Proceedings of the Fourth International Fluid Power Symposium

Sheffield 1975

ACKNOWLEDGEMENTS

The valuable assistance of the panel of referees and the Organising Committee in the successful organisation of the symposium is gratefully acknowledged.

ORGANISING COMMITTEE

Professor J.K. Royle, University of Sheffield (Chairman)

Dr. D.E. Bowns, University of Bath

Dr. K. Foster, University of Birmingham

J.D. Hamilton, National Engineering Laboratory

J.D. Kibble, National Coal Board

H.S. Stephens, BHRA Fluid Engineering

W.R. Taylor, Andrew Fraser & Co. Ltd.

N.G. Coles, BHRA Fluid Engineering (Secretary)

OPENING ADDRESS

First, I would like to welcome the delegates to Yorkshire, which was the birthplace (Stainborough, 10 miles or 16 km north of Sheffield) of Joseph Bramah (or Bramma as he was christened), the inventor of, or should one say the first to make, a hydraulic press to perform useful work (patented in 1795).

Pascal undoubtedly conceived this hydraulic press; and it was noted, that the 'pascal' had been adopted as an S.I. unit of pressure.

I notice that the 'bar' has been widely used in the papers submitted to this 4th International Fluid Power Symposium and I think that this is a wise choice in a period of transition from national to international units. The 'bar' has been widely used in the past by all nations as a unit of barometric pressure and it had been accepted as an S.I. unit as 10^5 x N/m^2 or 10^5 x pascal. In my opinion the unit of measurement should bear some relationship with what is being measured. The 'pascal' might be appropriate to measure blood-pressure, but is it a reasonable unit for those of us who were operating hydraulic presses at 300 to 500 bar (or 30 to 50 mega-pascal)? The Mega-Pascal (M.Pa) might seem appropriate, but what a mouthful to use in the workshop ! !

There is great practical advantage in having mono-syllabic units like: bar, volt, amp, watt.

I suggest that the absolute bar (10^5 N/m^2) could be known as the "bramah" or the "bra", or even "bhra".

I think that conferences such as this, attended by delegates from many nations and of diverse training and experience, provide an excellent meeting of minds for the cross-fertilization of ideas. The mechanical era of our civilization is facing a crisis. We are using up world stores of energy and rare minerals at a rapidly increasing rate; we may very soon reach a point of exhaustion. It is essential that engineers and scientists should meet regularly in order to discuss the problems which have to be solved to prolong human civilised life.

Frank Towler

F. Towler
Vice-President, BHRA Fluid Engineering

Proceedings of the Fourth International Fluid Power Symposium

CONTENTS

4th International

FLUID POWER SYMPOSIUM

April 16th-18th, 1975

PAPER A1

THE DIGITAL COMPUTATION OF PRESSURES AND FLOWS IN INTERCONNECTED FLUID VOLUMES, USING LUMPED PARAMETER THEORY

D. E. Bowns, B. Sc. (Eng), Ph. D., C. Eng., M. I. Mech. E.,

and

A. C. Rolfe, B. Sc.,

University of Bath, U. K.

Summary

This paper discusses one of the problems encountered in the simulation of the transient performance of fluid power systems.

Because of the relatively incompressible nature of oil and pipe systems, very small rates of change of flow can cause very high rates of change of pressure. If the digital computer simulation uses conventional integration techniques, these high rates of change of pressure necessitate the use of very small integration increments and consequently long computer running times. A pseudo-analytical technique is proposed here whereby these very small steps can be avoided. The analysis is developed and applied to both a single and a branched pipe system.

Two typical examples have been given, showing that the proposed method gives a marked reduction in computer running time compared with that obtained by conventional techniques, with the same accuracy of simulation.

Held at the University of Sheffield, England.
Organised and sponsored by BHRA Fluid Engineering, Cranfield, Bedford, England.

