Progress in Systems and Control Theory

Systems and Control in the Twenty-First Century

Christopher I. Byrnes
Biswa N. Datta
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Editors

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Preface

The mathematical theory of networks and systems has a long, and rich history, with antecedents in circuit synthesis and the analysis, design and synthesis of actuators, sensors and active elements in both electrical and mechanical systems. Fundamental paradigms such as the state-space realization of an input/output system, or the use of feedback to prescribe the behavior of a closed-loop system have proved to be as resilient to change as were the practitioners who used them.

This volume celebrates the resiliency to change of the fundamental concepts underlying the mathematical theory of networks and systems. The articles presented here are among those presented as plenary addresses, invited addresses and minisymposia presented at the 12th International Symposium on the Mathematical Theory of Networks and Systems, held in St. Louis, Missouri from June 24 - 28, 1996. Incorporating models and methods drawn from biology, computing, materials science and mathematics, these articles have been written by leading researchers who are on the vanguard of the development of systems, control and estimation for the next century, as evidenced by the application of new methodologies in distributed parameter systems, linear nonlinear systems and stochastic systems for solving problems in areas such as aircraft design, circuit simulation, imaging, speech synthesis and visionics.

We wish to thank these authors, and all the contributors to MTNS-96, for making this conference an outstanding intellectual celebration of this area and its ability to embrace and lead paradigm shifts, a celebration which will continue to grow in importance as we enter the next century. We also take great pleasure in thanking Rose Brower, Bijoy Ghosh, Michalina Karina, Susan McLaughlin, Giorgio Picci, Beth Scnettler, Elizabeth SoRelle and Sue Schenker for years of outstanding service to MTNS-96. This endeavor could not have succeeded without their dedication.

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Estimation Techniques for Distributed Parameter Systems
H.T. Banks and K. Kunisch

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Jean-Pierre Aubin and Hélène Frankowska

Weak Convergence Methods and Singularly Perturbed Stochastic Control and Filtering Problems
Harold J. Kushner

Methods of Algebraic Geometry in Control Theory: Part I
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Peter Falb

H^∞-Optimal Control and Related Minimax Design Problems
Tamer Başar and Pierre Bernhard

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Han-Fu Chen and Lei Guo

Viability Theory
Jean-Pierre Aubin

Representation and Control of Infinite Dimensional Systems, Vol. I
A. Bensoussan, G. Da Prato, M. C. Delfour and S. K. Mitter

Representation and Control of Infinite Dimensional Systems, Vol. II
A. Bensoussan, G. Da Prato, M. C. Delfour and S. K. Mitter

Mathematical Control Theory: An Introduction
Jerzy Zabczyk

H_∞-Control for Distributed Parameter Systems: A State-Space Approach
Bert van Keulen

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Alexander Asachenkov, Guri Marchuk, Ronald Mohler, Serge Zuev

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Michail I. Zelikin and Vladimir F. Borisov

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J. E. Lagnese, Günter Leugering, E. J. P. G. Schmidt

First Order Representations of Linear Systems
Margreet Kuijper

Hierarchical Decision Making in Stochastic Manufacturing Systems
Suresh P. Sethi and Qing Zhang

Optimal Control Theory for Infinite Dimensional Systems
Xunjing Li and Jiongmin Yong

Generalized Solutions of First-Order PDEs: The Dynamical Optimization Process
Andreĭ I. Subbotin

Finite Horizon H_∞ and Related Control Problems
M. B. Subrahmanyam

Control Under Lack of Information
A. N. Krasovskii and N. N. Krasovskii

H^∞-Optimal Control and Related Minimax Design Problems
A Dynamic Game Approach
Tamer Başar and Pierre Bernhard

Control of Uncertain Sampled-Data Systems
Geir E. Dullerud

Robust Nonlinear Control Design: State-Space and
Lyapunov Techniques
Randy A. Freeman and Petar V. Kokotović

Adaptive Systems: An Introduction
Iven Mareels and Jan Willem Polderman

Sampling in Digital Signal Processing and Control
Arie Feuer and Graham C. Goodwin

