

Robust Control Design with MATLAB®

**D.-W. Gu, P. Hr. Petkov and
M.M. Konstantinov**



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With 288 Figures



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Da-Wei Gu, PhD, DIC, CEng
Engineering Department, University of Leicester, University Road, Leicester,
LE1 7RH, UK

Petko Hristov Petkov, PhD
Department of Automatics, Technical University of Sofia, 1756 Sofia, Bulgaria

Mihail Mihaylov Konstantinov, PhD
University of Architecture, Civil Engineering and Geodesy,
1 Hristo Smirnenski Blvd., 1046 Sofia, Bulgaria

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To our families

Series Editors' Foreword

The topics of control engineering and signal processing continue to flourish and develop. In common with general scientific investigation, new ideas, concepts and interpretations emerge quite spontaneously and these are then discussed, used, discarded or subsumed into the prevailing subject paradigm. Sometimes these innovative concepts coalesce into a new sub-discipline within the broad subject tapestry of control and signal processing. This preliminary battle between old and new usually takes place at conferences, through the Internet and in the journals of the discipline. After a little more maturity has been acquired by the new concepts then archival publication as a scientific or engineering monograph may occur.

A new concept in control and signal processing is known to have arrived when sufficient material has evolved for the topic to be taught as a specialised tutorial workshop or as a course to undergraduate, graduate or industrial engineers. *Advanced Textbooks in Control and Signal Processing* are designed as a vehicle for the systematic presentation of course material for both popular and innovative topics in the discipline. It is hoped that prospective authors will welcome the opportunity to publish a structured and systematic presentation of some of the newer emerging control and signal processing technologies in the textbook series.

It is always interesting to look back at how a particular field of control systems theory developed. The impetus for change and realization that a new era in a subject is dawning always seems to be associated with short, sharp papers that make the academic community think again about the prevalent theoretical paradigm. In the case of the evolution of robust control theory, the conference papers of Zames (circa. 1980) on robustness and the very short paper of Doyle on the robustness of linear quadratic Gaussian control systems seem to stand as landmarks intimating that control theory was going to change direction again. And the change did come; all through the 1980s came a steady stream of papers re-writing control theory, introducing system uncertainty, H_∞ robust control and μ -synthesis as part of a new control paradigm.

Change, however did not come easily to the industrial applications community because the new theories and methods were highly mathematical. In the early stages even the classical feedback diagram which so often opened control engineering courses was replaced by a less intuitively obvious diagram. Also it

was difficult to see the benefits to be gained from the new development. Throughout the 1990s the robust control theory and methods consolidated and the first major textbooks and software toolboxes began to appear. Experience with some widely disseminated benchmark problems such as control design for distillation columns, the control design for hard-disk drives, and the inverted-pendulum control problem helped the industrial community see how to apply the new method and the control benefits that accrued.

This advanced course textbook on robust control system design using MATLAB[®] by Da-Wei Gu, Petko Petkov and Mihail Konstantinov has arrived at a very opportune time. More than twenty years of academic activity in the robust control field forms the bedrock on which this course book and its set of insightful applications examples are developed. Part I of the volume presents the theory – a systematic presentation of: systems notation, uncertainty modelling, robust design specification, the H_∞ design method, H_∞ loop shaping, μ -analysis and synthesis and finally the algorithms for providing the low-order controllers that will be implemented. This is a valuable and concise presentation of all the necessary theoretical concepts prior to their application which is covered in Part II.

Inspired by the adage “practice makes perfect”, Part II of the volume comprises six fully worked-out extended examples. To learn how to apply the complex method of H_∞ design and μ -synthesis there can be no surer route than to work through a set of carefully scripted examples. In this volume, the examples range from the academic mass-damper-spring system through to the industrially relevant control of a distillation column and a flexible manipulator system. The benchmark example of the ubiquitous hard-disk drive control system is also among the examples described. The MATLAB[®] tools of the Robust Control Toolbox, the Control System Toolbox and Simulink[®] are used in these application examples. The CD-ROM contains all the necessary files and instructions together with a pdf containing colour reproductions of many of the figures in the book.

In summary, after academic development of twenty years or so, the robust control paradigm is now fully fledged and forms a vital component of advanced control engineering courses. This new volume in our series of advanced control and signal processing course textbooks on applying the methods of H_∞ and μ -synthesis control design will be welcomed by postgraduate students, lecturers and industrial control engineers alike.

