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Fluid Mechanics and Its Applications

VIJAY GUPTA
SANTOSH K GUPTA

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And Its Applications

Vijay Gupta
Department of Aeronautical Engineering
Indian Institute of Technology, Kanpur

Santosh K. Gupta
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

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6 Shri B P Wadia Road, Basavangudi, Bangalore 560004
Abid House, Dr Bhadkamkar Marg, Bombay 400007
40/8 Ballygunge Circular Road, Calcutta 700019
Post Box No. 1124, Tiruvanmiyur, Madras 600041
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Preface

This book is written as a first course in fluid mechanics for undergraduate engineering students in all disciplines. It has been our experience over a number of years of teaching that the students consider fluid mechanics to be one of the most difficult subjects to grasp. In taking up this project we were vain enough to believe that this problem was partly created by the currently available textbooks and that we could help alleviate it!

Fluid mechanics as a subject is rather tricky, requiring a large number of concepts to be invoked for solving even simple-looking problems. Also, the wide variety of phenomena that are involved, with the flow behaviour changing suddenly with small changes in the values of parameters, make the subject frustrating to a beginner. Several popular textbooks, in their attempt to make concepts rigorous and elegant, delve to great depths into the mathematics of fluid flow using vector and even tensor calculus. But in doing so the emphasis shifts, often unintentionally, from the phenomena themselves to the mathematics. We believe that fluid-mechanical phenomena are complex enough requiring the full attention of the student without it being diverted by the demands of the newly-learnt mathematical tools which come in the way of developing a physical feel of the subject. It is because of this reason that we have kept the level of mathematics in this text to the minimum. We have used vector calculus only sparingly, and for this we have restricted most derivations to two-dimensions only, extending the results to the third dimension. We believe the elegance of vectorial or tensorial mathematics can be safely left to a second course, after the student has a firm grasp of the fundamental physical concepts.

In view of the importance we attach to the understanding of physical phenomena, we have brought forward in Chapter 1 an introduction to the various flow phenomena encountered in simple situations so that a student gets an idea of what kind of things he is to look for and becomes familiar with certain specialized vocabulary of fluid mechanics. We have found that this helps a student to become aware of the major flow phenomena and various fascinating applications of fluid mechanics and kindles an interest which sustains him through the following chapters. It should perhaps be pointed out that a beginning student should not spend too much time on

Chapter 1 trying to figure out the 'whys' of the phenomena. He should only become familiar with them and come back to this chapter after the Epilogue when, hopefully, he should be able to explain.

After a survey of fluid statics in Chapter 2, we develop the basic equations of fluid flow in Chapters 3 to 7. The treatment is as rigorous as possible within the constraints that we have explicitly set for ourselves. In the initial portions, the integral approach is favoured because it helps the student get a better feel of the subject. Later, we switch to the differential approach because the detailed information is provided only by it.

Similitude and modelling has been introduced in Chapter 9 where we have made the most radical departure from current textbooks. We have found in our teaching that with the scale-factor approach that we use, it is possible to make the previously mysterious dimensional analysis more understandable. This approach, which is much more physical than the Buckingham Π theorem route, permits the students to handle far more complex problems of modelling easily.

In Chapter 11 we have introduced a rational basis of making approximations in fluid mechanics. We find that our method of introducing boundary layers as a problem of multiple characteristic lengths finds greater favour with students than the traditional Prandtl approach or the currently-in-vogue singular-perturbation method. Our approach is more fundamental than the former but more physical than the latter.

In Chapters 12 and 13 we introduce potential flow and boundary-layer flows, along with associated concepts. In Chapter 15 the effects of compressibility are introduced. We concentrate on only the most dramatic effects and describe the whole series of flow phenomena that occur when a body accelerates through transonic speeds. In Chapter 16 we have introduced the basic structure of turbulence. This chapter is different from the rest in that it tends to explain and describe only the physical processes and does not attempt to build any problem-solving capability.