NOMENCLATURE

D	Differential operator $\left(\frac{d}{dt}\right)$	
t	time	(secs)
Q	flow rate	(Q/s)
p	pressure	(bar)
B	effective bulk modulus of the system	(bar)
C_D	coefficient of discharge of the orifice or restrictor	
A	area of discharge of the orifice or restrictor	(m^2)
ρ	density of the oil	(kg/m^3)
V	volume of oil in the pipe	(m^3)
K	constant defining orifice characteristics	
K	constant defining relief valve characteristics	(l/bar)
P_{RV}	relief valve cracking pressure	(bar)
T	time constant of relief valve	(secs)

SUFFIXES

S	source line
R	restrictor
L	load line
i	i^{th} pipe in the branched system
N	number of branches in the system
o	oil
p	pipe walls

INTRODUCTION

In recent years the digital computer has been used increasingly as a tool for the design and analysis of fluid power systems. Most of the work has been based on steady state analysis, but solutions to transient problems have been obtained and presented by authors such as SEBESTA and BOSE (1), ENRUH. ET. AL (2), EZEKIEL and PAYNTER (3), KNIGHT, B.E. (4), DRANSFIELD and ROGERS (5), KNIGHT G.C. ET. AL (6), BOWNS and WORTON-GRIFFITHS (7) and ZIELKE (8).

One of the difficulties in using a digital computer for the simulation of transients is that computation takes place in discrete steps, the smaller these steps then the closer the approximation. In a computer program for determining the dynamic characteristics of a hydraulic system it will invariably be necessary, within each step, to employ some form of numerical integration technique even though this leads to inaccuracy.

In order to improve the accuracy of the integration it is normal to reduce the integration increment thus increasing the computer running time.

One practical problem is the computation of pressure and flow changes in a simple hydraulic system with pipes connected by restrictors or orifices. Due to the relative inelasticity of the oil column a small step change in flow can cause very high rates of change of pressure within the system. These rates of change of pressure may be as high as 100 Kbar/sec and if simple integration techniques are used, increments as small as 10^{-6} seconds might be necessary, giving rise to very large computing times.

This paper proposes a method by which such small increments can be avoided for the general problem in which two or more volumes are connected together by restrictors or orifices. The method predicts the response of this type of system to a step change of flow rate, into or out of any one of more of the interconnected volumes. This response can then be used as a building block for the overall response of the system to continuously varying inputs.

THE PROBLEM

A generalised but simple system of the type to be discussed is the pipe system with a valve connecting a source S to a load L, fig. 1(a), a practical example being the pump-actuator system shown in fig. 1(b). Pipe friction has been ignored and all pressure losses assumed to take place in the valve. The fluid is assumed to be compressible and the pipe walls resilient. The system is subjected to a sudden increase in flow demand from the load. This will eventually result in a decrease of the pressure in the load line and, unless the source flow is increased, a subsequent decrease in the source line pressure.

In some cases, solution of this type of problem necessitates the use of distributed parameter theory since wave action may be significant. However, in many cases such an approach is unnecessarily complicated as the lengths of the pipes are small compared with the wavelengths of the propagating waves. It is these cases which are now being considered.

During the transient, the pressure downstream of the valve will initially reduce causing a change in the flow through the restrictor. This in turn will cause a change of the pressure in line S. At any instant the actuator is assumed to be taking a flow Q_L, the source flow is Q_S and the restrictor flow Q_R. The flow equations defining the system are:-

$$(Q_S - Q_R)\frac{B}{V_S} = \frac{dp_S}{dt} \qquad (1)$$

$$(Q_R - Q_L)\frac{B}{V_L} = \frac{dp_L}{dt} \qquad (2)$$

$$Q_R = C_D \cdot A \sqrt{\frac{2}{\rho}} \cdot \sqrt{(p_S - p_L)} \qquad (3)$$

$$= K\sqrt{(p_S - p_L)} \quad \text{(say)}$$

and if the relief valve is open:

$$Q_S = Q_{PUMP} - Q_{RV} \qquad (4)$$

where B is the effective bulk modulus of the fluid pipe system defined as:

$$\frac{1}{B} = \frac{1}{B_P} + \frac{1}{B_O}$$

Simultaneous solution of these equations will give the transient pressures and flows subsequent to any known disturbance.

METHODS AVAILABLE FOR DIGITAL INTEGRATION

The two non-linear simultaneous differential equations 1 and 2 necessitate the use of a numerical integration technique. Conventional methods, fully explained in references 9, 10, 11 and 12 can be summarised as follows:-

1) Euler methods:

   (a) Simple Euler integration: this method uses the slope of the dependent variable at the beginning of a time increment to predict its value at the end of the time increment and assumes that this slope remains constant throughout.

   (b) Modified Euler integration: uses the average slope during the time increment to predict the value at the end of the increment.

