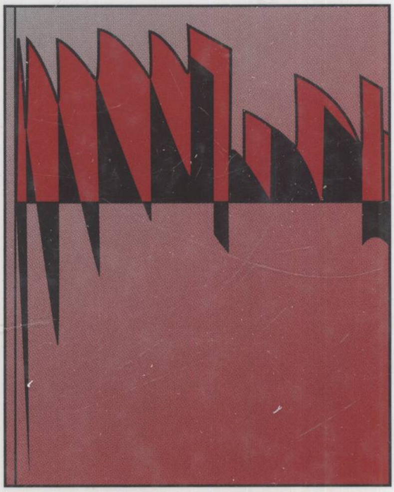

Statistical Inference

Second Edition

George Casella
Roger L. Berger

DUXBURY ADVANCED SERIES

Statistical Inference

Second Edition

George Casella
University of Florida

Roger L. Berger
North Carolina State University

DUXBURY
THOMSON LEARNING

Australia • Canada • Mexico • Singapore • Spain • United Kingdom • United States

DUXBURY
THOMSON LEARNING

Sponsoring Editor: *Carolyn Crockett*
Marketing Representative: *Tom Ziolkowski*
Editorial Assistant: *Jennifer Jenkins*
Production Editor: *Tom Novack*
Assistant Editor: *Ann Day*
Manuscript Editor: *Carol Reitz*
Permissions Editor: *Sue Ewing*
Cover Design: *Jennifer Mackres*
Interior Illustration: *Lori Heckelman*
Print Buyer: *Vena Dyer*
Typesetting: *Integre Technical Publishing Co.*
Cover Printing: *R. R. Donnelley & Sons Co., Crawfordsville*
Printing and Binding: *R. R. Donnelley & Sons Co., Crawfordsville*

For more information about this or any other Duxbury products, contact:
DUXBURY
511 Forest Lodge Road
Pacific Grove, CA 93950 USA
www.duxbury.com
1-800-423-0563 (Thomson Learning Academic Resource Center)

Printed in United States of America

10 9 8 7 6 5 4 3

Library of Congress Cataloging-in-Publication Data

Casella, George.
Statistical inference / George Casella, Roger L. Berger.—2nd ed.
p. cm.
Includes bibliographical references and indexes.
ISBN 0-534-24312-6
1. Mathematical statistics. 2. Probabilities. I. Berger, Roger L. II. Title.
QA276.C37 2001
519.5—dc21

2001025794

THIS BOOK IS PRINTED ON ACID-FREE RECYCLED PAPER

To Anne and Vicki

Duxbury titles of related interest

Daniel, *Applied Nonparametric Statistics* 2nd
Derr, *Statistical Consulting: A Guide to Effective Communication*
Durrett, *Probability: Theory and Examples* 2nd
Graybill, *Theory and Application of the Linear Model*
Johnson, *Applied Multivariate Methods for Data Analysts*
Kuehl, *Design of Experiments: Statistical Principles of Research Design and Analysis* 2nd
Larsen, Marx, & Cooil, *Statistics for Applied Problem Solving and Decision Making*
Lohr, *Sampling: Design and Analysis*
Lunneborg, *Data Analysis by Resampling: Concepts and Applications*
Minh, *Applied Probability Models*
Minitab Inc., *MINITAB™ Student Version 12 for Windows*
Myers, *Classical and Modern Regression with Applications* 2nd
Newton & Harvill, *StatConcepts: A Visual Tour of Statistical Ideas*
Ramsey & Schafer, *The Statistical Sleuth* 2nd
SAS Institute Inc., *JMP-IN: Statistical Discovery Software*
Savage, *INSIGHT: Business Analysis Software for Microsoft® Excel*
Scheaffer, Mendenhall, & Ott, *Elementary Survey Sampling* 5th
Shapiro, *Modeling the Supply Chain*
Winston, *Simulation Modeling Using @RISK*

To order copies contact your local bookstore or call 1-800-354-9706. For more information contact Duxbury Press at 511 Forest Lodge Road, Pacific Grove, CA 93950, or go to: www.duxbury.com

Preface to the Second Edition

Although Sir Arthur Conan Doyle is responsible for most of the quotes in this book, perhaps the best description of the life of this book can be attributed to the Grateful Dead sentiment, "What a long, strange trip it's been."

