Modelling, Robustness and Sensitivity Reduction in Control Systems

Edited by Ruth F. Curtain

NATO ASI Series

Series F: Computer and Systems Sciences, Vol. 34

Modelling, Robustness and Sensitivity Reduction in Control Systems

Edited by

Ruth F. Curtain

Mathematics Institute, University of Groningen
9700 AV Groningen, The Netherlands

Springer-Verlag
Berlin Heidelberg New York London Paris Tokyo
Published in cooperation with NATO Scientific Affairs Divison

Proceedings of the NATO Advanced Research Workshop on Modelling, Robustness and Sensitivity Reduction in Control Systems held in Groningen, The Netherlands, December 1–5, 1986

ISBN 3-540-17845-7 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-17845-7 Springer-Verlag New York Berlin Heidelberg

Library of Congress Cataloging in Publication Data. NATO Advanced Research Workshop on Modelling, Robustness, and Sensitivity Reduction in Control Systems (1986 : Groningen, Netherlands) Modelling, robustness, and sensitivity reduction in control systems. (NATO ASI series. Series F, Computer and systems sciences ; vol. 34) "Published in cooperation with NATO Scientific Affairs Division." "Proceedings of the NATO Advanced Research Workshop on Modelling, Robustness and Sensitivity Reduction in Control Systems held in Groningen, The Netherlands, December 1–5, 1986."—T.p. verso. 1. Automatic control—Congresses. 2. Control theory—Congresses. I. Curtain, Ruth F. II. North Atlantic Treaty Organization. Scientific Affairs Division. III. Title. IV. Series: NATO ASI series. Series F, Computer and systems sciences ; vol. 34. TJ212.2.N36 1986 629.8 87-12817
ISBN 0-387-17845-7 (U.S.)

Printed in Germany

Printing: Druckhaus Beltz, Hemsbach; Bookbinding: J. Schäffer GmbH & Co. KG, Grünstadt
2145/3140-543210

Modelling, Robustness and Sensitivity Reduction in Control Systems

NATO ASI Series

Advanced Science Institutes Series

A series presenting the results of activities sponsored by the NATO Science Committee, which aims at the dissemination of advanced scientific and technological knowledge, with a view to strengthening links between scientific communities.

The Series is published by an international board of publishers in conjunction with the NATO Scientific Affairs Division

A Life Sciences **B Physics**	Plenum Publishing Corporation London and New York
C Mathematical and Physical Sciences	D. Reidel Publishing Company Dordrecht, Boston, Lancaster and Tokyo
D Behavioural and Social Sciences **E Applied Sciences**	Martinus Nijhoff Publishers Boston, The Hague, Dordrecht and Lancaster
F Computer and Systems Sciences **G Ecological Sciences** **H Cell Biology**	Springer-Verlag Berlin Heidelberg New York London Paris Tokyo

Series F: Computer and Systems Sciences Vol. 34

PREFACE

This volume contains the proceedings of the NATO Advanced Research Workshop on "Modelling, Robustness and Sensitivity Reduction in Control Sytems" which was held at the University of Groningen, the Netherlands, during the first week of December, 1986.

Modelling is a fundamental and difficult problem in all the sciences; to design a controller one needs a model. While for some applications one has a good physical model, often one only has measurements of the inputs and outputs of the system available. **Modelling from measurement data** was one important theme of the workshop. To control theorists, this has traditionally meant the stochastic approach of "System Identification" but here the newer deterministic approaches shared the spotlight.

Of course all models are approximate, but one sometimes requires a lower order, simpler model which still retains the main features of the original model with respect to the problem of control design. **Approximation** in this sense is often called **model reduction** and this theme was discussed during the workshop. Given that we only have approximate models available, the concept of robustness has always played an important role in controller design. Robust controllers are those which can control not only the given nominal model, but also neighbouring perturbations while at the same time guaranteeing an acceptable performance; they are robust with respect to model uncertainties. Typical performance requirements are tracking ability, stability and the suppression of disturbances, usually with respect to certain frequency bands, and so another very desirable property of a controller is that its performance has a low sensitivity to external disturbances. **Robustness and sensitivity reduction** of controllers were two related themes of the workshop.

