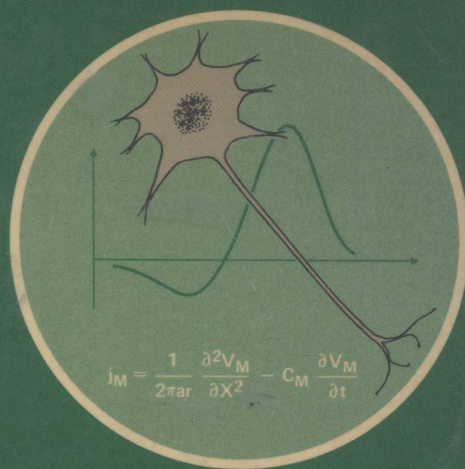


PHYSICS

WITH ILLUSTRATIVE EXAMPLES
FROM MEDICINE AND BIOLOGY

Volume 3:

ELECTRICITY AND MAGNETISM

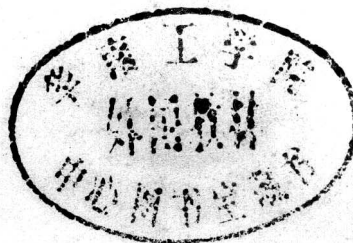


George B.
Benedek

Felix M. H.
Villars

8061603

5



PHYSICS

**WITH ILLUSTRATIVE EXAMPLES
FROM MEDICINE AND BIOLOGY**

Volume 3

ELECTRICITY AND MAGNETISM



E8051603

GEORGE B. BENEDEK

Professor of Physics,
Massachusetts Institute of Technology

FELIX M. H. VILLARS

Professor of Physics,
Massachusetts Institute of Technology



ADDISON-WESLEY PUBLISHING COMPANY
Reading, Massachusetts · Menlo Park, California
London · Amsterdam · Don Mills, Ontario · Sydney

5031887

This book is in the
ADDISON-WESLEY SERIES IN PHYSICS

Reproduced by Addison-Wesley from camera-ready copy prepared by
the authors.

Copyright © 1979 by Addison-Wesley Publishing Company, Inc.
Philippines copyright 1979 by Addison-Wesley Publishing Company,
Inc.

All rights reserved. No part of this publication may be reproduced,
stored in a retrieval system, or transmitted, in any form or by
any means, electronic, mechanical, photocopying, recording, or
otherwise, without the prior written permission of the publisher.
Printed in the United States of America. Published simultaneously
in Canada. Library of Congress Catalog Card No. 73-13556.

ISBN 0-201-00559-X
ABCDEFGHIJ-AL-79

PREFACE

With this volume on Electricity and Magnetism, we complete the third and final volume of our textbooks on Physics, with Illustrative Examples from Medicine and Biology. We believe that this volume is as unique as our previous books on Classical Mechanics (Vol. 1) and Statistical Physics (Volume 2). Here, we continue our program of interweaving into the rigorous development of classical physics, an analysis and clarification of a wide variety of important phenomena in physical chemistry, biology, physiology and medicine.

Chapter 1 contains a presentation of the fundamentals of electrostatics, including a thorough treatment of the microscopic theory of dielectric polarization and electric susceptibility of gases, non-polar and polar liquids and solids.

Chapter 2 on electric currents begins with a demonstration that the phenomenon of electrical conduction can be used to gain insight into the detailed microscopic properties of electrolyte solutions and metallic solids. A substantial analysis of electric networks having steady and time-dependent current flow then follows. Here we seek to provide the student with a fundamental and a working knowledge of practically important transducer devices and electrical circuits widely used in experimental physics and biology. The topic of circuits begins with a careful analysis of the response of the R C circuit to time varying currents and the use of this simple circuit as an integrator, a differentiator, as a source of trigger pulses. Its response to an a.c. source is used to introduce the ideas of impedance and rotating vector diagrams. An important feature of this chapter is also the extensive treatment of the principles and the applications of the operational amplifier.