Plans for the second edition started about six years ago, and for a long time we struggled with questions about what to add and what to delete. Thankfully, as time passed, the answers became clearer as the flow of the discipline of statistics became clearer. We see the trend moving away from elegant proofs of special cases to algorithmic solutions of more complex and practical cases. This does not undermine the importance of mathematics and rigor; indeed, we have found that these have become more important. But the manner in which they are applied is changing.

For those familiar with the first edition, we can summarize the changes succinctly as follows. Discussion of asymptotic methods has been greatly expanded into its own chapter. There is more emphasis on computing and simulation (see Section 5.5 and the computer algebra Appendix); coverage of the more applicable techniques has been expanded or added (for example, bootstrapping, the EM algorithm, p-values, logistic and robust regression); and there are many new Miscellanea and Exercises. We have de-emphasized the more specialized theoretical topics, such as equivariance and decision theory, and have restructured some material in Chapters 3–11 for clarity.

There are two things that we want to note. First, with respect to computer algebra programs, although we believe that they are becoming increasingly valuable tools, we did not want to force them on the instructor who does not share that belief. Thus, the treatment is "unobtrusive" in that it appears only in an appendix, with some hints throughout the book where it may be useful. Second, we have changed the numbering system to one that facilitates finding things. Now theorems, lemmas, examples, and definitions are numbered together; for example, Definition 7.2.4 is followed by Example 7.2.5 and Theorem 10.1.3 precedes Example 10.1.4.

The first four chapters have received only minor changes. We reordered some material (in particular, the inequalities and identities have been split), added some new examples and exercises, and did some general updating. Chapter 5 has also been reordered, with the convergence section being moved further back, and a new section on generating random variables added. The previous coverage of invariance, which was in Chapters 7–9 of the first edition, has been greatly reduced and incorporated into Chapter 6, which otherwise has received only minor editing (mostly the addition of new exercises). Chapter 7 has been expanded and updated, and includes a new section on the EM algorithm. Chapter 8 has also received minor editing and updating, and has a new section on p-values. In Chapter 9 we now put more emphasis on pivoting (having realized that "guaranteeing an interval" was merely "pivoting the cdf"). Also, the material that was in Chapter 10 of the first edition (decision theory) has been reduced, and small sections on loss function optimality of point estimation, hypothesis testing, and interval estimation have been added to the appropriate chapters.

Chapter 10 is entirely new and attempts to lay out the fundamentals of large sample inference, including the delta method, consistency and asymptotic normality, bootstrapping, robust estimators, score tests, etc. Chapter 11 is classic oneway ANOVA and linear regression (which was covered in two different chapters in the first edition). Unfortunately, coverage of randomized block designs has been eliminated for space reasons. Chapter 12 covers regression with errors-in-variables and contains new material on robust and logistic regression.

After teaching from the first edition for a number of years, we know (approximately) what can be covered in a one-year course. From the second edition, it should be possible to cover the following in one year:

Chapter 1:	Sections 1–7	Chapter 6:	Sections 1–3
Chapter 2:	Sections 1–3	Chapter 7:	Sections 1–3
Chapter 3:	Sections 1–6	Chapter 8:	Sections 1–3
Chapter 4:	Sections 1–7	Chapter 9:	Sections 1–3
Chapter 5:	Sections 1–6	Chapter 10:	Sections 1, 3, 4

Classes that begin the course with some probability background can cover more material from the later chapters.