During the last decade major advances have been made in the theory of approximation (model reduction) and robustness and sensitivity reduction of controllers by exploiting known results in two areas of mathematics: in classical mathematical analysis such as the work in interpolation theory by Nevanlinna, Pick, Fejér and Carathéodory, and in more recent developments in Operator Theory, such as the work of Adamjan, Arov and Krein in the seventies. This synthesis has resulted in a new research area in Systems and Control Theory known as **H^∞- Control** which was the main theme of this workshop and is closely related to the other themes of approximation, robustness and sensitivity reduction. These proceedings contain new contributions in these areas which range from abstract mathematical papers to some very concrete and challenging applications; an interesting interplay between mathematics and engineering.

As is well known, NATO workshops are primarily supported by the NATO Scientific Affairs Division and we are grateful to them for their sponsorship and generous financial support. This workshop was designated as belonging to the "Double Jump" programme, which means that the sectors: university, industry and government research institutions should all be involved in the workshop. A glance at the list of participants will verify that this was the case as far as participation in the scientific part of the workshop is concerned. With respect to the financial support, we have the pleasure of thanking the following long list of government agencies and companies: the Dutch Academy of Sciences, the Dutch Organization for the Advancement of Pure Scientific Research, the British Science and Engineering Research Council, the University and the Province of Groningen, de Nederlandse Aardolie Maatschappij (the Dutch Oil Company), de N.V. Nederlandse Gasunie (the Dutch Gas Company), Hollandse Signaalapparaten B.V. and Hoogovens Groep B.V.

Finally we would like to thank the Mathematics Institute of the University of Groningen for the support in organizing the workshop in particular the assistance of the workshop secretary, Janieta Schlukebir.

Ruth F. Curtain

for the International Committee:

Prof. R.F. Curtain, Mathematics Institute, University of Groningen
Dr. K. Glover, Dept. Engineering, University of Cambridge, U.K.
Prof. B. Francis, Dept. Engineering, University of Toronto, Canada
Prof. J.C. Doyle, Honeywell SRC, MN 17-2367, 2600 Ridgeway Parkway,
Minneapolis, U.S.A./ Dept. Electrical Engineering,
California Institute of Technology, Pasadena, CA 91125, USA.

TABLE OF CONTENTS

1. H^{∞}-Mathematical Foundations

2. Model Approximation and Robustness in Control Design

3. Modelling and Signal Processing

4. Modelling and System Identification

5. H^{∞}-techniques

A Guide To H^∞- Control Theory

Bruce A. Francis
Department of Electrical Engineering
University of Toronto
Toronto, Canada M5S 1A4
USA

