Mathematics and Its Applications

Alexander L. Fradkov,
Iliya V. Miroshnik and
Vladimir O. Nikiforov

Nonlinear and Adaptive Control of Complex Systems

Kluwer Academic Publishers

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by

Alexander L. Fradkov

Institute for Problems of Mechanical Engineering
of the Russian Academy of Sciences,
St Petersburg, Russia

Iliya V. Miroshnik

St Petersburg, State Institute of Fine Mechanics and Optics,
St Petersburg, Russia

and

Vladimir O. Nikiforov

St Petersburg, State Institute of Fine Mechanics and Optics,
St Petersburg, Russia

KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 0-7923-5892-9

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands.

Sold and distributed in North, Central and South America
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands.

Printed on acid-free paper

Printed in the Netherlands.

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Volume 491

PREFACE

This book presents a theoretical framework and control methodology for a class of complex dynamical systems characterized by high state space dimension, multiple inputs and outputs, significant nonlinearity, parametric uncertainty and unmodelled dynamics.

The book starts with an introductory Chapter 1 where the peculiarities of control problems for complex systems are discussed and motivating examples from different fields of science and technology are given.

Chapter 2 presents some results of nonlinear control theory which assist in reading subsequent chapters. The main notions and concepts of stability theory are introduced, and problems of nonlinear transformation of system coordinates are discussed. On this basis, we consider different design techniques and approaches to linearization, stabilization and passification of nonlinear dynamical systems.

Chapter 3 gives an exposition of the Speed-Gradient method and its applications to nonlinear and adaptive control. Convergence and robustness properties are examined. Problems of regulation, tracking, partial stabilization and control of Hamiltonian systems are considered.

In Chapter 4 we introduce the main notions related to the properties of regular hypersurfaces of being an invariant set and nontrivial attractor of a dynamical system. Then, we present a methodology of system analysis in the state space and design tools for solving the problems of equilibrium and set stabilization, as well as tracking control, for nonlinear multivariable systems having several controlling inputs.

In Chapter 5 we study multi-dimensional problems of outputs regulation, coordinating control and curve- (surface-) following, having the evident geometric nature similar to that of the problems considered in Chapter 4. However, unlike the previous parts, the emphasis is here placed on the output space where the majority of the real problems are originally stated.

In Chapter 6 the basic design methods of adaptive, robust adaptive and robust nonlinear control of uncertain plants are presented in the form of universal design tools. Various methodologies (including recursive design, augmented error based design, high-order tuner based design and reduced order reference model design), which allow one to overcome structural obstacles caused by violation of the matching condition or by high relative degree, are considered in the chapter. The practical applicability of the introduced design tools is illustrated by the example of output-feedback control of uncertain single-input/single-output linear systems.

Chapter 7 is devoted to decomposition methods in adaptive control based on separation of slow and fast motions in the system. Convergence and accuracy of decomposition for singularly perturbed and discretized systems are examined. The Speed-Gradient approach to decentralized adaptive control of nonlinear systems is presented.

In Chapter 8 we study applied nonlinear control problems of providing the required spatial motion of complex mechanical systems described by the Newton, Euler and Lagrange equations. The presentation begins with investigating the problem of motion of a rigid body, which is the basis for further consideration of multi-body mechanical systems such as multi-link manipulation robots and multi-drive wheeled mechanisms. Also applications to control of oscillatory mechanical systems, based on the material of Chapter 3, are presented.

Finally, in Chapter 9 the relations between control and physics are discussed. New concepts of "feedback resonance", "excitability index" are introduced with the purpose to better understand behavior of nonlinear nearly conservative systems under feedback action. The Speed-Gradient method of Chapter 3 is applied both to organize resonant system behavior and to reformulate the laws of dynamics for a wide class of physical systems. Applications to escape from a potential well, stabilization of unstable modes, feedback spectroscopy and derivation of the Onzagger principle are given. The chapter outlines a new field of research that may be called *cybernetical physics*.

A unique feature of the authors' approach is the combination of rigorous concepts and methods of modern nonlinear control such as goal sets, invariant and attracting submanifolds, Lyapunov functions, exact linearization and passification, the Kalman-Yakubovich lemma and so on, with approximate decomposition based methodologies related to partial linear approximation, averaging and singular perturbation techniques.

