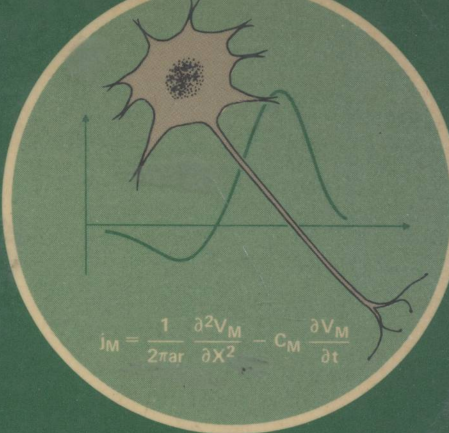
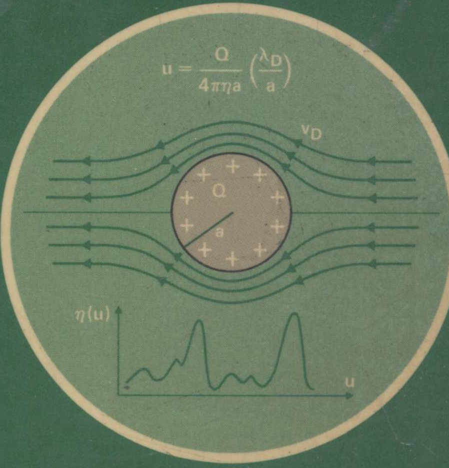
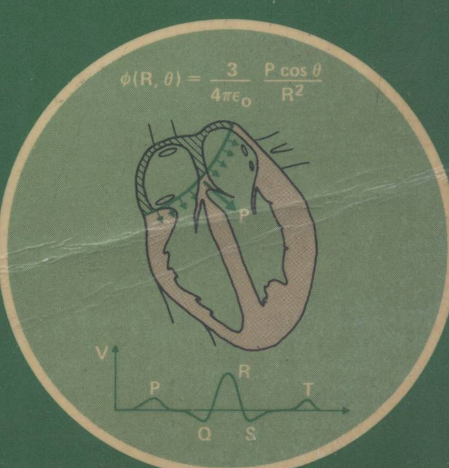
PHYSICS

WITH ILLUSTRATIVE EXAMPLES FROM MEDICINE AND BIOLOGY

Volume 3:
ELECTRICITY AND MAGNETISM







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PREFACE

With this volume on Electricity and Magnetism, we complete the third and final volume of our textbooks on Physics, with Illustrative Examples from Medicine and Biology. We believe that this volume is as unique as our previous books on Classical Mechanics (Vol. 1) and Statistical Physics (Volume 2). Here, we continue our program of interweaving into the rigourous development of classical physics, an analysis and clarification of a wide variety of important phenomena in physical chemistry, biology, physiology and medicine.

Chapter 1 contains a presentation of the fundamentals of electrostatics, including a thorough treatment of the microscopic theory of dielectric polarization and electric susceptibility of gases, non-polar and polar liquids and solids.

Chapter 2 on electric currents begins with a demonstration that the phenomenon of electrical conduction can be used to gain insight into the detailed microscopic properties of electrolyte solutions and metallic solids. A substantial analysis of electric networks having steady and time-dependent current flow then follows. Here we seek to provide the student with a fundamental and a working knowledge of practically important transducer devices and electrical circuits widely used in experimental physics and biology. The topic of circuits begins with a careful analysis of the response of the R C circuit to time varying currents and the use of this simple circuit as an integrator, a differentiator, as a source of trigger pulses. Its response to an a.c. source is used to introduce the ideas of impedance and rotating vector diagrams. An important feature of this chapter is also the extensive treatment of the principles and the applications of the operational amplifier.

Here various practically important op-amp circuits are systematically explained. The three final sections of this chapter are biological and medical applications. These are: the electrical determination of the thickness of the cell membrane by Fricke; the analysis of current flow and voltage distribution at the surface of the earth around a bolt of lightning, and finally a full account of the physical basis for scalar and vector electrocardiography as a means of measuring the electrical activity of the heart.

The subject matter of Chapter 3 is the physics of electrolyte solutions, and its application to the interpretation of bioelectric phenomena.

The chapter starts out; in section 3.1, with the development of the fundamental Nernst-Planck electro-diffusion equations. These are applied to determine diffusion- and liquid junction potentials, and to establish the concept of Debye shielding. The section concludes with a calculation of the electrophoretic mobility of macro-ions, and some illustrative examples of the use of this technique.

In part 3.2, we present and analyze the physical and chemical processes occurring in galvanic cells. After defining the concept of reversible electrodes, we analyze the factors contributing to the electromotive force of galvanic cells, and show how such emf measurements may be used to determine equilibrium constants of reactions, and to determine the pH of a solution.