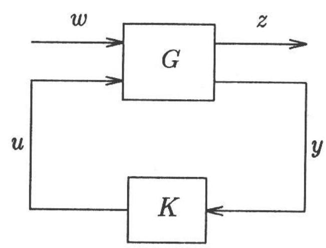

Figure 1. The standard block diagram

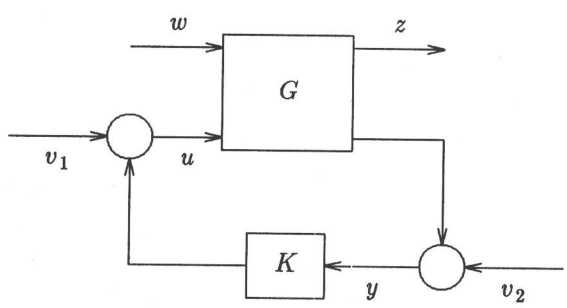

Figure 2. Diagram for stability definition

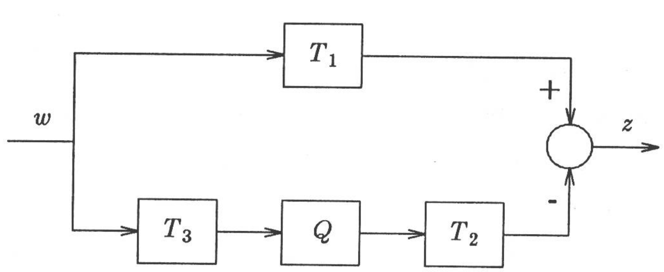

Figure 3. Model-matching

NATO ASI Series, Vol. F34
Modelling, Robustness and Sensitivity Reduction in Control Systems. Edited by R. F. Curtain

1. Introduction

This paper is intended as a tutorial on the most basic $\mathbf{H}_\infty$- control problem. The set-up is linear, time-invariant, finite-dimensional, continuous-time. The main theme is that the theory is most simply and elegantly developed in the framework of operators, while computations are most easily performed using state-space methods. (Thus state-space methods serve merely as slaves in an input-output setting.) The results are summarized in the form of algorithms, primarily to demonstrate that the computations can be done using off-the-shelf software.

Pioneered by Zames (1981), $\mathbf{H}_\infty$- optimization in control theory has been developed by many researchers and from several viewpoints. The state-space approach to computations was initiated primarily by Silverman and Bettayeb (1980) and Doyle (1984). The reader may consult Francis and Doyle (1987) and Dorato (1987) for reference lists and historical accounts.

The main text consists of five parts. In Section 2 the standard problem is posed and the model-matching problem (MMP)

$$\underset{Q}{\text{minimize}} \; \|T_1 - T_2 Q T_3\|_\infty$$

is offered as an example. Here T_i and Q are real-rational $\mathbf{H}_\infty$-matrices. The reader is then reminded that the standard problem can be reduced to MMP using the familiar parametrization of Youla, Jabr, and Bongiorno (1976).

The rest of the paper deals with MMP. The classification scheme of Limebeer and Hung (1986) is introduced, yielding three model-matching problems, MMP(i) (i=1-3), of increasing difficulty. In general one solves these problems by first computing the minimal model-matching error (the minimum norm above) and then computing an optimal Q.

Section 3 begins with a discussion of when an optimal Q exists. Mild sufficient conditions are given and then a three-step, high-level algorithm is developed for solving MMP(1). The most difficult step is the Nehari problem of approximating an $\mathbf{L}_\infty$-matrix by an $\mathbf{H}_\infty$-matrix.

In Section 4 the Nehari problem in the scalar-valued case (T_i and Q are scalar-valued functions) is solved completely using the theory of Sarason (1967) and

Adamjan, Arov, and Krein (1971), with state-space formulas by Silverman and Bettayeb (1980).

Section 5 deals with the factorization of a rational matrix. The canonical factorization theorem of Bart, Gohberg, Kaashoek, and van Dooren (1980) is presented and used to obtain spectral factorization, inner-outer factorization, and J-spectral factorization.

Finally, in Section 6 the Nehari problem in the matrix-valued case is solved using the theory of Ball and Helton (1983), with state-space formulas by Ball and Ran (1986).

The notation is fairly standard: $\mathbf{L}_\infty$ is the space of essentially-bounded matrix functions on the imaginary axis; $\mathbf{H}_2$ and $\mathbf{H}_\infty$ are the Hardy spaces for the right half-plane; and prefix $\mathbf{R}$ denotes real-rational. For a state-space realization, $[A, B, C, D]$ stands for the transfer matrix $D+C(sI-A)^{-1}B$.

