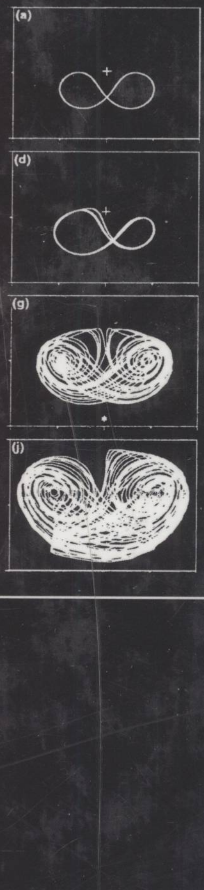
NONLINEAR INTERACTIONS

Analytical, Computational, and Experimental Methods



Ali H. Nayfeh

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NONLINEAR INTERACTIONS
Analytical, Computational, and Experimental Methods

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To the Memory of My Parents
Hasan and Khadrah
and to My Wife
Samirah
and Our Grandchildren
Ali, Layanne, and Ahmad

PREFACE

An understanding of the dynamic characteristics of a structural system is essential for its design and control. Many of the important characteristics can only be modeled by nonlinear governing equations. When the governing equations are nonlinear, the system and the characteristics of the solutions are also said to be nonlinear. There is a vast range of interesting, important, and potentially dangerous phenomena that are nonlinear. Nonlinearities can have important influences even while the amplitudes of the response are quite small (Nayfeh and Mook, 1979).

As an example, modeling a system that is subjected to a parametric excitation by linear equations and boundary conditions is unrealistic if the parametric excitation leads to instability because such a model predicts that the growth of the response is exponential. Consequently, it would be more realistic to include nonlinear terms, which limit the predicted response. Moreover, the linear model may predict stability (i.e., a decaying response), but the actual response may not decay under certain conditions. In this case, the parametric excitation produces a so-called subcritical instability that is only predictable by a nonlinear model.

Nonlinearity brings a whole range of phenomena that are not found in linear systems. In single-degree-of-freedom systems, these include multiple solutions and jumps; limit cycles; frequency entrainment; natural-frequency shifts; subharmonic, superharmonic, combination, and ultrasubharmonic resonances; period-multiplying and demultiplying bifurcations; and chaos. The devastating consequences of having one harmonic load with a frequency near the natural frequency might be lowered to a tolerable level by simply adding one or more nonresonant harmonic loads that produce a shift in the natural frequency. The large response produced by a primary resonant excitation can also be reduced significantly by simply adding other superharmonic-resonant loads having the proper amplitudes and phases. When the nonlinear terms are cubic and the sum of three frequencies in a multifrequency load equals or nearly equals the natural frequency of a system, it can experience a combination-resonant response in which the peak amplitudes are several times larger and appear more often than those predicted by linear theory. For such a response, the actual fatigue life can be much lower than what was predicted.

For multidegree-of-freedom and continuous systems, another example of a nonlinear phenomenon is an interaction among different modes, which can result in an energy exchange among them. These nonlinear interactions are the subject of this book. What makes the exchange of energy among modes dangerous is that typically energy is transferred from the low-amplitude high-frequency components of the motion associated with the high modes to the high-amplitude low-frequency components of the motion associated with the low modes. Thus, the modal interaction makes it possible for a high-frequency low-amplitude excitation, which is capable of doing a lot of work on the structure in a short period, to produce a large-amplitude low-frequency response. In the absence of modal interactions, the steady-state response of a damped structure will consist of only the directly excited modes. While the energy exchange may be dangerous, it can be useful if the energy can be transferred from the desired system to a secondary system, as in the case of autoparametric absorbers.

Nonlinear modal interactions have been the subject of a great deal of recent research. It has been found that, in weakly nonlinear systems where there exists a special relationship between two or more natural frequencies and an excitation frequency, the long-time response can contain large contributions from many linear modes (Nayfeh and Mook, 1979; Nayfeh and Balachandran, 1989; Sado, 1993; Ruijgrok, 1995). The large presence of more than one mode generally makes the actual response more complicated, increases the number of modal equations that must be treated, and causes the analysis to be more difficult. Modal interactions can lead to dangerously large responses in the very modes that are insignificant according to linear analysis. Consequently, the use of classical transfer functions and modal analysis techniques is inappropriate and may lead to erroneous conclusions about the system (Busby, Nopporn, and Singh, 1986; Zavodney, 1987a, 1991; Balachandran, Nayfeh, Smith, and Pappa, 1994).