Here various practically important op-amp circuits are systematically explained. The three final sections of this chapter are biological and medical applications. These are: the electrical determination of the thickness of the cell membrane by Fricke; the analysis of current flow and voltage distribution at the surface of the earth around a bolt of lightning, and finally a full account of the physical basis for scalar and vector electrocardiography as a means of measuring the electrical activity of the heart.

The subject matter of Chapter 3 is the physics of electrolyte solutions, and its application to the interpretation of bioelectric phenomena.

The chapter starts out; in section 3.1, with the development of the fundamental Nernst-Planck electro-diffusion equations. These are applied to determine diffusion- and liquid junction potentials, and to establish the concept of Debye shielding. The section concludes with a calculation of the electrophoretic mobility of macro-ions, and some illustrative examples of the use of this technique.

In part 3.2, we present and analyze the physical and chemical processes occurring in galvanic cells. After defining the concept of reversible electrodes, we analyze the factors contributing to the electromotive force of galvanic cells, and show how such emf measurements may be used to determine equilibrium constants of reactions, and to determine the pH of a solution.

Part 3.3 is devoted to the study of the electric properties of living cells. It starts out with the properties of a so called Donnan equilibrium between two fluid compartments, and introduces the concept of the Nernst potential. Data are there presented to demonstrate that most observed bio-membrane potentials require the assumption of ion pumping; the properties of

the steady state with pumping are established. In a final section, these results are applied to so called excitable cell membranes, and the main points of the Hodgkin-Huxley analysis of the action potential (nerve impulse) are presented.

Chapter 4 finally deals with magnetic and electromagnetic phenomena. This part is somewhat more condensed than it would be in a general text of this level. Nevertheless, we have attempted a careful presentation of the significant phenomena, starting with the magnetic field of steady currents, as expressed through the laws of Ampere and Biot-Savart. This is followed by a presentation of Faraday's law of induction, in its integral form. The role of electromagnetic induction for electro-mechanical energy conversion is emphasized. Self- and mutual inductance, and oscillating and coupled circuits are introduced, thus completing the discussion of circuits given in Chapter 2.

Chapter 4 closes with a brief but carefully reasoned statement of Maxwell's equations, based on a brief review of the evidence summarized by these equations. The logical and historical arguments surrounding the famous "displacement current" are briefly stated, and the existence of electromagnetic waves demonstrated. The main properties of these waves are discussed, but detailed applications were considered beyond the scope of this text.

Any lecturer in an introductory course in electricity and magnetism will realize that this text contains more material than it will be possible to present to a class in one semester. The lecturer may in fact omit several sections of his choice without disrupting the logical development of the basic

ideas.

Each chapter contains a set of problems. Most of these have been tested in the classroom at one time or another. Many of these problems are quite substantive. An attempt has been made to make them meaningful and interesting; some of them may be lengthy, but they are not exceedingly difficult.

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Professor Irving M. London, M.D., Director of the Harvard-M.I.T. Division of Health Sciences and Technology, who through his continued support has helped us to complete the writing of this text.

We would also like to thank our numerous colleagues at home and abroad, who by word or in writing have expressed their appreciation of the first two volumes of this text, and encouraged us to continue this effort by completing this third volume on Electricity and Magnetism.

Felix M. H. Villars

George B. Benedek

Cambridge, Mass.

January 12, 1979

PHYSICS: WITH ILLUSTRATIVE EXAMPLES FROM MEDICINE AND BIOLOGYVol. III Electricity and Magnetism

