

Tracts in Operations Research

Linear Programming and Theory of Games

P. K. GUPTA
MAN MOHAN

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LINEAR PROGRAMMING

AND

THEORY OF GAMES

(A Textbook for Students of Mathematics, Statistics, Engineering
and Management of all Indian Universities)

Second Revised Edition : July, 1978
Third Revised Edition : November, 1979

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LINEAR PROGRAMMING AND THEORY OF GAMES

[By P. K. Gupta & Man Mohan]

Notes of the Reader

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LINEAR PROGRAMMING AND THEORY OF GAMES



BOOK REVIEW (of first edition)

"This book is primarily of interest to students offering the paper in linear programming in Indian universities/institutions. It is divided into eleven chapters with an introductory chapter and an appendix. The introductory chapter and the appendix deal with the mathematical background on vector and matrix algebra needed for the development of the subject. Chapter 1 discusses graphical methods of solving two variable linear programming problems. There is no attempt to solve problems in three variables by graphical methods."

"The formulation of linear programming problems has been considered in Chapter 2. Chapters 3 and 4 discuss simplex algorithm, including Charnes method. Chapter 5 deals with degeneracy and cycling in linear programming. The fundamental theorem of duality and dual simplex methods are discussed in Chapter 6. The sensitivity analysis and the revised simplex algorithm are discussed in Chapters 7 and 8, while Chapters 9 and 10 discuss applications of linear programming to transportation, assignment and game theory."

"The book does not mention the advances that have been made in the last decade in developing new algorithms presumably because the book is not intended for research students. As a textbook in Indian universities and institutions, the book is very highly recommended. The book may also be of interest to practitioners interested in using linear programming to real-life problems."

— from OPSEARCH, VOL. 12 (1975), pp. 49-50.

Books by the same Authors :

1. **Operations Research** by Kanti Swarup, P. K. Gupta & Man Mohan
(Second Revised Edition : October, 1978)
2. **Problems in Operations Research Methods and Solutions** by P.K. Gupta &
Man Mohan
(First Edition : December, 1979)
3. **Introduction to Operations Research and Data Analysis** by P.K. Gupta &
(In Press) Man Mohan
4. **Linear Algebra** by P.K. Gupta & Man Man Mohan
(Under Preparation)
5. **Non-Linear and Dynamic Programming** by P.K. Gupta & Man Mohan
(Under Preparation)

उपदेनव शास्त्रं कुरुते धीमती यतः ।
तत्तु प्राप्यैव बिस्तारं स्ववमेवोपगच्छति ॥
जले तेलं खले गुह्यं पात्रे दामं मनागपि ।
प्राप्ते शास्त्रं स्वयं याति बिस्तारं वस्तुशक्ति ॥

"A little instruction and guidance in Science is sufficient for the intelligent student, for this alone will help him develop his knowledge of his own accord. Science instilled into the intelligent mind has sufficient vitality in it to grow and expand by its own force even as a drop of oil on a sheet of water, a piece of secret confined to a villian, or a little act of charity to the deserving person."

—Bhaskara



Dedicated
to
our Parents

TO THE THIRD EDITION

It is indeed encouraging for the authors to know that there is a pressing demand from esteemed readers for a new edition of the book *the second edition of which was released only a few months ago*. That the book has been found useful by the teachers and students at practically all major universities of India, is a matter of great satisfaction and encouragement for the authors. We express our sense of deep gratification at the universal appreciation of the value of the book by teachers of the subject and its students. We are happy to place in the hands of our readers the *revised and improved third edition* of the book. Some of the salient features of the new edition are :

● Nearly all the chapters are rewritten to make the treatment more exhaustive, rigorous and up-to-date.

● Some new typical problems have been added as solved examples and many old stereotype problems have been replaced by new up-to-date problems.

● Solved examples and unsolved problems have been added from latest (up to 1979) University examinations in India.

● A complete discussion of the following new topics has been included :

- (a) *Origin and development, requirements, scope, limitations, advantages and examples of Linear Programming Applications.*
(New Chapter 0)
- (b) *Fundamental Theorem of Game Theory* (in Chapter 8)
- (c) *Routing Problem* (in Chapter 10)
- (d) *Integer Programming* (in Chapter 11)
- (e) *Bounded Variable Programming* (New Chapter 12)
- (f) *Linear Fractional Programming* (New Chapter 13)
- (g) *Goal Programming, Stochastic Programming, Extreme Point Mathematical Programming, Flexible Programming and Decomposable Linear Programming.* (New Chapter 14)
- (h) *Non-Linear Programming* (New Chapter 15)
- (i) *A Computer Program for Simplex Method* (New Appendix)

● In addition, many new sections have been added at various places to fill the existing gaps. All algorithms are summarised in the *step-by-step* manner followed by the *Flow-Chart* for these steps.

It is hoped that the book will be found more useful in its present form.

We are grateful to Professor U.N. Singh, Pro-Vice-Chancellor, University of Delhi, Professor B.S. Yadav, Dean, Faculty of Mathematics, University of Delhi, and Professor R. N. Kaul, Department of Mathematics, University of Delhi, for their constant encouragement. A special word of appreciation is also due to Professor Kanti Swarup, I.I.P.A., of New Delhi, for his help and useful suggestions.

Finally we wish to thank M/s Sultan Chand & Sons, New Delhi, for their untiring efforts in bringing out this edition with speed and efficiency.

Suggestions and comments for further improvement shall be highly appreciated and duly acknowledged.

Delhi

November 13, 1979

P.K. GUPTA

MAN MOHAN

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(Third Revised Edition)

P.K. GUPTA ● MAN MOHAN

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Yours sincerely,

Prakash Chand

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