Part 3.3 is devoted to the study of the electric properties of living cells. It starts out with the properties of a so called Donnan equilibrium between two fluid compartments, and introduces the concept of the Nernst potential. Data are there presented to demonstrate that most observed biomembrane potentials require the assumption of ion pumping; the properties of the steady state with pumping are established. In a final section, these results are applied to so called excitable cell membranes, and the main points of the Hodgkin-Huxley analysis of the action potential (nerve impulse) are presented.

Chapter 4 finally deals with magnetic and electromagnetic phenomena. This part is somewhat more condensed than it would be in a general text of this level. Nevertheless, we have attempted a careful presentation of the significant phenomena, starting with the magnetic field of steady currents, as expressed through the laws of Ampere and Biot-Savart. This is followed by a presentation of Faraday's law of induction, in its integral form. The role of electromagnetic induction for electro-mechanical energy conversion is emphasized. Self- and mutual inductance, and oscillating and coupled circuits are introduced, thus completing the discussion of circuits given in Chapter 2.

Chapter 4 closes with a brief but carefully reasoned statement of Maxwell's equations, based on a brief review of the evidence summarized by these equations. The logical and historical arguments surrounding the famous "displacement current" are briefly stated, and the existence of electromagnetic waves demonstrated. The main properties of these waves are discussed, but detailed applications were considered beyond the scope of this text.

Any lecturer in an introductory course in electricity and magnetism will realize that this text contains more material than it will be possible to present to a class in one semester. The lecturer may in fact omit several sections of his choice without disrupting the logical development of the basic ideas.

Each chapter contains a set of problems. Most of these have been tested in the classroom at one time or another. Many of these problems are quite substantive. An attempt has been made to make them meaningful and interesting; some of them may be lengthy, but they are not exceedingly difficult.

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Professor Irving M. London, M.D., Director of the Harvard-M.I.T. Division of Health Sciences and Technology, who through his continued support has helped us to complete the writing of this text.

We would also like to thank our numerous colleagues at home and abroad, who by word or in writing have expressed their appreciation of the first two volumes of this text, and encouraged us to continue this effort by completing this third volume on Electricity and Magnetism.

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January 12, 1979

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Vol. III Electricity and Magnetism

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Chapter 1

THE COULOMB FORCE; ELECTRIC FIELD; ELECTROSTATIC POTENTIAL; ELECTROSTATIC ENERGY

1.1 ELECTRIC CHARGE AND COULOMB'S LAW

A. The Electric Structure of Matter

One of the principal achievements of physics in the first third of the 20th Century (1900 - 1930) is the realization that the structure and properties of matter can be understood--in great quantitative detail--in terms of the electric forces between the constituent particles: electrons and atomic nuclei.

Yet, the main body of electromagnetic theory as we use it at present was all established by the end of the 19th Century. The gap in time between these two achievements shows that the electric structure of matter is extremely well hidden from everyday observation, and in a way not needed for the description of large scale electric phenomena. We shall have to explain this apparent paradox.

The study of the atomic and molecular structure has shown that the laws of electromagnetism as derived from macroscopic (= large scale) observations remain valid at the level of atomic dimensions. In that domain, however, they must be combined with the laws of quantum dynamics rather than Newtonian mechanics to account for the properties of atoms, molecules, and condensed matter in general. Our present day ability to understand the structure of matter in these terms has greatly enriched the range of electromagnetic phenomena which can be presented even in an elementary course. In the presentation of bioelectric phenomena in particular, we shall often straddle the fence between the macroscopic and the microscopic, atomistic, and see how interesting this view on both sides can be.

The first step in the introduction of the subject of electricity is hampered by the difficulty that many "familiar" electric phenomena are in fact very complex and poorly understood. Frictional electricity, like the electric charge collected on a nylon shirt on a dry day, or the peculiar behavior of Saran Wrap, or electricity in a thundercloud, belong to the most complex and least understood electric phenomena. So we must begin elsewhere. The conceptually clearest way to begin is to start with the now established facts on the elementary constituent particles of matter: electrons, protons, neutrons. Protons and neutrons, collectively called nucleons, are the building blocks of atomic nuclei. These particles, of mass $m \sim 1.6 \times 10^{-24}$ gm, attract each other through a strong, short range force, the nuclear force. The term short range expresses the fact that, at separation $r > 10^{-13}$ cm, the force between two nucleons decreases rapidly to zero, roughly as $F \sim \frac{1}{r} e^{-r/r_o}$ with $r_o \sim 10^{-13}$ cm. A bound system of Z protons and N neutrons form an atomic nucleus. The integer Z is called the atomic number of the nucleus, and the integer A = N+Z the mass number. Z determines the chemical species of the atom.

In the neutral atom, a nucleus (Z,N) is surrounded by a "cloud" of Z electrons, within a radius of $\sim 10^{-8}$ cm from the central nucleus. The dynamics of this cloud must be described by the laws of quantum mechanics, in terms of which also the process of chemical bonding and molecular structure can be understood.