We have collected the major engineering applications into three chapters: 8, 10 and 14. This format was chosen after very careful and extensive thought. We found that each of these applications involved more than one concept and hence strained the logic of topic-oriented chapters, if incorporated there. In addition, the present format permits a bit more digression into aspects of problems which are not directly obtained from the usual level of mathematics used in the other chapters, and allows the instructor to select applications of his own interest in his teaching.

In an elementary book like this, we naturally do not claim any originality except in the method of presentation and organization. We think that certain novel (for an elementary textbook) features, like the recasting of the Euler acceleration formula into a physically more meaningful form, a simpler derivation of Reynolds transport theorem, a new approach to modelling, similitude and dimensional analysis, a new introduction to boundary layers and the basis of approximations, a simpler derivation of the

Bernoulli equation, etc., will benefit the student. We have borrowed heavily from our teaching experience to anticipate the difficulties, misunderstandings, misapplications and over-generalizations that a student is prone to, and have cautioned against them.

A large number of exercise problems have been included to help the student develop his grasp further. Many problems are new and few are repetitive or simple numericals. Many extend the scope of the treatment presented through carefully guiding the student along. While compiling the problems, we have drawn from the literature of diverse engineering disciplines and have thus provided a flavour of a multidisciplinary approach. *A Teacher's Manual is available on request from the publishers.*

It is possible to cover most of the material presented in this text in a 3-hour-per week, one-semester course, with tutorial support. It is also possible to use this text in a shorter course by omitting some of the material in Chapters 8, 10, 11 and 14, and possibly the whole of Chapters 15 and 16.

Finally, it is our great pleasure to express our most sincere gratitude to Prof. J. Srinivasan for the more than considerable amount of effort and time he spent sitting with us and carefully going through many earlier drafts of this text. We doubt if this text could have evolved into this format without the benefit of his uncompromising stand for exactitude. Prof. T. A. Wilson was kind enough to go through the typescript and offer several suggestions for improvement. We also thank Professors K. S. Gandhi, R. K. Malik, M. M. Oberoi and R. Singh for their critical review of various parts of the manuscript and to a large number of co-teachers and students at IIT Kanpur who have used this text and have given us their valuable comments. We also acknowledge the financial support received from the Curriculum Development Centre of the Quality Improvement Programme, IIT Kanpur for preparation of the manuscript, to Mr. S. J. Gupta and Mr. U. S. Misra for their excellent typing, Mr. B. L. Arora and Mr. S. S. Kushwaha for the drawings and Mr. S. S. Chauhan for help in preparation of the final typescript. And, most pleasantly, we must thank our wives for providing us tea and snacks and joining us at the right moment in order to cut short our often lengthy arguments and for keeping the children from tearing our manuscript to shreds.

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SANTOSH K. GUPTA

Contents

1. Introduction to Fluid Flows

1.1 Introduction

Fluid mechanics is concerned with the study of the motions of fluids (i.e., liquids and gases) and with the forces associated with these. The subject is of great interest for two reasons. First, an understanding of fluid mechanics helps us to explain a variety of fascinating phenomena around us. Second, an understanding of this subject is essential to solve many problems encountered by an engineer.

We live in a thin layer of air that blankets the surface of the earth. The local and global movements of the air determine our weather. The origins of tornadoes, hurricanes and the monsoon can be understood only through the use of the laws of fluid mechanics. The availability of water has been associated historically with the development and flourishing of many civilizations. To utilize the available water resources optimally, we need to predict the flow rates of water in the rivers during different seasons. The prediction of floods in rivers is equally important. In recent years, we have realized that large scale discharge of effluents into the atmosphere, sea, lakes and rivers has led to serious problems of pollution. In order to control pollution, one has to know the rates of mixing and dispersion of pollutants in these natural 'sinks' nature has provided us with. The rates of mixing are affected to a large extent by the local flow patterns and, therefore, an understanding of air and water movements in the atmosphere, rivers, etc., is required before these rates of dispersion can be calculated.