The authors present a number of original concepts and methods: set (submanifold) stabilization and coordinating control, Speed-Gradient control and adaptation algorithms, systems with implicit reference models, simplified robust modifications of high-order tuners and so on. Also some results published previously in the Russian literature and not well known in the West are exposed. Particularly, the book presents the most important results given in the authors' previous publications:

- Fomin, V.N., A.L. Fradkov and V.A. Yakubovich (1981) *Adaptive Control of Dynamic Objects*, Moscow, Nauka, (in Russian);

• Fradkov, A.L. (1990) *Adaptive Control in Complex Systems*, Moscow. Nauka, (in Russian);
• Drozdov, V.N., I.V. Miroshnik and V.I. Scorubsky (1989) *Automatic Control Systems with Microcomputers*, Leningrad, Mashinostroenie, (in Russian);
• Miroshnik, I.V. (1990) *Coordinating Control of Multivariable Systems*. Leningrad, Energoatomizdat, (in Russian);
• *Control of Complex Systems* (1995) Fradkov, A.L. and A.A. Stotsky (Eds.), St.Petersburg, Institute for Problems of Mechanical Engineering;
• *Proceedings of the Laboratory of Cybernetics and Control Systems* (1996) Miroshnik, I.V. and V.O. Nikiforov (Eds.), St.Petersburg, Institute of Fine Mechanics and Optics.

The prospective reader should have some degree of familiarity with standard university courses of calculus, linear algebra and ordinary differential equations. Knowledge of the basic course on linear control theory and the main concepts of differential geometry is also desirable. The book will be useful for researchers, engineers, university lecturers, and postgraduate students specializing in the fields of automatic control, mechanics and applied mathematics.

The efforts of the authors when writing the book have been shared in the following way:

• A.L. Fradkov wrote Chapters 3, 7 and 9, Sections 6.4, 8.4, and Appendix;
• I.V. Miroshnik wrote Chapters 4 and 5, Sections 1.1, 1.2, 8.1-8.3;
• V.O. Nikiforov wrote Chapter 6, Sections 1.3, 7.1 and 7.2.

Chapter 2 was written by all authors in close cooperation.

The authors would like to acknowledge the valuable help of their colleagues associated with the Laboratory "Control of Complex Systems" of the Institute for Problems of Mechanical Engineering of the Russian Academy of Sciences and the Laboratory "Cybernetics and Control Systems" of the Saint-Petersburg State Institute of Fine Mechanics and Optics (Technical University): B.R. Andrievsky, M.V. Druzhinina, P.Yu. Guzenko. S.M. Korolev, A.V. Lyamin, A.Yu. Pogromsky, I.G. Polushin, V.A. Shiegin, A.S. Shiriaev, O.I. Koroleva, K.V. Voronov.

We are also grateful to G.G. Levina, M.I. Miroshnik and D.A. Tomchin for their help in preparation of the manuscript.

The book contains the results of research supported by the Russian Foundation for Basic Research (grants 96-01-01151, 97-01-0432, 99-0100672, 99-01-00761) and by the St. Petersburg Scientific and Educational Center for the Problems of Machine-Building, Mechanics and Control Processes (Project 2.1-589 of the Russian Federal Program "Integration"). The work of A.L. Fradkov was also supported in part by the Dutch Organization for Pure Research (NWO). Important contributions stem from contacts with the students and from the lecture courses delivered by the authors in the Saint-Petersburg State Institute of Fine Mechanics and Optics (Technical University), Baltic State Technical University ("Voenmekh") and the Saint-Petersburg State University in 1995-1998.

Saint-Petersburg, Russia, 1999

NOTATIONS AND DEFINITIONS

Throughout the book we use the following notations and definitions.

The set of real numbers is denoted as $\mathbb{R}$ or $\mathbb{R}^1$, while $\mathbb{R}^n$ stands for the n-dimensional vector space. An element of $\mathbb{R}^n$ is the column vector composed of $x_1, x_2, \ldots, x_n$ and denoted as $x = \text{col}(x_1, x_2, \ldots, x_n)$ or $x = \{x_i\}$, $i = 1, 2, \ldots, n$. The set of positive real numbers and zero is denoted as $\mathbb{R}_+$ or $[0, \infty)$.

Euclidean norm of the vector $x \in \mathbb{R}^n$ is denoted as

$$|x| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} \; .$$

Let $A \in \mathbb{R}^n \times \mathbb{R}^n$ be a real $n \times n$ matrix. The eigenvalues of A are denoted as $\lambda_i\{A\}$, $i = 1, 2, \ldots, n$, and $|A|$ means a *matrix norm* induced by the Euclidean vector norm, i.e.,

$$|A| = \max_i \sqrt{\lambda_i\{A^T A\}} \; .$$

Let P be a symmetric real $n \times n$ matrix and $x^T P x$ is a quadratic form. If $x^T P x > 0$ for any $x \neq 0$, then the matrix P is called *positive definite* and denoted as $P > 0$. Matrices satisfying nonstrict inequality $x^T P x \geq 0$, for all $x \in \mathbb{R}^n$, are called *positive semidefinite* or nonnegative. The notation $|x|_P$ is used for the *weighted Euclidean norm* of x, i.e.,