Ellipsoidal Calculus for Estimation and Control
Alexander Kurzhanski and István Vályi

Minimum Entropy Control for Time-Varying Systems
Marc A. Peters and Pablo A. Iglesias

Chain-Scattering Approach to H^∞-Control
Hidenori Kimura

Systems and Control in the Twenty-First Century
Christopher I. Byrnes, Biswa N. Datta, Clyde F. Martin, and David S. Gilliam

Contents

State Space Method for Inverse Spectral Problems

D. Alpay and I. Gohberg

1 Introduction

Let H denote a differential operator of the form

$$(Hf)(t) = -iJ\frac{\mathrm{d}\, f}{\mathrm{d}\, t}(t) - V(t)f(t), \quad t \geq 0, \tag{1.1}$$

where

$$J = \begin{pmatrix} I_m & 0 \\ 0 & -I_m \end{pmatrix} \quad \text{and} \quad V(t) = \begin{pmatrix} 0 & k(t) \\ k(t)^* & 0 \end{pmatrix}. \tag{1.2}$$

Here, $k(t)$ is a $\mathbb{C}^{m\times m}$–valued function with entries in $L_1(0,\infty)$. It is sometimes called the potential of the differential operator, or the local reflexivity coefficient function (see [10] for this latter interpretation). Associated to the operator H are two important functions: the scattering function and the spectral function.

To define the scattering function, consider for real λ the $\mathbb{C}^{2m\times m}$–valued solution of the equation

$$-iJ\frac{\mathrm{d}}{\mathrm{d}\, t}X(t,\lambda) - V(t)X(t,\lambda) = \lambda X(t,\lambda), \tag{1.3}$$

subject to the boundary conditions

$$(I_m \ \ -I_m)X(0,\lambda) = 0, \qquad (I_m \ 0)X(t,\lambda) = e^{-i\lambda t}I_m + o(1) \ \ (t\rightarrow\infty).$$

Such a solution exists and is unique (see [20], [11]). It has the further property that there exists a $\mathbb{C}^{m\times m}$ matrix $S(\lambda)$ such that

$$(0 \ I_m)X(t,\lambda) = S(\lambda)e^{i\lambda t} + o(1) \ \ (t\rightarrow\infty).$$

The function $\lambda \mapsto S(\lambda)$ is called the scattering matrix function and it belongs to the Wiener algebra $\mathcal{W}^{m\times m}$. Recall that this algebra $\mathcal{W}^{m\times m}$ consists of the matrix–valued functions of the form

$$Z(\lambda) = D - \int_{-\infty}^{\infty} z(t)e^{i\lambda t}dt \tag{1.4}$$

where $D \in \mathbb{C}^{m\times m}$ and $z \in L_1^{m\times m}(\mathbb{R})$. Note that $D = \lim_{\lambda\rightarrow\pm\infty} Z(\lambda)$; we will use the notation $D = Z(\infty)$.

The scattering function S has the following properties: it takes unitary values, belongs to $\mathcal{W}^{m\times m}$, $S(\infty) = I_m$ and it admits a Wiener–Hopf factorization:

$$S(\lambda) = S_-(\lambda)S_+(\lambda), \tag{1.5}$$

where S_- and its inverse are in $\mathcal{W}_-^{m\times m}$, and S_+ and its inverse are in $\mathcal{W}_+^{m\times m}$. Here the subalgebra $\mathcal{W}_-^{m\times m}$ consists of the elements of the form (1.4) for which the support of $z(t)$ is in $\mathbb{R}_-$, and $\mathcal{W}_+^{m\times m}$ consists of the elements of the form (1.4) for which the support of $z(t)$ is in $\mathbb{R}_+$.

The inverse scattering problem consists in recovering the function k (and hence the potential) from the scattering function S. There is a rich literature about this problem. We follow the approach suggested in [18], [19], [20].