2) Runge-Kutta method: widely used method developed from the simple Euler technique, to give a more accurate, but less rapid predicition. The method uses weighted means of slopes, computed during the time increment, to predict the value of the dependent variable at the end of the increment. Modificiations of the method allow for variable step lengths.

3) Predictor-Corrector Methods: these use a polynomial extrapolation through already existing points to predict the result at the end of the time increment.

PSEUDO-ANALYTICAL SOLUTION

It will be shown by means of a simple numerical example that all of the above methods require very long computing times if a reasonably accurate solution is to be obtained.

However, by algebraic manipulation of the equations it is possible to use much coarser time steps and still achieve accurate solutions. Let us suppose that the flows Q_S, Q_R, Q_L and the pressures $p_{S\,1}$ and $p_{L\,1}$ are known at time t_1. We can obtain the variation of Q_R during the computing interval in the following manner:-

Rearrange equation (3):

$$\left(\frac{Q_R}{K}\right)^2 = P_S - P_L$$

differentiating with respect to time:

$$2\,\frac{Q_R}{K^2}\cdot\frac{dQ_R}{dt} = \frac{dP_S}{dt} - \frac{dP_L}{dt} \qquad (5)$$

and substituting values from equations (1) and (2):

$$2\,\frac{Q_R}{K^2}\cdot\frac{dQ_R}{dt} = \frac{B}{V_S}(Q_S - Q_R) - \frac{B}{V_L}(Q_R - Q_L)$$

Giving:

$$\frac{dQ_R}{dt} = \frac{BK^2}{2Q_R}\left\{\frac{Q_S}{V_S} + \frac{Q_L}{V_L}\right\} - \frac{BK^2}{2}\left\{\frac{1}{V_S} + \frac{1}{V_L}\right\}$$

or more simply:

$$\frac{dQ_R}{dt} = \frac{E}{Q_R} - F \qquad (6)$$

where:

$$E = \frac{BK^2}{2}\left\{\frac{Q_S}{V_S} + \frac{Q_L}{V_L}\right\}$$

and

$$F = \frac{BK^2}{2}\left\{\frac{1}{V_S} + \frac{1}{V_L}\right\}$$

If Q_S and Q_L are constant during the computing interval Δt, with values Q_S and Q_L respectively then this equation can be integrated between the limits t_2 at the end of the interval and t_1 at the beginning. The solution of this is:

$$\Delta t = t_2 - t_1 = \frac{1}{F}\left\{Q_{R_1} - Q_{R_2} + \frac{E}{F}\cdot\log_e\left[\frac{\frac{E}{F} - Q_{R_1}}{\frac{E}{F} - Q_{R_2}}\right]\right\} \qquad (7)$$

The value of Q_R at the end of the computing interval t can now be obtained using an iterative convergence routine, such as the Newton-Raphson technique. To obtain the pressure change at the load at the end of the computing interval equation (2) may also be integrated. This is best performed by separating the variables and equating equation (2) to equation (6).

ie. $$dt = \frac{dP_L}{(Q_R - Q_L)}\cdot\frac{V_L}{B} = \frac{Q_R}{(E - F.Q_R)}\cdot dQ_R$$

or more simply: $$dP_L = \left(\frac{B}{V_L}\right)\cdot\frac{(Q_R - Q_L)}{(E - F.Q_R)}\cdot Q_R\,dQ_R$$

With B, V_L, E, F and Q_L all remaining constant over the computing interval, Δt. Integration of this equation and substitution from equation (7) for Δt leads to a solution of p_L at the end of the computing interval, p_{L2}:

$$p_{L_2} = p_{L_1} + \frac{B}{V_L}\left(\frac{E}{F} - Q_L\right).\Delta t + \frac{B}{2V_L}.\frac{1}{F}\left(Q_{R_1}^2 - Q_{R_2}^2\right) \quad (8)$$

In a similar way an equation for the value at p_{S2} can be obtained:

$$p_{S_2} = p_{S_1} + \frac{B}{V_S}\left(Q_S - \frac{E}{F}\right).\Delta t - \frac{B}{2V_S}.\frac{1}{F}\left(Q_{R_1}^2 - Q_{R_2}^2\right) \quad (9)$$

Direct solutions of equations (7), (8) and (9) can, therefore, be obtained and the values of Q_{R2}, p_{L2} and p_{S2} at the time t_2 found. Using this method the maximum value of step size Δt is determined by the exigencies of the remainder of the system, rather than by the elasticity of the oil column.