Table of Contents

1.	THE COULOMB FORCE; ELECTROSTATIC FIELD; ELECTROSTATIC POTENTIAL; ELECTROSTATIC ENERGY	1-1
1.1	Electric Charge and Coulomb's Law.	1-1
A.	The Electric Structure of Matter.	1-1
	i) The electrostatic force between elementary charged particles.	1-2
	ii) The additivity of electrostatic forces.	1-4
	iii) Are atoms electrically neutral?	1-8
	iv) Quantum physics and the size of the hydrogen atom.	1-11
	v) Mobility of electric charges.	1-17
B.	Definition of the Unit of Electric Charge.	1-20
	i) The electrostatic unit of charge.	1-20
	ii) The Coulomb and the flow of charge.	1-22
	iii) Coulomb's Law in the MKS-system.	1-23
	iv) Charge conservation.	1-26
C.	Summary and Program for Study of Electrostatics.	1-28
1.2	The Electric Field.	1-32
A.	Test Charges and the Definition of E . The Electric Field Set Up by Some Simply Charge Distributions.	1-32
	i) Field of a single charge Q .	1-35
	ii) Field of two charges separated a distance a from the origin.	1-37
B.	Field Lines and Their Proper Geometrical Representation.	1-41
	i) Construction of field lines.	1-41
	ii) Electric field distribution for various arrangements of point charges.	1-44
	iii) The dipole field.	1-54
1.3	The Electrostatic Potential.	1-75
A.	Coulomb's Law and the Relation between the Electric Field and the Electrostatic Potential.	1-75
B.	Work Done by a Test Charge. Line Integrals and the Electrostatic Potential.	1-79



C.	Potential of Point Charges, Dipoles; Continuous Charge Distributions, Dipole or Double Layers.	1-83
i)	Potential of a dipole.	1-83
ii)	The potential set up by an infinite line of charge.	1-87
iii)	The potential set up by a double layer.	1-89
D.	Equipotential Surfaces and Field Lines. Boundary Conditions on the Electrostatic Field and the Potential on Electric Conductors.	1-92
E.	Laplace's Equation for the Electrostatic Potential in Free Space.	1-98
i)	Stockpile of skeleton solutions of Laplace's Equations.	1-99
ii)	Metal sphere in an initially uniform electric field.	1-101
1.4	Capacitors - Electrostatic Energy Storage.	1-109
A.	The Parallel Plate Capacitor.	1-112
i)	Electric field, plate charge and capacitance of the parallel plate capacitor.	1-113
ii)	Force between the plates of a parallel plate capacitor. Electrostatic field energy.	1-115
B.	Capacitors of Cylindrical and Spherical Symmetry.	1-119
1.5	The Electrostatics of Non Conductors. Dielectrics and Electric Polarizability.	1-122
A.	Definition of Dielectric Constant and Electric Susceptibility.	1-122
B.	Induced and Permanent Dipole Moments of Atoms and Molecules. Role of the Boltzmann Factor.	1-128
i)	Electronic polarizability of atoms and molecules.	1-128
ii)	Ionic polarizability.	1-130
iii)	Permanent electric dipoles and their orientation by an electric field.	1-132
C.	Bulk Polarization P. The Relation between P and the Polarization Surface Charge Density σ_{pol} .	1-138
D.	On the Connection Between the Dielectric Constant κ and the Microscopic Polarizability α .	1-141
E.	Polarizability of Gases.	1-143
i)	Experimental determination of induced and permanent electric dipole moments.	1-143
ii)	Classical model for the electronic polarizability	

