



Design with Constructal Theory

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Published by John Wiley & Sons, Inc., Hoboken, New Jersey Published simultaneously in Canada

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Library of Congress Cataloging-in-Publication Data

Bejan, Adrian.

Design with constructal theory / Adrian Bejan, Sylvie Lorente.

p. cm.

Includes index.

ISBN 978-0-471-99816-7 (cloth)

1. Design, Industrial. I. Lorente, Sylvie. II. Title.

TS171.4.B43 2008

745.2-dc22

2008003739

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Design with Constructal Theory

About the Authors

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Sylvie Lorente received all her degrees in civil engineering (BS, 1992; MS, 1992; PhD, 1996) from the National Institute of Applied Sciences (INSA), Toulouse. She is Professor of Civil Engineering at the University of Toulouse, INSA, and is affiliated with the Laboratory of Durability and Construction of Materials, LMDC. Her work covers several fields, including heat transfer in building structures, fluid mechanics, and transport mechanisms in cement based materials. She is the author of 70 peer-referred articles and three books. Sylvie Lorente received the 2004 Edward F. Obert Award and the 2005 Bergles-Rohsenow Young Investigator Award in Heat Transfer from the American Society of Mechanical Engineers, and the 2007 James P. Hartnett Award from the International Center of Heat and Mass Transfer.

Constructal theory advances are posted at www.constructal.org.

This book is the new design course that we have developed on several campuses during the past five years. The approach is new because it is based on constructal theory—the view that flow configuration (geometry, design) can be reasoned on the basis of a principle of configuration generation and evolution in time toward greater global flow access in systems that are free to morph. The generation of flow configuration is viewed as a physics phenomenon, and the principle that sums up its universal occurrence in nature (the *constructal law*, p. 2) is deterministic.

Constructal theory provides a broad coverage of "designedness" everywhere, from engineering to geophysics and biology. To see the generality of the method, consider the following metaphor, which we use in the introductory segment of the course. Imagine the formation of a river drainage basin, which has the function of providing flow access from an area (the plain) to one point (the river mouth). The constructal law calls for configurations with successively smaller global flow resistances in time. The invocation of this law leads to a balancing of all the internal flow resistance, from the seepage along the hill slopes to the flow along all the channels. Resistances (imperfection) cannot be eliminated. They can be matched neighbor to neighbor, and distributed so that their global effect is minimal, and the whole basin is the least imperfect that it can be. The river basin morphs and tends toward an equilibrium flow-access configuration.

The visible and valuable product of this way of thinking is the *configuration*: the river basin, the lung, the tree of cooling channels in an electronics package, and so on. The configuration is the big unknown in design: the constructal law draws attention to it as the unknown and guides our thoughts in the direction of discovering it.

In the river basin example, the configuration that the constructal law uncovers is a tree-shaped flow, with balances between highly dissimilar flow resistances such as seepage (Darcy flow) and river channel flow. The tree-shaped flow is the theoretical way of providing effective flow access between one point (source, sink) and an infinity of points (area, volume). The tree is a complex flow structure, which has multiple-length scales that are distributed nonuniformly over the available area or volume.

All these features, the tree shape and the multiple scales, are found in any other flow system whose purpose is to provide access between one point and an area or volume. Think of the trees of electronics, vascularized tissues, and city traffic, and you will get a sense of the universality of the principle that was used to generate and to discover the tree configuration.

Vascularized is a good name for the complex energy systems that the new thermal sciences is covering. Vascularized is everything, the animate, the inanimate, and the engineered, from the muscle and the river basin to the cooling of high-density electronics. The tissues of energy flows, like the fabric of society and all the tissues of biology, are designed (patterned, purposeful) architectures. The climbing to this high level of performance is the transdisciplinary effort: the balance between seemingly unrelated flows, territories, and disciplines. This balancing act—the optimal distribution of imperfection—generates the very design of the process, power plant, city, geography, and economics.

Trees are not the only class of configurations that result from invoking the constructal law. Straight tubes with round cross-sections are discovered when one favors the access for fluid flow between two points. Round tubes are found in many natural and engineered flow systems (blood vessels, subterranean rivers, volcanic shafts, piping, etc.). Optimal spacings between solid components are discovered by invoking the constructal law. Examples are the spacings between fins in heat exchangers and the spacings between heat-generating electronics in a package. Optimal intermittence (rhythm) is discovered in the same way, and, once again, the examples unite nature with engineering, from human respiration (in- and outflow), to the periodic shutdown and cleaning of heat exchangers in power plants.

Loops and grid-shaped flow patterns are useful because they add resilience and robustness to the tree-shaped flow configurations that they serve. Robustness and redundancy are precious properties in design, and our course teaches how to endow designs with such properties.

In summary, this design course provides the student with strategy for how to pursue and discover design (configuration, pattern) in both space and time. Constructal theory pushes design thinking closer to science and away from art. It tears down the walls between engineering and natural sciences. Because the design (the configuration generation phenomenon) has scientific principles that are now becoming known, it is possible to learn where to expect *opportunities* for discovering new configurations that are stepwise more effective. How to pursue the discovery with less effort and time (i.e., with strategy) is another merit of learning design generation as a scientific subject.