NUMERICAL EXAMPLE

To compare the above computational methods, the behaviour of the system of fig.1(b) will be examined, subsequent to a step decrease in flow at the load end. This could correspond to an instantaneous decrease in actuator speed, but other more realistic inputs could be readily simulated.

For the purpose of the example the pump and relief valve combination has been assumed to operate according to the following equation:

$$Q_S = Q_{PUMP} - \frac{K_2}{(1+TD)}\left(p_S - p_{RV}\right) \quad (10)$$

The relief valve is discharging a flow proportional to the difference between the line pressure p_S and its cracking pressure p_{RV}, and reacting to disturbances with a time constant T.

The sytem equations (1), (2) and (3) and the above equation describe the system parameters at any time t. Rewriting these for small increments in time:

$$T.\Delta Q_S = \left(Q_{PUMP} - Q_S - K_2(p_S - p_{RV})\right).\Delta t \qquad \text{from 10}$$

$$Q_R = K\sqrt{(p_S - p_L)} \qquad \text{from 3}$$

$$\Delta p_S = (Q_S - Q_R)\frac{B}{V_S}.\Delta t \qquad \text{from 1}$$

$$\Delta p_L = (Q_R - Q_L)\frac{B}{V_L}.\Delta t \qquad \text{from 2.}$$

The conventional techniques involve the simultaneous solution of all four equations, using an integration step, Δt. The pseudo-analytical method allows Q_R, p_S and p_L to be found at the end of the time step Δt, directly from equations (7), (8) and (9).

The following system parameters have been assumed:

Initial flow to actuator = 0.5 l/s

Actuator subjected to a step decrease in flow of 0.29 l/s

$Q_p = 1.0$ l/s	$K = 0.025$ l/bar	$B = 0.185 \times 10^5$ bar
$p_{si} = 250$ bar	$T = 0.1$ seconds	
$V_L = 0.1$ l	$A = 0.75 \times 10^{-4}$ m^2	
$V_S = 1$ l	$\rho = 875$ kg/m^3	
$p_{RV} = 230$ bar	$C_D = 0.7$	

The system was simulated using C.S.M.P. (Continuous System Modelling Program) a simulation program developed by I.B.M. and described in ref. (13).

The integration procedures used were:

(a) Runge-Kutta method, with a variable step length routine, to predict the result to a specified accuracy.

(b) Milne Predictor-Corrector method, with a variable step length facility.

Fortran programs were also written to simulate the system using the simple Euler technique and the pseudo-analytical technique. In order to check the accuracy of the results of the simulations, the system was simulated on an analogue computer which had the appropriate non-linear facilities. Because of its continuous nature, the results obtained were used as a standard of comparison. These results are shown on fig. (2). It can be seen that the flow through the valve oscillates about its new value and that the oscillations in pressure both upstream and downstream of the valve are extremely violent. It can be seen that they would eventually settle down. Fig. 3(a) shows the results of digital computation using the Euler technique. The computer used was the Bath University ICL 4/50 machine.

In one case a time step Δt of 0.001 sec. was taken, and it can be seen that 48 seconds of computer time was taken to simulate 0.6 seconds of real time, and the solution obtained was unstable. In the other case, a time step of 10^{-6} seconds gave a reasonably accurate prediction after a computing time of 548 seconds - an extremely expensive way of obtaining accuracy.
Very little improvement is shown in fig.3(b), the Runge-Kutta method taking 349 seconds, whilst the Milne method gave completely unsatisfactory results and still took 2,000 seconds computing time. Fig.3(c) shows the result using the pseudo-analytical method. This obtained good accuracy with a computing time of only 53 seconds.

The pseudo-analytical technique requires a numerical integration method to compute the values of system flow Q_S at each step by the integration of equation (10). In the result presented a simple Euler integration was used with a step length of 0.5×10^{-3} seconds. The use of the simple Euler technique for this purpose is acceptable because the rates of change of Q_S are relatively small, 5((l/s)/s) maximum, compared with 6×10^5 (bar/s) maximum for the load pressure changes. The overall response was predicted in 53 seconds with an error of no greater than 1%.