We are concerned with fluids in a more intimate sense as well. The flow of blood through our arteries and veins and the flow of air through our respiratory passages into the lungs, are examples of fluid motion which one needs to understand in order to deal with circulatory and pulmonary disorders. Artificial heart-lung machines (Fig. 1.1) have been made possible only after a thorough understanding of the fluid mechanics of blood flow in the heart, and the exchange of oxygen and carbon dioxide between the blood flowing on one side and inhaled air on the other, in the lungs. Similarly, artificial kidneys are now available, which duplicate the flow of blood through the kidneys with continuous removal of the waste products from the blood as it passes through the machine. Certain voice disorders can be treated, now, with a thorough understanding of how the exhaled air interacts with the vocal cords and makes them vibrate. The relatively new science named *tribology* is focussing attention, among other things, on the lubrication of various joints in the body.

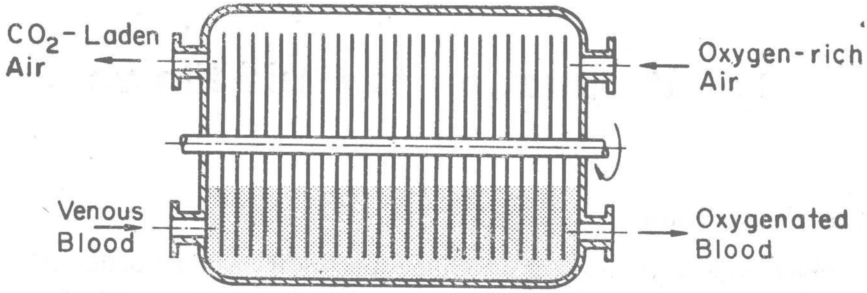


Fig. 1.1 A rotating disc artificial lung. The rotating discs pick up a thin layer of blood which is oxygenated as it comes in contact with oxygen-rich air.

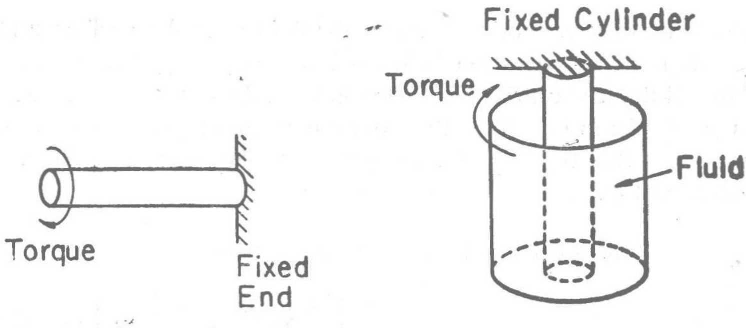
Most of the engineering applications of fluid mechanics are related to two aspects of fluid motion: one, the forces which cause or result from these motions, and the other, the effect of these fluid motions on the rates of transfer of heat and of mass (e.g., dispersion of pollutants) through the fluid body. The forces in fluids are put to such diverse uses as a sail boat, a wind mill, a hydraulic transmission, and in controlling the motion of an aircraft or a spacecraft, and even in the curving of the trajectory of a tennis ball.

Fluid motion is used to modify heat and mass transfer rates in heat exchangers, cooling towers, boilers, chimneys, artificial kidneys and heart-and-lung machines, and in the manufacture of semi-conductor devices, protection of spacecrafts from intense heating during re-entry in the earth's atmosphere, etc.

A knowledge of fluid mechanics is essential in such diverse branches of engineering as aeronautics, astronautics, automotive engineering, biomedical engineering, structural engineering, mining and metallurgical engineering, naval architecture and nuclear engineering.

1.2 Fluids

Fluids, as a class of matter, are distinguished from solids on the basis of their response to an applied shear force. If a solid bar is subjected to a torque, it twists (Fig. 1.2). The restoring elastic stresses in a solid (below the yield limit) are proportional to the strains, and therefore, a solid, when subjected to a torque, distorts through an angle θ (equilibrium distortion) such that internal stresses are developed which just balance the applied torque. The magnitude of the angle θ depends on the applied torque as well as on the elastic properties of the solid.



(a) (b)

Fig. 1.2 Difference between a solid and a liquid. The solid bar in (a) will acquire an equilibrium deformation, while the fluid in (b) will continue to deform under the action of a torque.

