Spin glasses

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Preface

Spin glasses are a fascinating new topic in condensed matter physics which developed essentially after the middle of the 1970's. The aim of this book is to give an introduction to it which will both attract the newcomer to the field (say, a student with a basic knowledge of solid state physics and statistical mechanics) and give a comprehensive survey to the expert who perhaps has worked on a very specific problem. It is a field which is still open to new ideas and concepts and in which important new experiments can certainly still be done.

Our understanding of spin glasses is based on three approaches: theory, experiment, and computer simulation. We have tried to present the most important developments in all of them. One possibility is to take the theory as a guide and to check it by comparison with experimental data and simulations. This is roughly what we do in the first part of this book (Chapters 3 to 6), after introducing the basic experiments, models and concepts which define what we are talking about. (Spin glasses are disordered systems, so we have to introduce several concepts which are unknown in the 'classical' theory of ideal solids.)

In Chapters 3 to 6 we discuss a mean field theory, which is so far the only well-established spin glass theory. It turns out to be highly nontrivial and has been developed over more than a decade. Its underlying ideas have also proved to be fruitful in optimization problems and the theory of neural networks. This led us to include a brief account of these subjects in Chapter 14 (entitled 'the physics of complexity').

However, the mean field theory gives only a hint about what happens in real spin glasses, and in Chapters 7 to 11 we rely more and more on experiment and computer simulation. Here the concepts of scaling and renormalization permit us considerable insight into the spin glass phase and the transition between it and the paramagnetic phase, and the idea of 'frustration' gives at least a feeling of the fundamental difference between ideal periodic solids and disordered ones.

This book is not a review. In the early and mid-1980's more than 400 papers per year were written on spin glasses (altogether more than 4000), and it would be completely hopeless to discuss or even mention them all. Rather, we have tried to present the most important ideas and developments in the field. Naturally, this is a very personal selection, and we want to apologize to the thousands of authors whose interesting papers we could not mention. Some of these papers have been discussed in the excellent review of Binder and Young (1986), in the somewhat older reviews by one of us (Fischer, 1983c, 1985), in the Heidelberg Colloquia on spin glasses and on glassy dynamics (van Hemmen and Morgenstern, 1983, 1986), and in the books by Chowdhury (1986) and Mézard et al (1987).

Our understanding of spin glasses has grown over many years, and it is a pleasure for us to thank the large number of our colleagues who contributed to it. We especially want to thank Philip Anderson, Alan Bray, Cyrano De Dominicis, Anil Khurana, Wolfgang Kinzel, Richard Klemm, Hans Maletta, Mike Moore, Richard Palmer, Hans-Jürgen Sommers, Peter Young, and Annette Zippelius.

We are also very grateful to Mrs Ch. Hake, who typed a large part of this book in TEX.

Contents

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Introduction

Questo é quel pezzo di calamita:
Pietra mesmerica, ch'ebbe l'origine nell'Alemagna,
Che poi sì celebre là in Francia fu.[1]

Lorenzo da Ponte, *Così fan Tutte*, Act I

One of the dominant themes in the history of physics in this century has been the effort to understand condensed states of matter. This began with very simple systems — the Van der Waals description of the liquid–gas transition and the Weiss mean field theory of ferromagnetism — and has gradually developed to include more and more complex and subtle states and phenomena. Spin glasses are the current frontier in this development, the most complex kind of condensed state encountered so far in solid state physics.

In trying to understand these systems, experimentalists have used a wide spectrum of probes in ingenious ways, and theorists have invented an equally wide variety of models and new theoretical concepts. The resulting developments have had an impact, not only on other parts of physics, but also on other fields such as computer science, mathematics, and biology. It is because of this widespread influence and the interest in spin glasses that it has aroused that we are writing this book.

We expect that many people who read this book will be condensed mat-

[1] This is a magnet: the mesmerizing stone discovered in Germany and then so famous in France.

ter physicists. However, we also have in mind as a typical reader someone from another area in physics, or perhaps a graduate student looking for a research topic, who wants to find out what all the excitement is about. She need not be a condensed matter physicist, or even a physicist at all, though we do assume a reasonable knowledge of basic statistical mechanics. With her in mind, we begin with some basic questions.