At the end of the day, this new *design as science* paradigm makes a solid contribution to physics, to predicting nature. The drawings made in this book are qualitatively the same as those of natural porous and vascularized materials. Most valuable are the similarities that emerge between natural structures and the ones derived here based on principle. They shed light on the natural tendency that *generates* multiple scales, hierarchy, complexity, and heterogeneity in flow systems such as hill slope drainage, forests, and living tissues. The fact that natural flow structures—the champions of flow perfection—have features similar to those discovered in constructal design lends confidence in the pursuit of better engineering design with constructal theory.

This book and solutions manual are based on an original fourth-year undergraduate and first-year graduate design course developed by the two of us at Duke University—course ME166 Constructal Theory and Design. We also taught constructal theory and design in short-course format at the University of Évora, Portugal; University of Lausanne, Switzerland; Yildiz University, Turkey; Memorial University, Canada; Shanghai Jiaotong University, People's Republic of China; and the University of Pretoria, South Africa.

We thank the students, who stoked the fire of our inquiry with questions and new ideas. In particular, we acknowledge the graphic contributions of our doctoral students: Sunwoo Kim, Kuan-Min Wang, Jaedal Lee, Yong Sung Kim, Luiz Rocha, Tunde Bello-Ochende, Wishsanuruk Wechsatol, Louis Gosselin, and Alexandre da Silva.

Our deepest gratitude goes to Deborah Fraze, who put the whole book together in spite of the meanness of the times.

During the writing of this book we benefited from research support for constructal theory from the Air Force Office of Scientific Research and the National Science Foundation. We thank Drs. Victor Giurgiutiu, Les Lee, and Hugh Delong of AFOSR; Drs. Rita Teutonico and Sandra Schneider of NSF; Dr. David Moorhouse of the Air Force Research Laboratory; and Professor Scott White and his colleagues at the University of Illinois.

Constructal theory and vascularization is a new paradigm and a worldwide activity that continues to grow (see www.constructal.org). We thank our friends and partners in the questioning of authority, in particular Heitor Reis, Antonio Miguel, Houlei Zhang, Stephen Périn, Gil Merkx, Ed Tiryakian, and Ken Land.

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January 2008

List of Symbols

```
acceleration, m/s<sup>2</sup>
a
             aspect ratios, Eq. (10.118)
a, b
a, b
             lengths, m
              area, m<sup>2</sup>
\boldsymbol{A}
             aspect ratio, Table 1.1
AR
              length, m
b
             dimensionless group, Eq. (9.26)
B
B_1, B_2
              global thermal resistances, Eqs. (5.22) and (5.50)
              brake power, W, Eq. (2.50)
Ė
              Bejan number, Eqs. (3.35) and (8.35)
Be
              Biot number, Eq. (1.45)
Bi
              concentration of one species per unit volume of solution, mol/m<sup>3</sup>
c
              specific heat, J/kg · K
c
              specific heat at constant pressure, J/kg · K
C_P
c, C
              constants
C
              pump work requirement, J, Eq. (1.33)
C
              thermal conductance, Eq. (6.16)
C_D
              drag coefficient, Eq. (1.37)
C_f
              skin friction coefficient
              coefficient of performance, Eqs. (2.18)–(2.19)
COP
              depth, smallest dimension, m
d
D
              diameter, spacing, m
              effective diffusion coefficient, m<sup>2</sup>/s
D
              hydraulic diameter, m, Eq. (1.21)
D_h
\boldsymbol{E}
              energy, J
\boldsymbol{E}
              modulus of elasticity, Pa
f
              flow resistance, dimensionless, Eq. (4.57)
f
f
f, F
              friction factor, Eq. (1.16)
              ratio, Eq. (6.59)
              function
              strength of concrete
 f_s
              strength of steel
              Faraday constant, 9 648 J/V mol
 \boldsymbol{F}
              force, N
              gravitational acceleration, m/s<sup>2</sup>
 g
              heat transfer coefficient, W/mK, Eq. (1.56)
 h
 h
               specific enthalpy, J/kg
```

```
latent heat of melting, J/kg
h_{sf}
h, H, H_m
             height, m
             area moment of inertia, m4
I
             current, A
             integral
I
             current density, A/m<sup>2</sup>
j
             diffusive flux, mol/m<sup>2</sup>s
J
             thermal conductivity, W/m \cdot K
k
k_{\rm s}
             roughness height, m
K
             local-loss coefficient, Eq. (1.31)
             permeability, m<sup>2</sup>
K
             length, m
l
             mean free path, m
L
             length, thickness, m
m, M
             mass, kg
             number
m
             mass flow rate, kg/s
m
              dimensionless mass flow rate, Eqs. (7.14) and (7.38)
M
M
              moment, Nm
n, N
              number
N
              number of heat loss units, Eq. (7.100)
Nu
              Nusselt number, Eq. (1.60)
              number of pairing (or bifurcation) levels
p
              porosity
p
              wetted perimeter, m
p
P
              force, N
P
              pressure, Pa
Ŷ
              dimensionless pressure drop, Eq. (7.49); see also Be, Eq. (3.35)
Po
              Poiseuille constant, Eq. (1.23)
Pr
              Prandtl number, Eq. (1.60)
              heat current, W
q
q',
     Q'
              heat current per unit length W/m
q''
              heat flux, W/m<sup>2</sup>
q'''
              volumetric heat generation rate, W/m<sup>3</sup>
Q
Q
Q'
Q'
Q'
              heat transfer, J
              volumetric fluid flow rate, m<sup>3</sup>/s
              heat source per unit length, J/m
              heat source per unit area, J/m<sup>2</sup>
              heat transfer rate, W
              radial position, m
              ratio
r
              pipe radius, m
r_0
```