$$|x|_P = \sqrt{x^T P x} \; .$$

Let $f(t)$ be a measurable vector function defined on $\mathbb{R}_+$, i.e., $f : \mathbb{R}_+ \to \mathbb{R}^n$. The $\mathcal{L}_p$ norm, where $1 \leq p < \infty$, is introduced as

$$||f||_p = \left(\int_0^\infty |f(t)|^p dt \right)^{\frac{1}{p}} ,$$

while $\mathcal{L}_\infty$ norm is defined as

$$||f||_\infty = \text{ess} \sup_t |f(t)|,$$

where "ess sup" is taken over $\mathbb{R}_+$ with possible exception of a set of zero Lebesgue measure. If the norm $||f||_p$ is finite, we write $f \in \mathcal{L}_p$. The spaces of all functions that are globally bounded and square-integrable on $[0, \infty)$ are denoted by $\mathcal{L}_\infty$ and $\mathcal{L}_2$, respectively. The vector space of continuous functions $f : \mathbb{R}_+ \to \mathbb{R}^n$ with the uniform norm

$$||f||_c = \sup_t |f(t)|$$

is denoted $\mathcal{C}[0, \infty)$.

A scalar function $v : \mathbb{R}^n \to \mathbb{R}_+$ is called *positive definite* if $v(0) = 0$ and $v(x) > 0$ for all $x \neq 0$. A scalar function $v : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}$ is called *radially unbounded* if

$$\lim_{|x|\to\infty} \inf_{t\geq 0} v(x,t) \;=\; \infty.$$

A function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is called a *$\mathcal{K}$-function* if it is continuous, strictly increasing and $\gamma(0) = 0$; it is referred to as a *$\mathcal{K}_\infty$-function* if it is a radially unbounded $\mathcal{K}$-function.

The function $f(x,t) : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is called *Lipschitz in x in the set $\mathcal{D} = \mathcal{X} \times \mathcal{T} \subset \mathbb{R}^n \times \mathbb{R}_+$ uniformly in t* if there exists a constant $L = L(\mathcal{X}) > 0$ such that for all $(x,t) \in D$ and $(x^*, t) \in \mathcal{D}$ the following inequality is valid

$$|f(x,t) - (f(x^*,t)| \;\leq\; L\,|x - x^*|, \tag{N.1}$$

and the constant L does not depend on $t \in \mathcal{T}$. The function $f(x,t)$ is called *locally Lipschitz in x uniformly in t* if it is Lipschitz in $\mathcal{D} = \mathcal{X} \times \mathbb{R}_+$ uniformly in t for any compact set $\mathcal{X} \subset \mathbb{R}^n$. Finally, the function $f(x,t)$ is called *globally Lipschitz in x* (or, simply, Lipschitz) if it is Lipschitz in $\mathbb{R}^n \times \mathbb{R}_+$, i.e., inequality (N.1) holds for all $x \in R^n$, $x^* \in R^n$ and $t \geq 0$, while the constant L does not depend on t.

Let $x \in \mathcal{X}$, where $\mathcal{X} \subset \mathbb{R}^n$ is an open set, and a scalar real-valued function $f(x) = f(x_1, x_2, \ldots, x_n)$ is the mapping $\mathcal{X} \to \mathbb{R}$. The function f is a *function of a class* C^k, $k = 1, 2, \ldots, \infty$ (or $f(x) \in C^k$) when it is k times continuously differentiable. It is *smooth* when $f \in C^\infty$ or $f \in C^k$, where k is the necessary order of its derivatives.

The vector function $f(x) = \mathrm{col}(f_1(x), f_2(x), \ldots, f_m(x))$, or the mapping $\mathcal{X} \to \mathbb{R}^m$, is *smooth* ($f \in C^\infty$ or $f \in C^k$ for some large k) when all scalar functions f_j are of the class C^∞ or C^k, respectively.

Let $x \in \mathcal{X}$, $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{Y} \subset \mathbb{R}^m$ be open sets. A *Jacobian matrix* of the smooth vector function $f(x)$, or of the mapping $f : \mathcal{X} \to \mathcal{Y}$, is an $m \times n$

matrix of its partial derivatives $\partial f_j/\partial x_i$ defined as

$$\frac{\partial f}{\partial x} = \begin{vmatrix} \partial f_1/\partial x_1 & \partial f_1/\partial x_2 & \dots & \partial f_1/\partial x_n \\ \partial f_2/\partial x_1 & \partial f_2/\partial x_2 & \dots & \partial f_2/\partial x_n \\ \vdots & \vdots & & \vdots \\ \partial f_m/\partial x_1 & \partial f_m/\partial x_2 & \dots & \partial f_m/\partial x_n \end{vmatrix}.$$

The smooth mapping f is called *nonsingular at the point* $x = x^* \in \mathcal{X}$ when rank $f(x^*) = m$, i.e.,

$$\text{rank } \frac{\partial f}{\partial x}\bigg|_{x^*} = m.$$

Let $m = n$. The smooth mapping $f : \mathcal{X} \to \mathcal{Y}$ is called a *diffeomorphism* when it is one-to-one and there exists a smooth inverse mapping $f^{-1} : \mathcal{Y} \to \mathcal{X}$.

