems
5. The supplemental text material described above
6. PowerPoint lecture slides
7. Figures from the text in electronic format, for easy inclusion in lecture slides
8. Data sets from the book examples and homework problems, in electronic form
9. Sample Syllabus
10. Sample Student Projects

The instructor's section is for instructor use only, and is password-protected. Visit the Instructor Companion Site portion of the website, located at www.wiley.com/college/montgomery, to register for a password.

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I express my appreciation to the many students, instructors, and colleagues who have used the six earlier editions of this book and who have made helpful suggestions for its revision. The contributions of Dr. Raymond H. Myers, Dr. G. Geoffrey Vining, Dr. Brad Jones, Dr. Christine Anderson-Cook, Dr. Connie M. Borrer, Dr. Scott Kowalski, Dr. Dennis Lin, Dr. John Ramberg, Dr. Joseph Pignatiello, Dr. Lloyd S. Nelson, Dr. Andre Khuri, Dr. Peter Nelson, Dr. John A. Cornell, Dr. George C. Runger, Dr. Bert Keats, Dr. Dwayne Rollier, Dr. Norma Hubele, Dr. Murat Kulahci, Dr. Cynthia Lowry, Dr. Russell G. Heikes, Dr. Harrison M. Wadsworth, Dr. William W. Hines, Dr. Arvind Shah, Dr. Jane Ammons, Dr. Diane Schaub, Mr. Mark Anderson, Mr. Pat Whitcomb, Dr. Pat Spagon, and Dr. William DuMouche were particularly valuable. My current and former Department Chairs, Dr. Ron Askin and Dr. Gary Hogg, have provided an intellectually stimulating environment in which to work.

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DOUGLAS C. MONTGOMERY
TEMPE, ARIZONA

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