radial distance, radius, m

R

```
ideal gas constant, J/kg · K
R
R
            resistance
R
             universal gas constant, 8.314 J/K mol
            Rayleigh number based on y, Eq. (1.76)
Ra_v
Ren
            Reynolds number based on D, Eq. (1.14)
             thermal resistance, K/W, Eq. (1.40)
R_t
S
            specific entropy, J/kg · K
             stress, Pa
S
S
            entropy, J/K
S
             spacing, m
S
             sum
S
             surface, m
Sc
             Schmidt number, v/D
S_{gen}
             entropy generation, J/K
St
             Stanton number, Eq. (1.70)
Sv
             Svelteness number, Eq. (1.1)
t
             thickness, m
             time, s
t
T
             temperature, K
             velocity components, m/s
u, v
             average longitudinal velocity, m/s
U
U
             overall heat transfer coefficient, W/m<sup>2</sup>K
U
             potential, V
             specific volume, m<sup>3</sup>/kg
v
             velocity, m/s
V
             volume, m<sup>3</sup>
V
             width, m
W
W
             work, J
Ŵ
             power, W
\dot{W}'
             power per unit length, W/m
             Cartesian coordinates, m
x, y, z
             flow entrance length, m
X
X_T
             thermal entrance length, m
             charge number
\mathbf{Z}
             thickness, m
```

Greek Letters

| α, β | angles, rad |
|----------------------|--|
| α | thermal diffusivity, m ² /s |
| $\boldsymbol{\beta}$ | coefficient of volumetric thermal expansion, K ⁻¹ |
| γ | ratio, Eq. (8.41) |
| δ | deflection, m |

xx List of Symbols

```
δ
       thickness, m
       pressure difference, Pa
\Delta P
       temperature difference, K
\Delta T
       effectiveness, Eq. (7.75)
       small quantity
ε
       fin efficiency, Eq. (1.44)
η
       first-law efficiency, Eq. (2.15)
\eta_{
m I}
       second-law efficiency, Eq. (2.16)
\eta_{	ext{II}}
        angle, rad
\theta
       dimensionless temperature difference, Eq. (7.123)
\theta
       temperature difference, K
\theta
       critical length scale, m
λ
       Lagrange multiplier
λ
       thickness, m
λ
       viscosity, kg/s m
\mu
       kinematic viscosity, m<sup>2</sup>/s
ν
       radius of curvature, m
ρ
       density, kg/m<sup>3</sup>
ρ
        stress, Pa
α
        shear stress, Pa
τ
ξ
        aspect ratio, Eq. (4.44)
ξ
        pressure loss, Eq. (1.32)
φ
        volume fraction, porosity; see also p
        electrical potential, V
\varphi
        dimensionless global flow resistance, Eq. (8.51)
ψ
```

Subscripts

| a | air |
|------------------|----------------|
| avg | average |
| b | base |
| b | body |
| b | brick |
| b | bulk |
| В | branch |
| \boldsymbol{c} | canopy |
| c | central |
| c | channels |
| C | compressor |
| C | conduction |
| D | diffuser, drag |
| \boldsymbol{E} | east |

exposed expfluid, frontal fFC forced convection ground g Н high inner, species, rank i in inlet \boldsymbol{L} low log-mean lm maximum m mean m melting m minimized m maximum allowable ma minimized twice mm minimized three times mmm max maximum min minimum N north N nozzle NC natural convection optimized 0 optimized twice 00 optimized three times 000 outer 0 optimum opt out outlet path p pipes p pump p radial r reference ref reversible rev sector, solid, steel S S south thermal t t trunk Tturbine Wwest wall \boldsymbol{w} longitudinal Z Σ total, summed free stream, far field

xxii List of Symbols

Superscripts

| b | bulk |
|------------------------|---------------------|
| n | nano-size |
| ()* | optimized |
| (-) | averaged |
| $(^{\sim}), (^{\sim})$ | dimensionless |
| ()' | per unit length |
| ()" | per unit area |
| ()"" | per unit volume |
| (.) | rate, per unit time |
| \boldsymbol{P} | power plant |
| R | refrigeration plant |

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