2. The standard problem and the model-matching problem

The standard set-up is shown in Figure 1. In this figure w, u, z, and y are vector-valued signals: w is the exogenous input, typically consisting of command signals, disturbances, and sensor noises; u is the control signal; z is the output to be controlled, its components typically being tracking errors, filtered actuator signals, etc.; and y is the measured output. The transfer matrices G and K are, by assumption, real-rational and proper: G represents a generalized plant, the fixed part of the system, and K represents a controller. Partition G as

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}.$$

Then Figure 1 stands for the algebraic equations

$$z = G_{11}w + G_{12}u$$

$$y = G_{21}w + G_{22}u$$

$$u = Ky.$$

To define what it means for K to stabilize G, introduce two additional inputs, v_1 and v_2, as in Figure 2. It simplifies the theory to guarantee that the nine transfer matrices from w, v_1, v_2 to z, u, y exist and are proper for every proper real-rational K. A simple sufficient condition for this is that G_{22} be strictly proper. Accordingly, this will be *assumed* hereafter. If these nine transfer matrices are stable, i.e. they belong to $\mathbf{RH}_\infty$, then we say that *K stabilizes G*. (This is the usual notion of internal stability.)

The *standard problem* is this: find a real-rational proper K to minimize the $\mathbf{H}_\infty$-norm of the transfer matrix from w to z under the constraint that K stabilize G. Observe that the transfer matrix from w to z is a linear-fractional, hence nonlinear, transformation of K:

$$z = [G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}]w.$$

There are several well-studied special cases of the standard problem, for example the weighted sensitivity, the mixed sensitivity, and the robust stability problems, but perhaps the simplest special case is the model-matching problem, abbreviated MMP. In Figure 3 the transfer matrix T_1 represents a "model" which is to be matched by the cascade $T_2\ Q\ T_3$ of three transfer matrices T_2, T_3, and Q. Here, T_i (i=1–3) are given and the "controller" Q is to be designed. It is assumed that $T_i \in \mathbf{RH}_\infty$ (i=1–3) and it is required that $Q \in \mathbf{RH}_\infty$. Thus the four blocks in Figure 3 represent stable linear systems.

For our purposes the *model-matching criterion* is

$$\sup\ \{\|z\|_2 :\ w \in \mathbf{H}_2,\ \|w\|_2 \le 1\} = \text{minimum}.$$

Since the $\mathbf{H}_2$-induced norm equals the $\mathbf{H}_\infty$-norm of the transfer matrix, this is equivalent to

$$\|T_1 - T_2\ Q\ T_3\|_\infty = \text{minimum}.$$

This model-matching problem can be recast as a standard problem by defining

$$G := \begin{bmatrix} T_1 & T_2 \\ T_3 & 0 \end{bmatrix}$$

$$K := -Q,$$

so that Figure 3 becomes equivalent to Figure 1. The constraint that K stabilize G

is then simply that $Q \in \mathbf{RH}_\infty$.

This version of the model-matching problem is not so important *per se*; its significance for us arises from the fact that the standard problem can in fact be transformed into the model-matching problem, which is considerably simpler. How to do this is by now standard: one parametrizes all K 's stabilizing G as a linear-fractional transformation of a free parameter matrix Q in $\mathbf{RH}_\infty$; then the transfer matrix from w to z is an affine function of Q, i.e. it's of the form $T_1 - T_2 Q T_3$. The theory behind this conversion is omitted; however we summarize the procedure in the form of a state-space algorithm, due primarily to Doyle (1984), to compute T_i (i=1–3) from G.

The algorithm starts with a minimal realization of G:

$$G(s) = [A,\ B,\ C,\ D]\,.$$

Since the input and output of G are partitioned as

$$\begin{bmatrix} w \\ u \end{bmatrix},\ \begin{bmatrix} z \\ y \end{bmatrix},$$

the matrices B, C, and D have corresponding partitions:

$$B = [B_1\ \ B_2]$$

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

$$D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}.$$

Then

$$G_{ij}(s) = [A,\ B_j,\ C_i,\ D_{ij}]\,,\quad i,j=1,2\,.$$

Note that D_{22}=0 because G_{22} is strictly proper.

Procedure 1

Step 1. Choose F and H so that

$$A_F := A + B_2 F\,,\ A_H := A + H C_2$$

are stable.