Finally, it is almost impossible to thank all of the people who have contributed in some way to making the second edition a reality (and help us correct the mistakes in the first edition). To all of our students, friends, and colleagues who took the time to send us a note or an e-mail, we thank you. A number of people made key suggestions that led to substantial changes in presentation. Sometimes these suggestions were just short notes or comments, and some were longer reviews. Some were so long ago that their authors may have forgotten, but we haven't. So thanks to Arthur Cohen, Sir David Cox, Steve Samuels, Rob Strawderman and Tom Wehrly. We also owe much to Jay Beder, who has sent us numerous comments and suggestions over the years and possibly knows the first edition better than we do, and to Michael Perlman and his class, who are sending comments and corrections even as we write this.

This book has seen a number of editors. We thank Alex Kugashev, who in the mid-1990s first suggested doing a second edition, and our editor, Carolyn Crockett, who constantly encouraged us. Perhaps the one person (other than us) who is most responsible for this book is our first editor, John Kimmel, who encouraged, published, and marketed the first edition. Thanks, John.

George Casella
Roger L. Berger

Preface to the First Edition

When someone discovers that you are writing a textbook, one (or both) of two questions will be asked. The first is "Why are you writing a book?" and the second is "How is your book different from what's out there?" The first question is fairly easy to answer. You are writing a book because you are not entirely satisfied with the available texts. The second question is harder to answer. The answer can't be put in a few sentences so, in order not to bore your audience (who may be asking the question only out of politeness), you try to say something quick and witty. It usually doesn't work.

The purpose of this book is to build theoretical statistics (as different from mathematical statistics) from the first principles of probability theory. Logical development, proofs, ideas, themes, etc., evolve through statistical arguments. Thus, starting from the basics of probability, we develop the theory of statistical inference using techniques, definitions, and concepts that are statistical and are natural extensions and consequences of previous concepts. When this endeavor was started, we were not sure how well it would work. The final judgment of our success is, of course, left to the reader.

The book is intended for first-year graduate students majoring in statistics or in a field where a statistics concentration is desirable. The prerequisite is one year of calculus. (Some familiarity with matrix manipulations would be useful, but is not essential.) The book can be used for a two-semester, or three-quarter, introductory course in statistics.

The first four chapters cover basics of probability theory and introduce many fundamentals that are later necessary. Chapters 5 and 6 are the first statistical chapters. Chapter 5 is transitional (between probability and statistics) and can be the starting point for a course in statistical theory for students with some probability background. Chapter 6 is somewhat unique, detailing three statistical principles (sufficiency, likelihood, and invariance) and showing how these principles are important in modeling data. Not all instructors will cover this chapter in detail, although we strongly recommend spending some time here. In particular, the likelihood and invariance principles are treated in detail. Along with the sufficiency principle, these principles, and the thinking behind them, are fundamental to total statistical understanding.

Chapters 7–9 represent the central core of statistical inference, estimation (point and interval) and hypothesis testing. A major feature of these chapters is the division into methods of *finding* appropriate statistical techniques and methods of *evaluating* these techniques. Finding and evaluating are of interest to both the theorist and the practitioner, but we feel that it is important to separate these endeavors. Different concerns are important, and different rules are invoked. Of further interest may be the sections of these chapters titled Other Considerations. Here, we indicate how the rules of statistical inference may be relaxed (as is done every day) and still produce meaningful inferences. Many of the techniques covered in these sections are ones that are used in consulting and are helpful in analyzing and inferring from actual problems.

The final three chapters can be thought of as special topics, although we feel that some familiarity with the material is important in anyone's statistical education. Chapter 10 is a thorough introduction to decision theory and contains the most modern material we could include. Chapter 11 deals with the analysis of variance (oneway and randomized block), building the theory of the complete analysis from the more simple theory of treatment contrasts. Our experience has been that experimenters are most interested in inferences from contrasts, and using principles developed earlier, most tests and intervals can be derived from contrasts. Finally, Chapter 12 treats the theory of regression, dealing first with simple linear regression and then covering regression with "errors in variables." This latter topic is quite important, not only to show its own usefulness and inherent difficulties, but also to illustrate the limitations of inferences from ordinary regression.