If, however, the torque is applied to a fluid, the behaviour is entirely different. The fluid does not acquire an equilibrium distortion but continues to deform as long as the torque acts. This behaviour is used to define a fluid. Thus, a fluid is a substance which cannot be in equilibrium under the action of any shear force, howsoever small.

Although a fluid does not resist a shear force by acquiring an equilibrium deformation, that is, the outer cylinder in Fig. 1.2 does not have an *equilibrium position* under the action of a torque, it, however, has an *equilibrium velocity*. This equilibrium value increases with the applied torque. This suggests that a fluid does resist a shear force*, not by acquiring an equilibrium deformation but by acquiring an equilibrium *rate* of deformation. Thus, a fluid deforms continuously under the action of a shear force, but at a finite rate determined by the applied shear force and the fluid properties.

1.3 Viscosity 粘度

The property which characterizes the resistance that a fluid offers to applied shear forces is termed *viscosity*. This resistance does not depend upon the deformation itself (as is the case with solids) but on the rate of deformation. Consider a fluid confined between two parallel plates, with the lower plate stationary and the upper plate moving with a velocity V_0 (Fig. 1.3). The upper plate sets the fluid in motion with a velocity V_x, which is a function of y, the vertical distance measured from the lower plate. Extensive experiments have shown that, for all real fluids possessing any viscosity, however small, the fluid particles in immediate contact with any solid surface, move with the velocity of the surface itself. That

*Otherwise the cylinder will continue to accelerate.

is, there is no relative motion between the fluid near the surface and solid surface itself. This condition is termed the *no-slip condition,* and holds good for all fluids except super-cooled helium. We shall require here that $V_x=0$ at $y=0$ and $V_x=V_0$ at the moving plate and thus, V_x changes with y. It can be seen that the rate of deformation of the fluid, in such a simple geometry, is (Fig. 1.3)

$$\text{Rate of deformation} = \frac{\partial}{\partial t}(\text{shear strain})$$
$$= \frac{\partial}{\partial t}\left[\frac{1}{\delta y}\left\{\left(V_x + \frac{dV_x}{dy}\delta y\right)\delta t - V_x\delta t\right\}\right]$$
$$= \frac{dV_x}{dy} \tag{1.1}$$

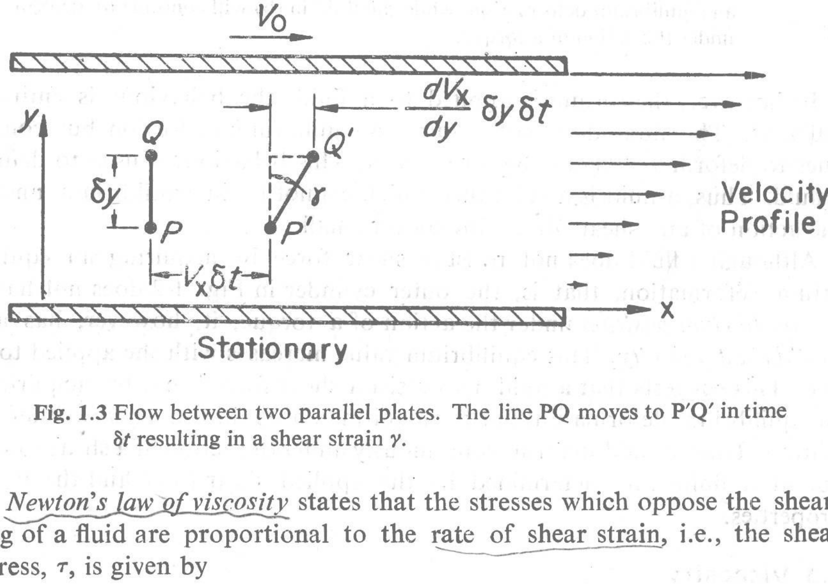


Fig. 1.3 Flow between two parallel plates. The line PQ moves to P′Q′ in time δt resulting in a shear strain γ.

Newton's law of viscosity states that the stresses which oppose the shearing of a fluid are proportional to the rate of shear strain, i.e., the shear stress, τ, is given by

$$\tau = \mu\frac{dV_x}{dy} \tag{1.2}$$

for such simple flows. The coefficient μ is termed the *viscosity** (or the dynamic viscosity) and plays an important role in the study of forces in fluid flows.