PSEUDO-ANALYTICAL TECHNIQUE APPLIED TO BRANCHED SYSTEMS

The pseudo-analytical technique can be readily applied to branched systems of the type shown in fig.4. In this system a source flow Q_S passes through a pipe and is divided between N branches with an upstream restrictor on each and a flow being taken from each. Each line in the system is assumed to have capacitance as before.

Carrying out the analysis as before:

For the ith branch:

$$\left(\frac{dp_L}{dt}\right) = \frac{B}{V_L} \cdot \left(Q_{Ri} - Q_{Li}\right) \tag{11}$$

For the feeder line:

$$\left(\frac{dp_S}{dt}\right) = \frac{B}{V_S} \cdot \left(Q_S - \sum_{x=1}^{N} Q_{Rx}\right) \tag{12}$$

$$Q_{Ri} = K_i \sqrt{(p_S - p_{Li})} \tag{13}$$

It is helpful to establish values of Q_R, $\frac{dp_L}{dt}$ and $\frac{dp_S}{dt}$

which occur when the quantity $\frac{dQ_R}{dt}$ becomes zero. In order to distinguish

these, they will be denoted by an asterisk, ie. $Q_R{}^*$, $\frac{dp_L}{dt}{}^*$ and $\frac{dp_S}{dt}{}^*$

From equation (5) if $\frac{dQ_R}{dt}$ is set to zero then:

$$\frac{dp_S}{dt}^* = \frac{dp_L}{dt}^* = \frac{dp}{dt}^*$$

and it can be shown App. 1 that:

$$\frac{dp}{dt}^* = B. \frac{\left(Q_S - \sum_{x=1}^{N} Q_{Lx}\right)}{\left(V_S + \sum_{x=1}^{N} V_{Lx}\right)} \tag{14}$$

and also:

$$Q_{Ri}^* = Q_{Li} + V_{Li} \frac{\left(Q_S - \sum_{x=1}^{N} Q_{Lx}\right)}{\left(V_S + \sum_{x=1}^{N} V_{Lx}\right)} \tag{15}$$

Before any analysis of this more complicated system takes place it is helpful to establish the sort of response which would be expected due to load demand changes in each of the branches. The response of the simple pipe system in fig. 1(a) due to a step change in load demand can be found by direct solution of equations (7), (8) and (9). The restrictor flow Q_R will change very quickly and then steady out at a value defined by the system parameters and denoted as $Q_R{}^*$, in this steady condition the system and load pressures may still be changing, but at a steady rate defined by $\frac{dp}{dt}^*$.

It would be expected then that each of the branches of the more complicated system would respond in a similar manner although the magnitudes of the responses would be different. The response of each branch cannot be predicted by applying the simple, single pipe theory directly because of the dependency of each line on the source pressure, which is also a function of all the other branches. The expected response of the pressures in a three branch system, assuming an increase in source flow at the same time as the load changes occur, is shown in fig.5. The source and load flows change instantaneously at the time t_1 and the analysis is to predict the values of system pressures and restrictor flows at time t_2 a time Δt seconds later. The system pressures change during the time Δt by an amount which has been denoted by p_S and p_{Li}. The restrictor flows at both t_1 and t_2 are shown in fig.5 as functions of the pressures at that time.

From fig.5 it can be seen that:

$$\left(\frac{Q_{Ri}}{K_i}\right)^2_{t_2} - \left(\frac{Q_{Ri}}{K_i}\right)^2_{t_1} = \Delta p_S + \Delta p_{Li} \qquad (16)$$

equations (11) and (12) can be re-written as:

$$dp_{Li} = \frac{B}{V_{Li}} Q_{Ri}\, dt - \frac{B}{V_{Li}} Q_{Li}\, dt$$

$$dp_S = \frac{B}{V_S} Q_S\, dt - \frac{B}{V_S} \sum_{x=1}^{N} Q_{Rx}\, dt$$

and these three equations can be solved simultaneously to give:

$$\Delta p_S = \left(\frac{dp}{dt}\right)^* \Delta t - \frac{1}{\left(V_S + \sum_{x=1}^{N} V_{Lx}\right)} \left\{ \sum_{x=1}^{N} \left(Q_{Rx_1}^2 - Q_{Rx_2}^2\right) \cdot \frac{V_{Lx}}{K_x^2} \right\} \qquad (17)$$