We now turn to the spectral function. The operator H defined by (1.1) is selfadjoint when restricted to the space D_H of $\mathbb{C}^{2m}$–valued functions f which are absolutely continuous and which satisfy the initial value $(I_m \ -I_m)f(0) = 0$. Let W be a $\mathbb{C}^{m\times m}$–valued function which is continuous on the real line and for which $W(\lambda) > 0$ for all real λ. It is called a spectral function for the operator H if there is a unitary mapping $U : L_2^{2m}(0,\infty) \to L_2^m(W)$ such that $(UHf)(\lambda) = \lambda(Uf)(\lambda)$ for $f \in D_H$, where $L_2^m(W)$ is the Hilbert space of $\mathbb{C}^m$–valued measurable functions g such that $\int_{-\infty}^{\infty} g(t)^*W(t)g(t)dt < \infty$. If S given by (1.5) is the scattering function of the operator (1.1), then the function

$$W(\lambda) = S_-(\lambda)^{-1}S_-(\lambda)^{-*} \tag{1.6}$$

is a spectral function of H, and the map U is given in terms of the continuous orthogonal polynomials of M.G. Krein (see [17], [11]). The definitions of these functions and of the map U are given in the next section. We will call this function the spectral function of the operator H; it is uniquely determined from the scattering function S and the condition $W(\infty) = I_m$. Let $W \in \mathcal{W}^{m\times m}$, with $W(\infty) = I_m$. The function W admits Wiener–Hopf factorizations $W = W_+W_+^* = W_-W_-^*$, where W_- and its inverse are in $\mathcal{W}_+^{m\times m}$ and W_- and its inverse are in $\mathcal{W}_-^{m\times m}$. The function W is the spectral function of the differential operator (1.1) with scattering matrix–function $S = W_-^{-1}W_+$. The inverse spectral problem consists of recovering the function k from the spectral function W.

We will also consider the case where the reflection coefficient matrix function $R(\lambda)$ is known and rational. We recall that $R(\lambda) = X_{21}\,(0,\lambda)X_{11}\,(0,\lambda)^{-1}$ where $X = (X_{ij})$ is the (unique) $\mathbb{C}^{2m\times 2m}$ solution of equation (1.3) subject to the asymptotic property

$$X(t,\lambda) = \begin{pmatrix} e^{-i\lambda t}I_m & 0 \\ 0 & -e^{i\lambda t}I_m \end{pmatrix} + o(1) \ \ (t\to\infty).$$

In this paper we present explicit formulas for k when the spectral matrix function (or equivalently the scattering matrix function or the reflection coefficient function) is rational, and give applications of these formulas to the equivalence between Kreĭn's and Marchenko's approach to inverse problems in the rational case. This leads us to a new relationship between the coefficient matrix functions of the Carathéodory–Toeplitz and Nehari extension problems. We also discuss a solution of the direct scattering problem in the rational case which turns out to be related to a problem of partial realization considered in [14]. In general, the results of this paper are obtained by a method which is based on the state space method from system theory. The main results with complete proofs can be found in the papers [2], [1], [3], [5], [4] and [6]. A topic not discussed here is the discrete case (see [1]).

Some words on notation: we denote by $\mathbb{C}^{m\times n}$ the space of m–rows and n–columns matrices with complex entries, and $\mathbb{C}^m$ is short for $\mathbb{C}^{m\times 1}$; the identity matrix of $\mathbb{C}^{m\times m}$ is denoted by I_m, or simply by I. The adjoint of a matrix A is denoted by A^*.

2 The Approaches of Kreĭn and Marchenko

The approach of M.G. Kreĭn's to the inverse spectral problem is as follows: let

$$W(\lambda) = I_m - \int_{-\infty}^{\infty} h(u)e^{i\lambda u}du$$

with $h \in L_1^{m\times m}(\mathbb{R})$ be the spectral function. Since $W(\lambda) > 0$ for all real λ, the integral equation

$$\gamma_\tau(t,s) - \int_0^\tau h(y-u)\gamma_\tau(u,s)du = h(t-s), \qquad t,s \in [0,\tau] \tag{2.1}$$

has a unique solution $\gamma_\tau(t,s)$ for every $\tau > 0$. Then, the potential $k(t)$ is given by the formula

$$k(t) = -2i\gamma_{2t}(0,2t). \tag{2.2}$$

Let

$$P(t,\lambda) = e^{i\lambda t}\left(I_m + \int_0^{2t} \gamma_{2t}(u,0)e^{-i\lambda u}du\right) \tag{2.3}$$

and

$$R(t,\lambda) = e^{i\lambda t}\left(I_m + \int_0^{2t} \gamma_{2t}(2t-u,2t)e^{-i\lambda u}du\right). \tag{2.4}$$