d.	The R-C circuit as integrator, time averager or filter.	2-108
e.	The R-C circuit as a differentiator.	2-110
f.	Response to a sinusoidal signal.	2-111
ii)	The elements of alternating current circuit theory.	2-120
a.	Rotating vector diagrams.	2-120
b.	The parallel R-C circuit.	2-125
c.	The compensated attenuator.	
D.	Operational Amplifiers	2-133
i)	Basic properties of an ideal operational amplifier.	2-133
ii)	Departure of actual op-amps from ideality.	2-138
iii)	Linear operational amplifier circuits using feedback.	2-139
a.	Inverting amplifier circuit.	2-139
b.	Non-inverting amplifier; the follower.	2-143
c.	Wave form addition and subtraction circuits.	2-149
d.	Circuits for integration and idfferentiation.	2-152
2.6	The Electrical Determination of the Capacitance and Thickness of the Cell Membrane.	2-158
A.	The Electrostatic Capacitance and Energy Stored in the Cell Membrane.	2-160
B.	Method of Measurement of the Cell Membrane Capacitance. Connection with Alternating Current Circuit Theory.	2-167
2.7	Charge and Current Flow in Three Dimensions: The Continuity Equation and Charge Neutrality: Quasi-Stationary Flow. Application to Lightning Bolts and Current Flow in the Earth.	2-171
A.	The Continuity Equation and Charge Neutrality in a Conducting Medium.	2-172
B.	Quasi-Stationary Current Flow.	2-177
C.	Lightning Bolts and Current Flow in the Earth.	2-179
2.8	Electrocardiography	2-186
A.	Elementary Anatomy and Electrophysiology of Heart Nerve and Muscle Cells. Double Layers and Transcellular Potential Differences.	2-187
B.	Depolarization of the Heart. The Heart Vector $\vec{P}(t)$.	2-190
C.	Electrical Activity of the Heart. $\vec{P}(t)$ for the Normal Heart.	2-193
D.	Electric Potential Lines on the Body Surface.	

The Spherical Model	2-197
E. Scalar and Vector Electrocardiography.	2-207
References	2-209
Problems on Electric Currents	2-211
 3. ELECTROCHEMISTRY AND BIO-ELECTRICITY:	3-1
3.1 Diffusion of Ions and the Nernst-Planck Equation.	3-2
A. Ion Flow Due to Diffusion and Electric Fields; the Nernst-Planck Electrodifffusion Equation.	3-2
B. Diffusion Potentials, Liquid Junctions and Salt Bridges.	3-9
i) Diffusion potential in a single electrolyte of non uniform concentration.	3-9
ii) Junction potential across boundary of two distinct electrolytes.	3-13
iii) Salt bridges; measurement of cell membrane potentials.	3-17
C. Debye Shielding. The Ion Cloud Around an Electrically Charged Surface.	3-20
i) The equilibrium ion concentrations and equili- brium electrostatic potential. The Boltzmann Factor.	3-22
ii) The Poisson-Boltzmann Equation; Debye length defined.	3-25
iii) Solution of the linearized Poisson-Boltzmann Equation.	3-27
iv) Magnitude of the Debye shielding distance. The ionic strength of a solution.	3-30
v) Can the Debye length be measured?	3-32
vi) Debye shielding of a charged sphere.	3-35
D. Electrophoresis.	3-38
i) Some observations on protein structure and electric charge.	3-39
ii) The electrophoretic mobility of a charged particle.	3-44
iii) Electrophoretic measurements.	3-53
3.2 Galvanic Cells; Electrodes; Electrochemical Measurements; pH Determination.	3-60
A. Introduction. Galvanic Cells.	3-60

B.	Self Inductance; Mutual Inductance; Electric Circuits with Inductance. Electrical Resonance.	4-49
i)	Definition of self inductance.	4-49
ii)	The inductor as a circuit element. The simple R-L circuit.	4-50
iii)	The series R-L circuit driven by an alternating voltage source.	4-56
iv)	Circuits containing resistance, capacitance and inductance. Electrical resonance.	4-60
a)	Resonant L-C circuit: oscillations.	4-62
b)	The driven series R-L-C circuit.	4-67
v)	Mutual inductance of two current loops.	4-73
vi)	The transformer as a circuit element.	4-76
4.3	Maxwell's Equations and Electromagnetic Waves.	4-83
A.	Summing Up. Statement of Maxwell's Equations.	4-83
i)	The field concept.	4-84
ii)	Charge, current and charge conservation.	4-87
iii)	Properties of the electric field.	4-89
iv)	Properties of the magnetic field.	4-94
B.	Electromagnetic Waves.	4-105
i)	Waves and the wave equation.	4-105
ii)	Electromagnetic plane waves.	4-109
iii)	Sinusoidal waves. Linear and circular polarization.	4-113
iv)	The electromagnetic spectrum.	4-117
	Problems on the Magnetic Field and Magnetic Force.	4-119
	TABLE OF BASIC CONSTANTS	4-136
	TABLE OF MKSA UNITS	4-137