and

$$\Delta p_{Li} = \left(\frac{dp}{dt}\right)^* \Delta t - \frac{1}{\left(V_S + \sum_{x=1}^{N} V_{Lx}\right)} \left\{ F_{Li} V_S + \sum_{x=1}^{N} \left(F_{Li} - F_{Lx}\right) \cdot V_{Lx} \right\} \qquad (18)$$

where:

$$F_{Li} = \frac{1}{K_i^2} \cdot \left(Q_{Ri_2}^2 - Q_{Ri_1}^2\right)$$

In order to find the load and source pressures at time t_2 the equations (17) and (18) must be solved. This is done with the substitution of the system data and also $\frac{dp}{dt}$ and Q_{Ri_2}, the former being found quite simply from equation (14).

The latter, Q_{Ri2} can be found from an equation derived, as for the simple pipe system, from the three basic system equations, (11), (12) and (13) re-written as follows:

$$\left(\frac{dp_L}{dt}\right)_i = \frac{B}{V_{Li}} \cdot (Q_{Ri} - Q_{Li}) \quad (11)$$

$$\left(\frac{dp_S}{dt}\right) = \frac{B}{V_S} \cdot \left(Q_S - \sum_{x=1}^{N} Q_{Rx}\right) \quad (12)$$

$$Q_{Ri} = K_i \sqrt{(p_S - p_{Li})} \quad (13)$$

Differentiation of equation (13) with respect to time, and substitution for $\frac{dp_{Li}}{dt}$ and $\frac{dp_S}{dt}$ from equations (11) and (12) results in:

$$\frac{2}{B} \cdot \frac{Q_{Ri}}{K_i^2} \cdot \frac{dQ_{Ri}}{dt} = \frac{Q_S}{V_S} - \frac{\sum_{x=1}^{N} Q_{Rx}}{V_S} - \frac{Q_{Ri}}{V_{Li}} + \frac{Q_{Li}}{V_{Li}}$$

which can be arranged to give:

$$\frac{2}{BK_i^2} \cdot \frac{dQ_{Ri}}{dt} = \frac{1}{Q_{Ri}} \left\{ \frac{Q_S}{V_S} + \frac{Q_{Li}}{V_{Li}} - \sum_{x=1}^{(i-1)} \left(\frac{Q_{Rx}}{V_S}\right) - \sum_{x=i+1}^{N} \left(\frac{Q_{Rx}}{V_S}\right) \right\} - \left\{ \frac{1}{V_{Li}} + \frac{1}{V_S} \right\}$$

or

$$\frac{dQ_{Ri}}{dt} = \frac{E_i}{Q_{Ri}} - F_i \quad (19)$$

where:

$$E_i = \frac{BK_i^2}{2} \left\{ \frac{Q_S}{V_S} + \frac{Q_{Li}}{V_{Li}} - \frac{1}{V_S} \left[\sum_{x=1}^{(i-1)} Q_{Rx} + \sum_{x=i+1}^{N} Q_{Rx} \right] \right\}$$

and

$$F_i = \frac{BK_i^2}{2} \left\{ \frac{1}{V_S} + \frac{1}{V_{Li}} \right\}$$

This is of the same form as the simple single pipe system, equation (6), which was then solved, and for use in the branched system can be written as:

$$\Delta t = \frac{1}{F_i} \left\{ Q_{Ri_1} - Q_{Ri_2} + \frac{E_i}{F_i} \log_e \left[\frac{\frac{E_i}{F_i} - Q_{Ri_1}}{\frac{E_i}{F_i} - Q_{Ri_2}} \right] \right\} \quad (20)$$

and if $\frac{dQ_{Ri}}{dt}$ is set ot zero in equation (19) then it can be seen that:

$$Q_{Ri}^* = \frac{E_i}{F_i}$$

and equation (20) can be re-written more easily as:

$$\Delta t = \frac{1}{iF_i}\left\{ Q_{Ri_1} - Q_{Ri_2} + Q_{Ri}^{*} \log_e\left[\frac{Q_{Ri}^{*} - Q_{Ri_1}}{Q_{Ri}^{*} - Q_{Ri_2}}\right]\right\} \quad (21)$$

Q_{Ri}* is found from equation (15) and equation (21) is solved in the same way as before, using an iterative convergence routine.