The viscosity of some fluids like air, water and glycerine is almost constant over a wide range of rates of deformation. This implies that the shear stress varies linearly with the rate of strain. Such fluids are termed as *Newtonian fluids* and in this book we shall confine our attention to such fluids only. The fluids in which the shear stress does not vary linearly with the rate

*From Equation 1.2, it can be seen that the units of viscosity are Pa s (which is the same as kg/m s). A commonly used unit is centipoise (cp) which is equal to 10^{-3} Pa s.

of strain are termed as non-Newtonian. Blood, grease and sugar solutions are some common non-Newtonian fluids. There are also some substances which cannot be classified as either fluids or solids, but show intermediate behaviour. These are called viscoelastic fluids. Both non-Newtonian and viscoelastic fluids fall outside the scope of this text. Fig. 1.4 gives a partial classification of substances based on their rheological (i.e., shear stress *vs.* rate of strain) behaviour. An ideal fluid with zero viscosity plays an important part in the study of fluids. Such a fluid offers no resistance at any rate of strain, and therefore, the upper plate in Fig. 1.3 will move with an ever increasing velocity even with the slightest of forces, if the gap between the two plates is filled with an ideal fluid.

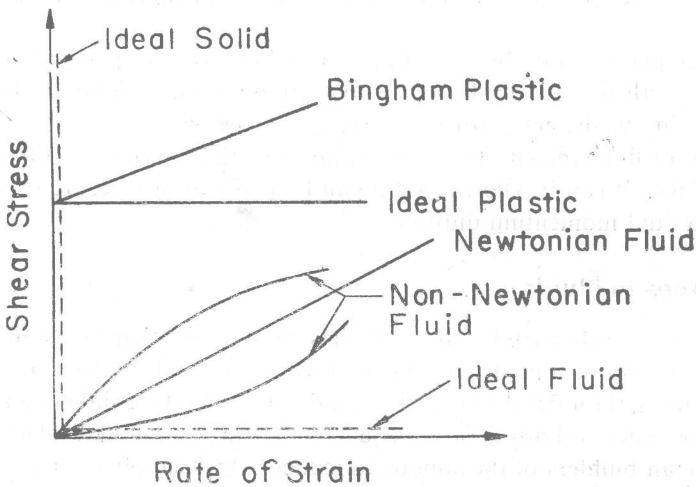


Fig. 1.4 Rheological classification of matter.

1.4 Effect of Viscosity

Consider a very large plate initially at rest in a large expanse of a stationary fluid. At time $t = 0$, the plate starts to move with a constant velocity V_0. The layer of fluid in the immediate vicinity of the plate moves with it, so that there is no relative motion between the solid and the fluid in immediate contact with it (by the no-slip condition).

As soon as the fluid in immediate contact with the plate starts moving with a finite velocity, the action of viscosity comes into play. The viscous forces tend to drag other layers of fluid along as well. Fig. 1.5 shows the velocity variations normal to the plate at various times. It is noticed that at any given time, the velocity decreases rapidly from its value V_0 as we move away from the plate and soon becomes negligible. The distance over which its value reduces to a fixed fraction of V_0 (usually 1%) is termed as the *penetration depth* and signifies the distance through which the effect of the

透入深度.

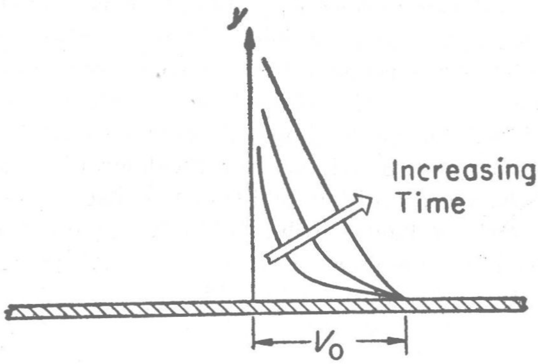


Fig. 1.5 Velocity profiles over a flat plate which is set in motion impulsively.

impulsive plate motion has penetrated into the fluid. The penetration depth increases with time. Note that this penetration (of the motion of the plate) is solely due to the action of viscosity, and if the viscosity were zero, this diffusion of fluid velocity (or momentum) into the interior would not have taken place. It can be shown that μ can be taken as a measure of the rate at which fluid momentum diffuses.