First: What is a spin glass? The simplest answer (which we will naturally have to improve on in the course of succeeding paragraphs and chapters) is that it is a collection of spins (i.e. magnetic moments) whose low-temperature state is a frozen disordered one, rather than the kind of uniform or periodic pattern we are accustomed to finding in conventional magnets. It appears that in order to produce such a state, two ingredients are necessary: There must be competition among the different interactions between the moments, in the sense that no single configuration of the spins is uniquely favoured by all the interactions (this is commonly called 'frustration'), and these interactions must be at least partially random. These facts suggest that the spin glass state is intrinsically different from conventional forms of order and requires new formal concepts to describe it. This challenge has been the fundamental motivation for theorists in this field.

Experimentally, it does not seem to be hard to find spin glasses. Quite the contrary, spin glass behaviour has been seen in virtually every kind of system which people have been able to make that satisfies these requirements.

The first kind of system to be studied widely consisted of dilute solutions of magnetic transition metal impurities in noble metal hosts. The impurity moments produce a magnetic polarization of the host metal conduction electrons around them which is positive at some distances and negative at others. Other impurity moments then feel the local magnetic field produced by the polarized conduction electrons and try to align themselves along it. Because of the random placement of the impurities, some of interactions are positive (i.e. favouring parallel alignment of the moments) and some are negative. Thus we clearly have random, competing interactions.

At one time, many people believed that spin glass behaviour was sensitively dependent on particular features of this special class of systems. But we now know that this is not so. Spin glass states have also been found in magnetic insulators and in amorphous alloys, where the dependence of the interactions on the distance between the moments is entirely different from that in the above crystalline metallic systems. The 'spin' degrees of freedom need not even be magnetic. Properties analogous to those of spin glasses, with the electric dipole moment taking the place of the magnetic one, have been seen in ferroelectric–antiferroelectric mixtures, and a kind of orientational freezing has been observed in disordered molecular crystals in which the electric quadrupole moment plays the role of the spin. This universal nature of the observed phenomena is another reason for thinking this is an important problem to study. The next fundamental question we ask is how we observe such a state.

The description 'frozen disorder' suggests that we are dealing with a state where the local spontaneous magnetization $m_i = \langle S_i \rangle$ at a given site i is nonzero, though the average magnetization $M = N^{-1} \sum_i m_i$, as well as any 'staggered' magnetization $M_K = N^{-1} \sum_i e^{-i\mathbf{K}\cdot\mathbf{r}_i} m_i$, vanishes. That the low-temperature state was not an antiferromagnet was indicated by neutron scattering experiments, which showed no magnetic Bragg peaks which would have indicated long range order. (Here and henceforth $\langle S_i \rangle$ means the conventional thermal average, and the magnetization is in units of $-g\mu_B$.)

The local spontaneous magnetizations make their presence felt in an experiment because they reduce the susceptibility from the value it otherwise would have. This effect is in fact familiar from antiferromagnets, where a sharp reduction in the susceptibility from its extrapolated high-temperature form signals the onset of antiferromagnetic order. The same thing happens in spin glasses, and Fig. 1.1 shows examples of some susceptibility measurements that played a key role in arousing the interest in this field that exploded in the mid-1970's. They exhibit a marked cusp at a temperature which is rather sharply defined, suggesting a second-order phase transition between the disordered paramagnetic state and a spin glass state characterized by nonvanishing local spontaneous magnetizations m_i. The difference between the measured susceptibility and the extrapolation of the high-temperature form should be some measure of the degree of freezing. Immediately, people wanted to know in what ways this transition (if, indeed, it was a sharp transition) was like ordinary second-order phase transitions and in what ways it might be different.

The connection between the susceptibility and the existence of frozen moments can be made more explicit by supposing we have a system of Ising spins ($S_i = \pm 1$) and considering the single-site susceptibility χ_{ii} defined as the amount of magnetization m_i induced at site i by an external field $B_i = -h_i/g\mu_B$ acting only on this site:

$$\chi_{ii} \equiv \frac{\partial m_i}{\partial h_i} \tag{1.1}$$

A fundamental theorem of classical statistical mechanics (see, e.g. Landau and Lifshitz, 1969) relates the equilibrium fluctuations of any thermodynamic variable to the mean amount of this variable induced by a conjugate field. The present case affords the simplest possible example of this relation. It says (in units where the Boltzmann constant $k_B = 1$)