The unitary map U between the spaces $L_2^{2m}[0,\infty)$ and $L_2^m(W)$ alluded to in the introduction is given by

$$(Uf)(\lambda) = \sqrt{2\pi}\int_0^\infty (P(t,-\lambda) \quad R(t,\lambda))f(t)dt. \tag{2.5}$$

Marchenko's approach is concerned with inverse scattering. Let

$$S(\lambda) = I_n - \int_{-\infty}^{\infty} \sigma(u)e^{-i\lambda u}du \quad \lambda \in \mathbb{R}$$

be the scattering matrix function where $\sigma \in L_1^{m\times m}(\mathbb{R})$, and set

$$\xi(u) = \begin{pmatrix} 0 & \sigma(u)^* \\ \sigma(u) & 0 \end{pmatrix}. \tag{2.6}$$

Marchenko's approach consists in solving the equation

$$M(t,s) - \xi(t+s) - \int_t^\infty M(t,u)\xi(u+s)du = 0 \tag{2.7}$$

for $0 \le t \le s < \infty$ (see [20, equation (1.10)]) with the unknown matrix $M(t,s) = (m_{ij}(t,s))_{i,j=1,2}$ (where the block m_{ij} are $\mathbb{C}^{n\times n}$–valued).

The potential is then given by $k(t) = -2im_{21}(t,t)$.

3 Review of the State Space Technique

We recall a number of facts from the theory of realization of matrix–valued rational functions. Any $\mathbb{C}^{m\times m}$–valued rational function W, analytic on the real line and at infinity with $W(\infty) = I_m$, can be written as

$$W(\lambda) = I_m + C(\lambda I_n - A)^{-1}B, \tag{3.1}$$

where $A \in \mathbb{C}^{n\times n}$, $B \in \mathbb{C}^{n\times m}$ and $C \in \mathbb{C}^{m\times n}$.

Such an expression (3.1) is called a realization of W. The realization is called minimal if the number n in (3.1) is as small as possible and the minimal such n is called the McMillan degree of W. Two minimal realizations of W are similar: namely, if $W(\lambda) = I_m + C_i(\lambda I_n - A_i)^{-1}B_i,\ i = 1,2$ are two minimal realizations of W, there exists a (uniquely defined and invertible) matrix $S \in \mathbb{C}^{n\times n}$ such that

$$A_2 = SA_1S^{-1} \quad B_2 = SB_1 \quad C_2 = C_1S^{-1}. \tag{3.2}$$

For these facts and more information on the theory of realization of matrix–valued functions, we refer to [8] and [21].

If W is a $\mathbb{C}^{m\times m}$–valued function analytic on the real line and at infinity with $Z(\infty) = I_m$, with minimal realization (3.1), it is of the form (1.4) with

$$z(u) = \begin{cases} iCe^{-iuA}(I_m - P)B & u > 0 \\ -iCe^{-iuA}PB & u < 0 \end{cases} \tag{3.3}$$

where P is the Riesz projection corresponding to the eigenvalues of A in $\mathbb{C}_+$. The function z has absolutely summable entries and thus Z is in the Wiener algebra and it is therefore meaningful to consider the case of rational scattering and spectral functions.

A factorization $R = R_-R_+$ of R into two $\mathbb{C}^{n\times n}$–valued functions analytic at infinity is called a (right) canonical (Wiener-Hopf or spectral) factorization if R_- and its inverse are analytic in the closed lower half plane