The forces which come into play here are almost exclusively the electrostatic or Coulomb force. This force operates between any two electrons, any two protons and between protons and electrons. We now describe its main attributes:

i) The Electrostatic Force Between Elementary Charged Particles

a) The strength of the force decreases with the inverse square of the distance between the two partners.

b) Two electrons, and two protons, repel each other; an electron and a proton attract each other.

c) For equal distances, the magnitude of the forces between two electrons, or two protons, or an electron-proton pair, is exactly equal.

This can be summarized in the algebraic statement, expressing the force $\vec{F}_{12}$ on a particle 1 due to a particle 2 a distance r apart:

$$\vec{F}_{12} = \pm K \frac{\hat{r}_{12}}{r^2} \qquad 1.1\text{-}1$$

where $\hat{r}_{12}$ is a unit vector ($|\hat{r}_{12}| = 1$) along the line joining particles 1 and 2. The constant K is the same, irrespective of whether particles 1 and 2 are an electron-electron, or proton-proton, or electron-proton pair. It is in fact a universal constant, whose numerical value is

$$K = 2.31 \times 10^{-19} \text{ erg cm}$$

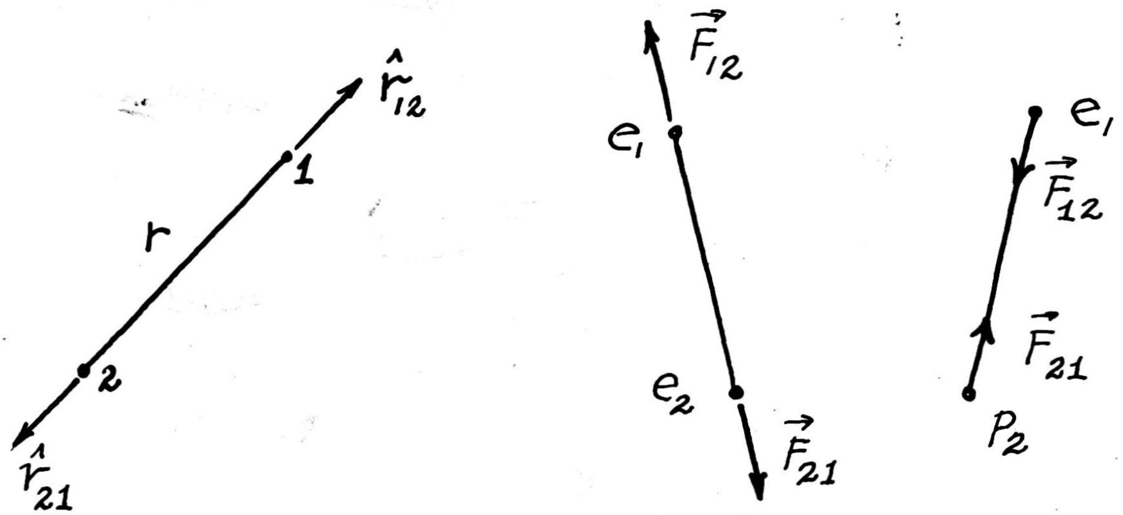

FIG. 1.1-1: DEFINITION OF $\hat{r}_{12}$, $\hat{r}_{21}$, AND THE FORCES $\vec{F}_{12}$, $\vec{F}_{21}$ BETWEEN A PAIR OF CHARGED PARTICLES

Particles which attract or repel each other by the Coulomb force are called electrically charged, or simply charged. There is no Coulomb force acting between electrons and neutrons, or protons and neutrons, nor between two neutrons. Particles such as the neutron are called electrically neutral or uncharged. For a Coulomb force to be exerted between two particles, both particles have to be charged.

The facts expressed so far can be further condensed by introducing the concept of the charge number z of a particle: This is an integer, with the following assignments

$$\text{proton:} \quad z = z_p = 1$$
$$\text{electron:} \quad z = z_e = -1$$
$$\text{neutron:} \quad z = z_o = 0$$

We can then express the Coulomb law of force between any pair formed from the three types of particles by writing

$$\vec{F}_{12} = K \quad z_1 z_2 \frac{\hat{r}_{12}}{r^2} \qquad 1.1\text{-}2$$

For 2 protons or 2 electrons, $z_1 z_2 = +1$, and equation [1.1-2] expresses their repulsion; for an electron-proton pair, $z_1 z_2 = -1$, and [1.2] expresses their attraction. If one partner (or both) is a neutron, then $z_1 z_2 = 0$, and [1.1-2] expresses the absence of a Coulomb force between the 2 partners. We will deal in a later section with the question of how the value of K is actually determined, and what the implications are of this value for the properties of matter.

ii) The Additivity of Electrostatic Forces

The question now arises: How does this above formulation generalize to a case where more than two charged particles are present?

All evidence points to the fact that in this case, the net electrostatic force on a charged particle is just the vector sum of the forces exerted by every other charge that is present: To illustrate this, let us consider an electron ($z_e = -1$), subject to the Coulomb force of another