Chapter 1

THE COULOMB FORCE; ELECTRIC FIELD; ELECTROSTATIC
POTENTIAL; ELECTROSTATIC ENERGY

1.1 ELECTRIC CHARGE AND COULOMB'S LAWA. The Electric Structure of Matter

One of the principal achievements of physics in the first third of the 20th Century (1900 - 1930) is the realization that the structure and properties of matter can be understood--in great quantitative detail--in terms of the electric forces between the constituent particles: electrons and atomic nuclei.

Yet, the main body of electromagnetic theory as we use it at present was all established by the end of the 19th Century. The gap in time between these two achievements shows that the electric structure of matter is extremely well hidden from everyday observation, and in a way not needed for the description of large scale electric phenomena. We shall have to explain this apparent paradox.

The study of the atomic and molecular structure has shown that the laws of electromagnetism as derived from macroscopic (= large scale) observations remain valid at the level of atomic dimensions. In that domain, however, they must be combined with the laws of quantum dynamics rather than Newtonian mechanics to account for the properties of atoms, molecules, and condensed matter in general. Our present day ability to understand the structure of matter in these terms has greatly enriched the range of electromagnetic phenomena which can be presented even in an elementary course. In the presentation of bioelectric phenomena in particular, we shall often straddle the fence between the macroscopic and the microscopic, atomistic, and see how interesting this view on both sides can be.

The first step in the introduction of the subject of electricity is

hampered by the difficulty that many "familiar" electric phenomena are in fact very complex and poorly understood. Frictional electricity, like the electric charge collected on a nylon shirt on a dry day, or the peculiar behavior of Saran Wrap, or electricity in a thundercloud, belong to the most complex and least understood electric phenomena. So we must begin elsewhere. The conceptually clearest way to begin is to start with the now established facts on the elementary constituent particles of matter: electrons, protons, neutrons. Protons and neutrons, collectively called nucleons, are the building blocks of atomic nuclei. These particles, of mass $m \sim 1.6 \times 10^{-24}$ gm, attract each other through a strong, short range force, the nuclear force. The term short range expresses the fact that, at separation $r > 10^{-13}$ cm, the force between two nucleons decreases rapidly to zero, roughly as $F \sim \frac{1}{r} e^{-r/r_0}$ with $r_0 \sim 10^{-13}$ cm. A bound system of Z protons and N neutrons form an atomic nucleus. The integer Z is called the atomic number of the nucleus, and the integer $A = N+Z$ the mass number. Z determines the chemical species of the atom.

In the neutral atom, a nucleus (Z,N) is surrounded by a "cloud" of Z electrons, within a radius of $\sim 10^{-8}$ cm from the central nucleus. The dynamics of this cloud must be described by the laws of quantum mechanics, in terms of which also the process of chemical bonding and molecular structure can be understood.

The forces which come into play here are almost exclusively the electrostatic or Coulomb force. This force operates between any two electrons, any two protons and between protons and electrons. We now describe its main attributes:

i) The Electrostatic Force Between Elementary Charged Particles

- a) The strength of the force decreases with the inverse square of the distance between the two partners.

- b) Two electrons, and two protons, repel each other; an electron and a proton attract each other.
- c) For equal distances, the magnitude of the forces between two electrons, or two protons, or an electron-proton pair, is exactly equal.