Step 2. Set

$$\underline{A} = \begin{bmatrix} A_F & -B_2 F \\ 0 & A_H \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} B_1 \\ B_1 + HD_{21} \end{bmatrix}$$

$$\underline{C} = [C_1 + D_{12}F \quad -D_{12}F]$$

$$T_1(s) = [\underline{A}, \underline{B}, \underline{C}, D_{11}]$$

$$T_2(s) = [A_F, B_2, C_1 + D_{12}F, D_{12}]$$

$$T_3(s) = [A_H, B_1 + HD_{21}, C_2, D_{21}] .$$

Limebeer and Hung (1986) introduced a useful classification scheme for MMP. It involves the relative dimensions of the matrices T_1 and T_2. Let's say a matrix is wide if the number of its rows is $\leq$ the number of its columns, and strictly wide if the inequality is $<$. Similarly for tall and strictly tall. The classification scheme is this:

MMP(1): T_1 is wide and T_2 is tall

MMP(2): T_1 is strictly tall or T_2 is strictly wide (exclusive or)

MMP(3): T_1 is strictly tall and T_2 is strictly wide.

It turns out that MMP(2) is harder than MMP(1), and MMP(3) harder than MMP(2). Difficulty here refers to the complexity of computing optimal Q's.

3. Existence and a high-level algorithm for MMP(1)

To each Q in $\mathbf{RH}_\infty$ there corresponds a *model-matching error*, $\|T_1 - T_2 Q T_3\|_\infty$. Let α denote the *infimal model-matching error*:

$$\alpha := \inf\{\|T_1 - T_2 Q T_3\|_\infty : Q \in \mathbf{RH}_\infty\} . \tag{1}$$

A matrix Q in $\mathbf{RH}_\infty$ satisfying

$$\alpha = \|T_1 - T_2 Q T_3\|_\infty$$

will be called *optimal.*

This section is first concerned with the question of when an optimal Q exists. This theorem provides a sufficient condition:

> An optimal Q exists if the ranks of the two matrices $T_2(j\omega)$ and $T_3(j\omega)$ are constant for all $0 \leq \omega \leq \infty$.

These rank conditions are not necessary for existence, but from now on we assume they hold. To see the underlying idea in this theorem, first note from (1) that

$$\alpha = \text{dist}\,(T_1,\ T_2 \mathbf{RH}_\infty T_3) \tag{2}$$

where

$$T_2 \mathbf{RH}_\infty T_3 := \{T_2 Q T_3 : Q \in \mathbf{RH}_\infty\}.$$

Since $\mathbf{RH}_\infty \subset \mathbf{H}_\infty$ we have

$$\alpha \geq \text{dist}\,(T_1,\ T_2 \mathbf{H}_\infty T_3). \tag{3}$$

But since T_1 is itself real-rational, the two distances in (2) and (3) are equal, so

$$\alpha = \text{dist}\,(T_1,\ T_2 \mathbf{H}_\infty T_3). \tag{4}$$

The rank conditions above guarantee that $T_2 \mathbf{H}_\infty T_3$ is weak-star closed in $\mathbf{H}_\infty$ (see for instance Theorem II.7.5 in Garnett (1981)), and this in turn implies that the distance in (4) is achieved, i.e.

$$\alpha = \|T_1 - T_2 Q T_3\|_\infty$$

for some Q in $\mathbf{H}_\infty$. Proving that the distance is achieved by a real-rational Q is harder, and in fact hasn't been completely worked out. In the general case, the constructive procedure for MMP yields a Q in $\mathbf{RH}_\infty$ so that $T_2 Q T_3$ is arbitrarily close to T_1.

Deep results on uniqueness have recently been obtained by Foias and Tannenbaum (1986).

There's a simple formula due to Young (1986) for α as the norm of a certain operator. Define two subspaces $\mathbf{X}$ and $\mathbf{Y}$ of $\mathbf{L}_2$,

$$\mathbf{X} := T_3^{-1} \mathbf{H}_2$$