1.5 Forces in Fluids

For over two thousand years, man has been aware that fluids in motion or even at rest are capable of exerting forces on solid subjects in contact with them. Archimides discovered around 250 BC that a solid immersed in a fluid experiences a buoyant force equal to the weight of fluid displaced by it. The Roman builders of the famous aqueducts which supplied water to Rome across large distances were familiar with the relationship between the rate of flow of water in a channel and the slope of its bed. But it was only in the seventeenth century that the French mathematician B. Pascal clarified the nature of pressure—the force (measured per unit area) which stationary fluids exert on a surface. He postulated that the pressure at a point in a fluid is the same in all directions and this led to the development of the hydraulic press (Fig. 1.6).

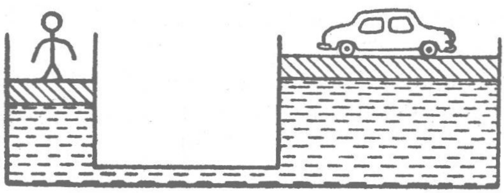


Fig. 1.6 Hydraulic ram or press.

It is evident from our common experience of walking against a strong breeze that fluid streams moving past a solid body exert a force on it in the direction of fluid flow. Similarly, a body moving through a stationary fluid experiences a force opposite to the direction of motion. The existence of this force, termed as *drag,* has been known to man for a long time. He had to overcome the drag of water when he propelled his boat or ship. He also found by experience that the shape of the hull, the portion of the boat in contact with the water, controls the drag to a large extent, and that a cusped hull gave the lowest drag (Fig. 1.7). An engineer is often called upon to calculate drag forces and to control them. A ship designer wants a hull leading to the lowest drag. An aeroplane or a racing car must also have the minimum possible drag corresponding to its size, for one has to expend power for maintaining motion of the vehicle against the drag.* Also, the designer must know the magnitude of the drag to prescribe the power of the engine required.

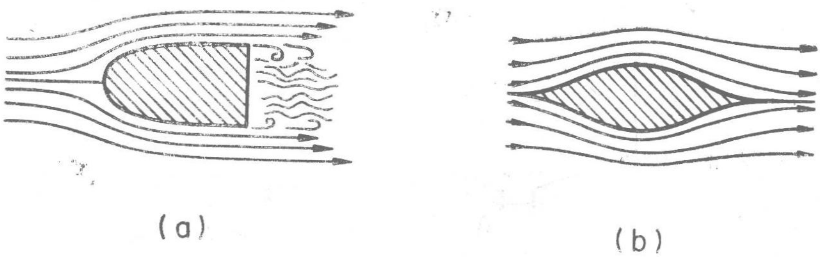


Fig. 1.7 The cusped hull in (b) gives a lower drag. Lines represent the pattern of water flow as observed from the boat.

There are several applications where an engineer wants to *maximise* the drag. A parachute exploits the large drag its canopy experiences. Its designer must prescribe a canopy diameter large enough to provide sufficient drag to overcome most of the weight, but not so large that the downward velocity becomes frustratingly low. One part of the wings of aeroplanes is raised on landing so as to be perpendicular to the direction of motion and thus, substantially increase the drag force and reduce the landing run (Fig. 1.8). Ships use a similar drag-increasing device for applying brakes.

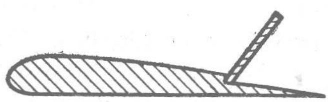


Fig. 1.8 Air brakes in an aircraft. The figure represents the cross-section of the wing. A pivoted flap is raised while landing. It increases the drag and shortens the landing run.

Some equipment used for separating light from heavy solids (or solids of different densities) in chemical and metallurgical industries rely on differen-

*The drag becomes more and more important at high speeds as will be discussed in Chapter 13.