A smooth mapping $f : \mathcal{X} \to \mathbb{R}^m$ that assigns to each point $x \in \mathcal{X} \subset \mathbb{R}^n$ a vector $f \in \mathbb{R}^m$ is called a *smooth vector field.* Let $g_1(x), g_2(x), \dots, g_\nu(x)$ be the smooth vector fields defined on the set $\mathcal{X}$. A mapping $\mathcal{G}(x)$ assigning to each point $x \in \mathcal{X}$ a vector space that spans $g_1, g_2, \dots, g_\nu$, or

$$\mathcal{G}(x) = \text{span}\{g_1(x), g_2(x), \dots, g_\nu(x)\}$$

is called a *smooth distribution* on the set $\mathcal{X}$.

Let $\phi(x) \in C^1$ be a scalar function $\mathbb{R}^n \to \mathbb{R}$. Then $\nabla\phi(x)$ denotes the column vector of its first derivatives calculated as

$$\nabla\phi(x) = \left(\frac{\partial \phi}{\partial x}\right)^T.$$

If $x = \text{col}(x_1, x_2)$ and ϕ is a function of two vector variables, then

$$\nabla_{x_1}\phi(x_1, x_2) = \left(\frac{\partial \phi}{\partial x_1}\right)^T.$$

Let $f(x)$ be a smooth vector field defined on $\mathcal{X} \subset \mathbb{R}^n$. The scalar function $\mathcal{X} \to \mathbb{R}^1$ introduced as

$$\mathcal{L}_f\phi(x) = \frac{\partial \phi}{\partial x} f(x)$$

is a derivative of ϕ along f, often called a (scalar) *Lie derivative.* Let $f(x)$ and $g(x)$ be smooth vector fields defined on $\mathcal{X}$. The mapping $[f, g] : \mathcal{X} \times \mathcal{X} \to \mathbb{R}^n$ (a vector *Lie derivative*) introduced as

$$[f(x), g(x)] = \mathcal{L}_f g(x) = \frac{\partial g}{\partial x} f(x) - \frac{\partial f}{\partial x} g(x)$$

is called a *Lie bracket*. Throughout the book the following notations are also used

$$\mathcal{L}_g\mathcal{L}_f\phi(x) = \frac{\partial}{\partial x}(\mathcal{L}_f\phi)\, g(x), \quad \mathcal{L}_f^k\phi(x) = \frac{\partial}{\partial x}(\mathcal{L}_f^{k-1}\phi)\, g(x)$$

$$\mathrm{ad}_f^0 g(x) = g(x), \quad \mathrm{ad}_f^1 g(x) = [f(x), g(x)], \quad \mathrm{ad}_f^k g(x) = [f(x), \mathrm{ad}_f^{k-1} g(x)],$$

where $k = 2, 3, \ldots$.

A smooth distribution $\mathcal{G}(x) = \mathrm{span}\{g_1(x), g_2(x), \ldots, g_\nu(x)\}$ defined on the set $\mathcal{X}$ is called *nonsingular* when $\dim \mathcal{G}(x) = m = \mathrm{const}$ for all $x \in \mathcal{X}$, and *involutive* when

$$[g_i(x), g_j(x)] \in \mathcal{G}(x)$$

for all vectors $g_i(x), g_j(x) \in \mathcal{G}(x)$.

A polynomial $\beta(p)$ is called *Hurwitz* if all roots of the equation $\beta(p) = 0$ have negative real parts. A real $n \times n$ matrix A is called Hurwitz if all its eigenvalues $\lambda_i\{A\}$, $i = 1, 2, \ldots, n$, have negative real parts.

The *degree* of a polynomial $\beta(p)$ is denoted as $n = \deg\beta(p)$. The *relative degree* of a rational function $\beta(p)/\alpha(p)$ is the integer $\rho = \deg\alpha(p) - \deg\beta(p)$. The rational function is called:

i) *proper* if $\rho \geq 0$;
ii) *strictly proper* if $\rho > 0$;
iii) *minimum phase* if $\beta(p)$ is Hurwitz;
iv) *asymptotically stable* if $\alpha(p)$ is Hurwitz.

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