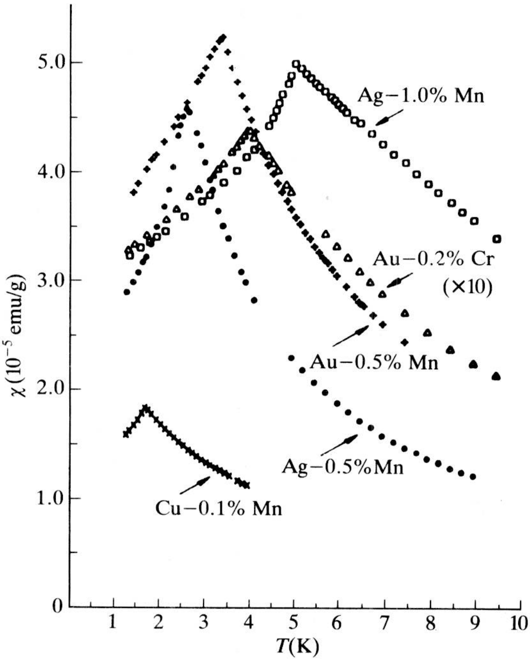

Figure 1.1: The ac susceptibility of Cu-0.1% Mn (×), Ag-0.5% Mn (•), Au-0.5% Mn (+), Au-0.2% Cr (△), and Ag-1.0% Mn (□) versus temperature for a magnetic field $H = 20$ Oe and 100 Hz (from Cannella and Mydosh, 1972, 1974).

$$T\chi_{ii} = \langle (S_i - \langle S_i \rangle)^2 \rangle = 1 - m_i^2 \tag{1.2}$$

where the last step uses explicitly the fact that $S_i^2 = 1$. Averaging over all the sites in the system gives

$$\chi_{loc} \equiv \frac{1}{N} \sum_i \chi_{ii} = \frac{1 - N^{-1} \sum_i m_i^2}{T} \tag{1.3}$$

That is, the reduction of the average local susceptibility χ_{loc} from the Curie law characteristic of free moments is a direct measure of the mean square local spontaneous magnetization in the frozen state. Although the experiments of Fig. 1.1 do not measure the local susceptibility, but rather the so-called uniform susceptibility (which we denote by χ without any subscript)

$$\chi = \frac{1}{N} \sum_{ij} \chi_{ij} = \frac{1}{N} \sum_{ij} \frac{\partial m_i}{\partial h_j} \tag{1.4}$$

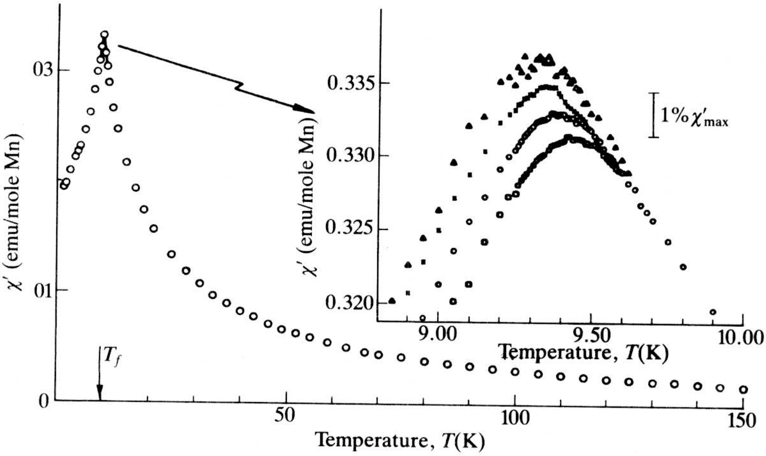

Figure 1.2: The ac susceptibility as a function of temperature for Cu-0.9% Mn for the frequencies 1.33 kHz (□), 234 Hz (○), 10.4 Hz (x), and 2.6 Hz (△) (from Mulder et al, 1981, 1982).

it can be shown (Fischer, 1976) that the off-diagonal elements of χ_{ij} vanish (in zero field) if the interactions between different spins are symmetrically distributed. More generally, the uniform χ will have a cusp if χ_{loc} does, so the experiments really do indicate the existence of a nonzero frozen spontaneous magnetization — a spin glass state.