EXAMPLE OF THE USE OF THE BRANCHED SYSTEM ANALYSIS

A simple illustration of the use of the branched system theory is the response of the system shown in fig.6. A pump and relief valve combination drives two hydraulic motors A and B through restrictors on the inlet line of each motor. Assume that the relief valve is set much higher than the system pressure and the pump flow remains constant. The motors are of equal displacement and are running at the same constant speed. In steady conditions equal load torques are applied to each motor and the restrictor flows are equal. A sudden load torque, increasing the load by 10% is applied to motor B, and it is required to predict the changes in the system pressures and motor speeds, over a given period. The system parameters used are given below and the load on the motors is assumed to be of the form:

$$T_L = J.\frac{d\omega}{dt} + f.\omega + C(t)$$

where: J is the moment of inertia of motor and load.
f is the viscous friction coefficient.
C is the speed independent load torque (a function of time).
w the angular velocity of the motor.

Data for one motor:

Motor displacement	:	0.1 l/rad
Initial running speed	:	10 rad/s
Volume of line connected to each motor	:	10 litres
Volume of supply line	:	10 litres
Initial supply pressure	:	200 bar
Initial motor pressure	:	198.4 bar
Inertia of motor and load, J	:	15 kg m^2
Viscous friction coefficient	:	50 Nm/(rad/s)
Initial speed independent component of load torque, C	:	1484 Nm, applied to each motor

Fig.7 shows the predicted change in speeds and line pressures subsequent to the step changes in load of C, applied to motor B. As might be expected the speed of motor A rises whilst that of motor B falls. The oscillations due to the effects of compressibility are clearly shown. The corresponding pressure changes can also be seen on fig.7.

CONCLUSIONS

The two systems described are just two examples of many systems incorporating restrictors where pipe line compressibility affects the transient response. The two examples were taken from the oil hydraulics field but other examples can readily be found in water systems and pneumatic systems, and also in other mechanical engineering problems involving elastic deflection.

Digital simulation of such systems may be expensive of computer time if conventional integration techniques are used and a method is suggested which is both accurate and economical of computer time. It has been shown that the suggested method is unlikely to lead to computational instability and its accuracy is within 1% for each of the two problems worked out as examples.

References

1. Sebesta, H.R. and Rose, J.E: 'Using Computers to Design Fluid Power Circuits' Machine Design, 42, 3, pp. 129-129, (December 10th, 1970).

2. Unruh, D.R., Fitch, E.C. and Sebesta, H.R: 'How Computers Aid Analysis of Fluid Power Systems'. Hydraulics & Pneumatics, 24, 3, pp. 81-83 (March 1971).

3. Ezekiel, F.D. and Paynter, H.M: 'Computer Representations of Systems Involving Fluid Transients'. Trans. ASME. Paper No. 56-A-120, (1956).

4. Knight, B.E: 'Fuel Injection System Calculations'. Proc. I.Mech.E. Automobile Division, pp. 25, (1960-61)

5. Dransfield, P. and Rogers, K.J: 'Simulating Hydraulic Control Systems' National Conference on Fluid Power, Chicago, U.S.A. 13-15 September 1972.

6. Knight, G.C., McCallion, H. and Dudley, B.R: 'Connection Capacitance Effects in Hydrostatic Transmissions and Their Prediction by Mathematical Model'. Proc. I.Mech.E. Vol. 186 (1972).

7. Bowns, D.E. and Worton-Griffiths, J. 'Dynamic Characteristics of a Hydrostatic Transmission System'. Proc. I.Mech.E. Fluid Plant and Machinery Group, Vol. 186, 55/72 (1972).

8. Zielke, W. 'Digital Simulation of Airplane Hydraulic Systems' Trans. ASME, Paper No. 71-WA/FE-21, (1971).

9. Khabaza, I.M: 'Numerical Analysis' Pergaman Press, (1969).

10. Gerald, C.F: 'Applied Numerical Analysis'. Addison Wesley, (1970).

11. Kildebrand, F.B: 'Introduction to Numerical Analysis'. McGraw Hill, (1956).

12. Users Reference Manual: Digital Simulation Language, 'An I.B.M. 1800 Program for the Simulation of Continuous Process Dynamics', (1968).

13. McArthur, C.D: 'A Users Guide to C.S.M.P.'. University of Edinburgh, Catalogue No. 19.410.200.