As more concrete guidelines for basing a one-year course on this book, we offer the following suggestions. There can be two distinct types of courses taught from this book. One kind we might label "more mathematical," being a course appropriate for students majoring in statistics and having a solid mathematics background (at least $1\frac{1}{2}$ years of calculus, some matrix algebra, and perhaps a real analysis course). For such students we recommend covering Chapters 1–9 in their entirety (which should take approximately 22 weeks) and spend the remaining time customizing the course with selected topics from Chapters 10–12. Once the first nine chapters are covered, the material in each of the last three chapters is self-contained, and can be covered in any order.

Another type of course is "more practical." Such a course may also be a first course for mathematically sophisticated students, but is aimed at students with one year of calculus who may not be majoring in statistics. It stresses the more practical uses of statistical theory, being more concerned with understanding basic statistical concepts and deriving reasonable statistical procedures for a variety of situations, and less concerned with formal optimality investigations. Such a course will necessarily omit a certain amount of material, but the following list of sections can be covered in a one-year course:

Chapter	Sections
1	All
2	2.1, 2.2, 2.3
3	3.1, 3.2
4	4.1, 4.2, 4.3, 4.5
5	5.1, 5.2, 5.3.1, 5.4
6	6.1.1, 6.2.1
7	7.1, 7.2.1, 7.2.2, 7.2.3, 7.3.1, 7.3.3, 7.4
8	8.1, 8.2.1, 8.2.3, 8.2.4, 8.3.1, 8.3.2, 8.4

9	9.1, 9.2.1, 9.2.2, 9.2.4, 9.3.1, 9.4
11	11.1, 11.2
12	12.1, 12.2

If time permits, there can be some discussion (with little emphasis on details) of the material in Sections 4.4, 5.5, and 6.1.2, 6.1.3, 6.1.4. The material in Sections 11.3 and 12.3 may also be considered.

The exercises have been gathered from many sources and are quite plentiful. We feel that, perhaps, the only way to master this material is through practice, and thus we have included much opportunity to do so. The exercises are as varied as we could make them, and many of them illustrate points that are either new or complementary to the material in the text. Some exercises are even taken from research papers. (It makes you feel old when you can include exercises based on papers that were new research during your own student days!) Although the exercises are not subdivided like the chapters, their ordering roughly follows that of the chapter. (Subdivisions often give too many hints.) Furthermore, the exercises become (again, roughly) more challenging as their numbers become higher.

As this is an introductory book with a relatively broad scope, the topics are not covered in great depth. However, we felt some obligation to guide the reader one step further in the topics that may be of interest. Thus, we have included many references, pointing to the path to deeper understanding of any particular topic. (The *Encyclopedia of Statistical Sciences*, edited by Kotz, Johnson, and Read, provides a fine introduction to many topics.)

To write this book, we have drawn on both our past teachings and current work. We have also drawn on many people, to whom we are extremely grateful. We thank our colleagues at Cornell, North Carolina State, and Purdue—in particular, Jim Berger, Larry Brown, Sir David Cox, Ziding Feng, Janet Johnson, Leon Gleser, Costas Goutis, Dave Lansky, George McCabe, Chuck McCulloch, Myra Samuels, Steve Schwager, and Shayle Searle, who have given their time and expertise in reading parts of this manuscript, offered assistance, and taken part in many conversations leading to constructive suggestions. We also thank Shanti Gupta for his hospitality, and the library at Purdue, which was essential. We are grateful for the detailed reading and helpful suggestions of Shayle Searle and of our reviewers, both anonymous and non-anonymous (Jim Albert, Dan Coster, and Tom Wehrly). We also thank David Moore and George McCabe for allowing us to use their tables, and Steve Hirdt for supplying us with data. Since this book was written by two people who, for most of the time, were at least 600 miles apart, we lastly thank Bitnet for making this entire thing possible.

George Casella
Roger L. Berger

"We have got to the deductions and the inferences," said Lestrade, winking at me. "I find it hard enough to tackle facts, Holmes, without flying away after theories and fancies."

Inspector Lestrade to Sherlock Holmes

The Boscombe Valley Mystery

Contents

List of Tables