The freezing temperature T_f, defined by the cusp in the ac susceptibility as seen in Fig. 1.1, actually turns out to depend on the frequency of the applied magnetic field. The 'true' T_f should therefore be defined by the limit of vanishing frequency. Furthermore,the cusp is not completely sharp, as shown in Fig. 1.2 for CuMn (which is one of the best investigated spin glass systems). In this more precise experiment one has to distinguish between the in-phase or real part $\chi'(\omega, T)$ and the out of phase or imaginary part $\chi''(\omega, T)$ of the complex susceptibility $\chi(\omega, T) = \chi'(\omega, T) + i\chi''(\omega, T)$.

There is a crude phenomenology for describing these slow dynamics (Lundgren et al, 1981; van Duyneveldt and Mulder, 1982) for frequencies in the range shown in Fig. 1.2. Below $T_f(\omega)$, the real part of $\chi(\omega)$ varies approximately logarithmically with frequency:

$$\chi'(\omega) = \chi_0 + a \ln\left(\frac{1}{|\omega|}\right) \tag{1.5}$$

Then the Kramers–Kronig relations imply a roughly frequency-independent χ'':

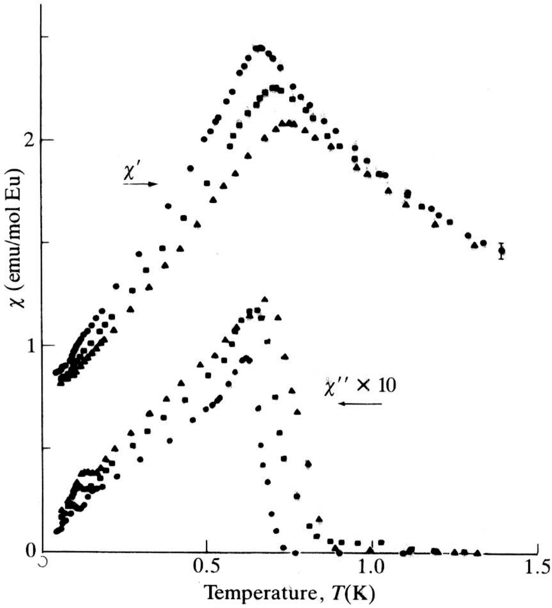

Figure 1.3: Temperature dependence of the real (solid symbols) and imaginary (open symbols) parts of the susceptibility for $Eu_{0.2}Sr_{0.8}S$ at an applied field $H \approx 0.1$ Oe (from Hüser et al, 1983).

$$\chi'' = \frac{\pi a}{2} \operatorname{sgn} \omega \tag{1.6}$$

which is independently measurable (see Fig. 1.3).

The logarithmic dependence is not exact, just a rather good fit, and other functional forms such as a power law with a small exponent work as well. For much lower frequencies (i.e. times sufficiently longer than those characteristic of these experiments) the frequency dependence of χ' seems to disappear, indicating that a true equilibrium limit $\chi(0)$ has been reached. For much higher frequencies, the simple approximate frequency dependence of (1.5)–(1.6) breaks down, but the qualitative feature of frequency dependence extending over many orders of magnitude in frequency, from microscopic characteristic frequencies to the inverse of the longest experimental measuring times, has been found in a wide variety of experiments and in essentially all spin glass systems. This universal feature sets spin glasses apart from conventional magnets, where no significant frequency dependence is observed for frequencies much lower than the characteristic microscopic frequencies of the system.