Most of the research on modal interactions is focused on autoparametric, also called internal, resonances in systems where the linear natural frequencies ω_i are commensurate or nearly commensurate; that is, the ω_i are related by $\sum_{i=1}^{n} k_i\omega_i \approx 0$, where the k_i are positive or negative integers. This relationship is called an internal or autoparametric resonance condition. The vector $\mathbf{k} = \{k_1, k_2, \cdots, k_n\}$ is called the resonance vector, whereas the number $k = |k_1| + |k_2| + \cdots + |k_n|$ is called the order of the internal resonance. Internal or autoparametric resonances may be activated during free as well as driven oscillations and are responsible for the redistribution of energy among the various natural modes. Autoparametric resonances have been treated successfully with perturbation methods. There also exists a large body of experimental results which are in good general agreement with the perturbation results. Autoparametric resonances may provide coupling and energy exchanges among the modes. Consequently, exciting a high-frequency mode may produce a large-amplitude response in a low-frequency mode involved with it in an autoparametric resonance. Autoparametric resonances are discussed in Chapters 2-5.

In externally excited multidegree-of-freedom and continuous systems, combination resonances may occur in response to a simple-harmonic excitation. Consequently, a high-frequency excitation may produce large-amplitude responses in the low-frequency modes that participate in the combination resonance. In parametrically excited systems, multimode interactions can occur when the excitation frequency is near the sum or difference of two or more linear natural frequencies. These so-called combination resonances have been studied extensively in the literature (Evan-Iwanowski, 1976; Nayfeh and Mook, 1979). Again, these combination resonances can lead to interactions between high- and low-frequency modes. Combination resonances are discussed in Chapter 5.

Several recent experimental studies suggest that another type of interaction may occur between high- and low-frequency modes. In these experiments, a high-frequency mode was directly excited either parametrically or externally, yet the response contained a large contribution from the first mode. The presence of the first mode was accompanied by a slow modulation of the amplitude and phase of the high-frequency mode with the frequency of the modulation being nearly equal to the natural frequency associated with the first mode. The results indicate that the mechanism for the excitation of the first mode is neither a classical internal resonance nor an external or a parametric combination resonance involving the first mode. Rather, the appearance of the first mode is accompanied by a slow modulation of the high-frequency modes. This mechanism is the subject of Chapter 6.

The interaction between high- and low-frequency modes observed experimentally is of great practical importance. In many manufacturing structures and dwellings, high-frequency excitations can be caused by rotating machinery; in large floating structures, high-frequency excitations can be caused by waves; in ships, high-frequency excitations can be caused by the propeller blades passing the rudder; etc. Through the mechanisms discussed above, energy from high-frequency sources can be transferred to the low-frequency modes of supporting structures or foundations, and the result can be harmful large-amplitude oscillations. For example, certain parts of an airplane can be violently excited by an engine running at an angular speed that is much larger than their natural frequencies (von Kármán, 1940). Moreover, some preliminary results (S. Nayfeh and Nayfeh, 1993) indicate that use of conventional methods for decreasing modal interactions, such as increasing the dissipation or decreasing the forcing amplitude, may have undesirable effects. The interaction between high- and low-frequency modes is discussed in Chapter 6.

Modern flexible structures typically have many modes with low natural frequencies. Because these structures are rather flexible, large-amplitude vibrations may occur, and geometric and other nonlinearities become significant. The nonlinearities can couple the modes and produce strong, often dangerous, exchanges of energy between modes. As discussed in Chapters 2-5, modal interactions are greatly enhanced when a special relationship between the natural frequencies of two or more modes and the excitation frequency exists. The nature of these frequency relationships depends on the degree of the nonlinearity present in the system, the number of modes involved, and the character of the excitation.

In Chapter 7, we discuss interactions resulting from the simultaneous presence of more than one mechanism, such as two two-to-one internal resonances involving three modes, a two-to-one and a combination internal resonance, a one-to-one and a two-to-one internal resonance, two combination resonances involving three modes, a one-to-one internal resonance and widely spaced modes, and two one-to-one internal resonances involving three modes, which are in turn involved in a two-to-one resonance with a fourth mode. Lefschetz (1956) described a commercial airplane in which the propellers induced a subharmonic vibration of order one-half in the wings, which in turn induced a subharmonic vibration of order one-half in the rudder. The oscillations were so violent that the airplane broke up.

In Chapter 8, we discuss the construction of nonlinear normal modes of discrete and continuous systems. We start with a discussion of the notions of nonlinear normal modes. Then, we describe methods for constructing nonlinear normal modes for discrete as well as continuous systems with quadratic and cubic nonlinearities. We describe the method of multiple scales, the method of normal forms, the real-valued invariant-manifold approach, the complex-valued invariant-manifold approach, and an energy-based formulation.

Interactions between internal and parametric resonances are discussed throughout the book. Most of the cited references appeared in the English literature. For a review of the research in Japan on nonlinear oscillations of elastic structures, we refer the reader to Yasuda (1996).

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