This can be summarized in the algebraic statement, expressing the force \vec{F}_{12} on a particle 1 due to a particle 2 a distance r apart:

$$\vec{F}_{12} = \pm K \frac{\hat{r}_{12}}{r^2} \quad 1.1-1$$

where \hat{r}_{12} is a unit vector ($|\hat{r}_{12}| = 1$) along the line joining particles 1 and 2. The constant K is the same, irrespective of whether particles 1 and 2 are an electron-electron, or proton-proton, or electron-proton pair. It is in fact a universal constant, whose numerical value is

$$K = 2.31 \times 10^{-19} \text{ erg cm}$$

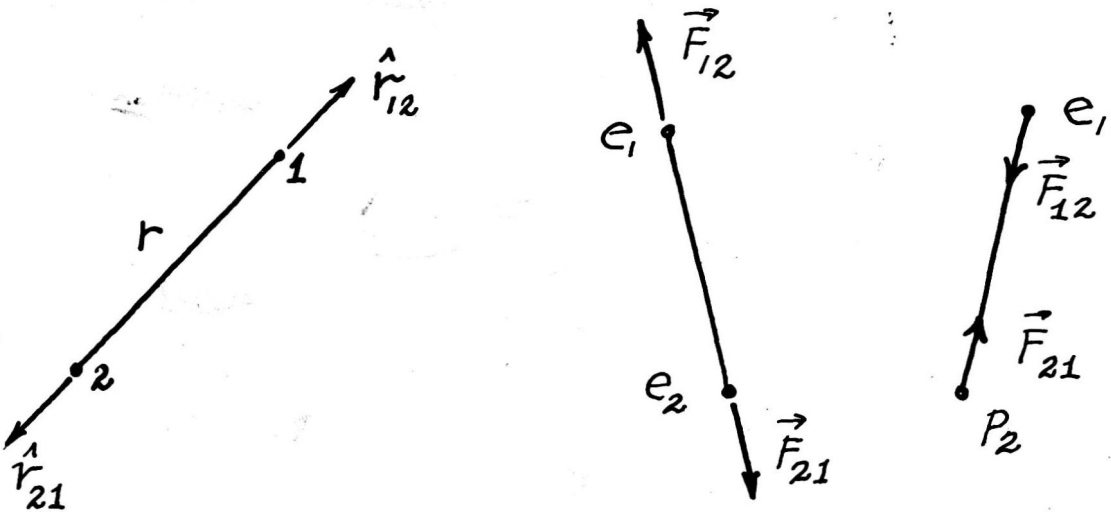


FIG. 1.1-1: DEFINITION OF \hat{r}_{12} , \hat{r}_{21} , AND THE FORCES \vec{F}_{12} , \vec{F}_{21} BETWEEN A PAIR OF CHARGED PARTICLES

Particles which attract or repel each other by the Coulomb force are called electrically charged, or simply charged. There is no Coulomb force acting between electrons and neutrons, or protons and neutrons, nor between two

neutrons. Particles such as the neutron are called electrically neutral or uncharged. For a Coulomb force to be exerted between two particles, both particles have to be charged.

The facts expressed so far can be further condensed by introducing the concept of the charge number z of a particle: This is an integer, with the following assignments

$$\text{proton: } z = z_p = 1$$

$$\text{electron: } z = z_e = -1$$

$$\text{neutron: } z = z_o = 0$$

We can then express the Coulomb law of force between any pair formed from the three types of particles by writing

$$\vec{F}_{12} = K z_1 z_2 \frac{\hat{r}_{12}}{r^2} \quad 1.1-2$$

For 2 protons or 2 electrons, $z_1 z_2 = +1$, and equation [1.1-2] expresses their repulsion; for an electron-proton pair, $z_1 z_2 = -1$, and [1.2] expresses their attraction. If one partner (or both) is a neutron, then $z_1 z_2 = 0$, and [1.1-2] expresses the absence of a Coulomb force between the 2 partners. We will deal in a later section with the question of how the value of K is actually determined, and what the implications are of this value for the properties of matter.

ii) The Additivity of Electrostatic Forces

The question now arises: How does this above formulation generalize to a case where more than two charged particles are present?

All evidence points to the fact that in this case, the net electrostatic force on a charged particle is just the vector sum of the forces exerted by every other charge that is present: To illustrate this, let us consider an electron ($z_e = -1$), subject to the Coulomb force of another