The presence of this 'glassy' behaviour with such long characteristic

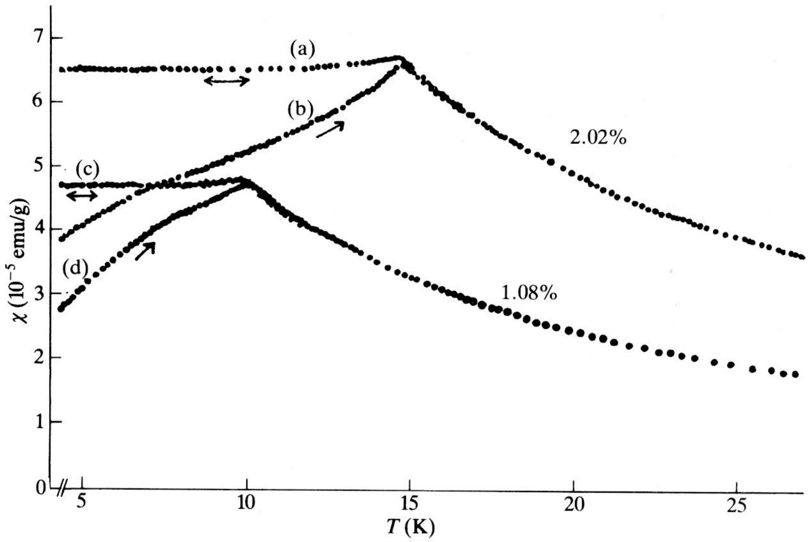

Figure 1.4: The static susceptibility of CuMn vs temperature for 1.08 and 2.02% Mn. After zero-field cooling ($H < 0.05$ Oe), initial susceptibilities (b) and (d) were taken for increasing temperature in a field of $H = 5.90$ Oe. The susceptibilities (a) and (c) were obtained in the field $H = 5.90$ Oe, which was applied above T_f before cooling the samples (from Nagata et al, 1979).

times suggests the possible presence of many metastable spin configurations with a distribution of energy barriers separating them. (Assuming that the typical time to cross a barrier depends exponentially on its height ΔE ($\tau \propto \exp(\beta\Delta E)$), we do not require too broad a distribution of barrier heights in order to get a very wide relaxation time distribution at low temperatures.)

Another important feature of all spin glasses is the onset of remanence effects below T_f. This is illustrated in Fig. 1.4 for the dc susceptibility of CuMn as measured in extremely small fields (0.05 Oe $\leq$ H $\leq$ 5.9 Oe). Even in these small fields χ_{dc} for $T < T_f$ depends strongly on the way the experiment is performed: $\chi_{dc}(T)$ is largest (and roughly temperature-independent) after 'field-cooling', i.e. if the field is applied above T_f and the sample subsequently cooled in this field to a temperature below T_f. This measurement is, to a very good approximation, reversible; that is, one can go up and down in temperature and measure the same magnetization, independent of history. This is in contrast to the 'zero-field-cooled' susceptibility χ_{zfc}, obtained by cooling the sample below T_f in zero field and then applying the field. After applying a field at a temperature below T_f, the magnetization jumps to a finite value, followed by a slow additional increase. This 'irreversible' contribution to χ_{zfc} decays only very slowly if the field is suddenly switched off.

The difference between χ_{ac} (Figs. 1.1 and 1.2) and χ_{dc} and the remanence effects in χ_{dc} have the same origin as the frequency dependence of χ_{ac} discussed above: the glass-like nature of the system below T_f. There are many roughly equivalant spin configurations, and the state which is reached depends crucially on details of the experiment such as the frequency and magnitude of the applied field, the speed with which one cools down, and whether one cools in zero or finite field.

There is also a difference at higher fields between the zero-field-cooled remanent magnetization, in which the field is applied at the measuring temperature and then switched off again ('isothermal' remanent magnetization (IRM)), and the 'thermoremanent' magnetization (TRM), that is, the magnetization remaining when the field is switched off after field-cooling. Fig. 1.5 shows the field dependence of these remanent magnetizations in $\underline{\text{Au}}\text{Fe}$. Again, both of them are time-dependent: Fig. 1.6a shows the decay of the IRM plotted against $\ln t$, which suggests a decay law

$$M_R(t) = M_0 - S_{RM} \ln t \tag{1.7}$$

This is in contrast to EuSrS (which again is a 'standard' spin glass), in which the power law

$$M_R(t) \propto t^{-a(T,H)} \tag{1.8}$$

indicated in Fig. 1.6b is found. Finally, one can also fit data by an exponential function of a power law:

$$M_R(t) \propto \exp[-(t/\tau)^{\beta}] \tag{1.9}$$

as indicated in Fig. 1.6c for $\underline{\text{Ag}}\text{Mn}$, the exponent β being about 1/3 for T not too close to T_f. Any of these very slow decay laws is consistent with our qualitative ideas about the glassiness of the spin glass state, with many possible configurations separated by barriers of varying heights. However, this variety of fits in different systems (if it is meaningful) suggests that perhaps not all spin glass properties are universal and that the glass-like structure might vary in its details from system to system.

A similar non-universal property is the hysteresis of the magnetization. An example ($\underline{\text{Cu}}\text{Mn}$) is shown in Fig. 1.7. As in ferromagnets, hysteresis effects are due to *anisotropy*, which might be extremely different in the various spin glass systems. The origin of this anisotropy will be discussed in Section 6.3.