M.J. Grimble and M.A. Johnson
Glasgow, Scotland, U.K.
February 2005

Preface

Robustness has been an important issue in control-systems design ever since 1769 when James Watt developed his flyball governor. A successfully designed control system should be always able to maintain stability and performance level in spite of uncertainties in system dynamics and/or in the working environment to a certain degree. Design requirements such as gain margin and phase margin in using classical frequency-domain techniques are solely for the purpose of robustness. The robustness issue was not that prominently considered during the period of 1960s and 1970s when system models could be much more accurately described and design methods were mainly mathematical optimisations in the time domain. Due to its importance, however, the research on robust design has been going on all the time. A breakthrough came in the late 1970s and early 1980s with the pioneering work by Zames [170] and Zames and Francis [171] on the theory, now known as the $\mathcal{H}_\infty$ optimal control theory. The $\mathcal{H}_\infty$ optimisation approach and the μ -synthesis/analysis method are well developed and elegant. They provide systematic design procedures of robust controllers for linear systems, though the extension into nonlinear cases is being actively researched.

Many books have since been published on $\mathcal{H}_\infty$ and related theories and methods [26, 38, 65, 137, 142, 145, 174, 175]. The algorithms to implement the design methods are readily available in software packages such as MATLAB[®] and Slicot [119]. However, from our experience in teaching and research projects, we have felt that a reasonable percentage of people, students as well as practising engineers, still have difficulties in applying the $\mathcal{H}_\infty$ and related theory and in using MATLAB[®] routines. The mathematics behind the theory is quite involved. It is not straightforward to formulate a practical design problem, which is usually nonlinear, into the $\mathcal{H}_\infty$ or μ design framework and then apply MATLAB[®] routines. This hinders the application of such a powerful theory. It also motivated us to prepare this book.

This book is for people who want to learn how to deal with robust control-system design problems but may not want to research the relevant theoretic developments. Methods and solution formulae are introduced in the first part

of the book, but kept to a minimum. The majority of the book is devoted to several practical design case studies (Part II). These design examples, ranging from teaching laboratory experiments such as a mass-damper-spring system to complex systems such as a supersonic rocket autopilot and a flexible-link manipulator, are discussed with detailed presentations. The design exercises are all conducted using the new *Robust Control Toolbox v3.0* and are in a hands-on, tutorial manner. Studying these examples with the attached MATLAB[®] and Simulink[®] programs (170 plus M- and MDL-files) used in all designs will help the readers learn how to deal with nonlinearities involved in the system, how to parameterise dynamic uncertainties and how to use MATLAB[®] routines in the analysis and design, *etc.*. It is also hoped that by going through these exercises the readers will understand the essence of robust control system design and develop their own skills to design real, industrial, robust control systems.

The readership of this book is postgraduates and control engineers, though senior undergraduates may use it for their final-year projects. The material included in the book has been adopted in recent years for MSc and PhD engineering students at Leicester University and at the Technical University of Sofia. The design examples are independent of each other. They have been used extensively in the laboratory projects on the course *Robust and Optimal Control Systems* taught in a masters programme in the Technical University of Sofia.

The authors are indebted to several people and institutions who helped them in the preparation of the book. We are particularly grateful to The MathWorks, Inc. for their continuous support, to Professor Sigurd Skogestad of Norwegian University of Science and Technology who kindly provided the nonlinear model of the Distillation Column and to Associate Professor Georgi Lehov from Technical University of Russe, Bulgaria, who developed the uncertainty model of the Flexible-Link Manipulator.

Using the CD ROM

The attached CD ROM contains six folders with M- and MDL-files intended for design, analysis and simulation of the six design examples, plus a pdf file with colour hypertext version of the book. In order to use the M- and MDL-files the reader should have at his (her) disposition of MATLAB[®] v7.0.2 with Robust Control Toolbox v 3.0, Control System Toolbox v6.1 and Simulink[®] v6.1. Further information on the use of the files can be found in the file Readme.m on the disc.

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Basic Methods and Theory

Introduction

Robustness is of crucial importance in control-system design because real engineering systems are vulnerable to external disturbance and measurement noise and there are always differences between mathematical models used for design and the actual system. Typically, a control engineer is required to design a controller that will stabilise a plant, if it is not stable originally, and satisfy certain performance levels in the presence of disturbance signals, noise interference, unmodelled plant dynamics and plant-parameter variations. These design objectives are best realised *via* the feedback control mechanism, although it introduces in the issues of high cost (the use of sensors), system complexity (implementation and safety) and more concerns on stability (thus internal stability and stabilising controllers).

Though always being appreciated, the need and importance of robustness in control-systems design has been particularly brought into the limelight during the last two decades. In classical single-input single-output control, robustness is achieved by ensuring good gain and phase margins. Designing for good stability margins usually also results in good, well-damped time responses, *i.e.* good performance. When multivariable design techniques were first developed in the 1960s, the emphasis was placed on achieving good performance, and not on robustness. These multivariable techniques were based on linear quadratic performance criteria and Gaussian disturbances, and proved to be successful in many aerospace applications where accurate mathematical models can be obtained, and descriptions for external disturbances/noise based on white noise are considered appropriate. However, application of such methods, commonly referred to as the linear quadratic Gaussian (LQG) methods, to other industrial problems made apparent the poor robustness properties exhibited by LQG controllers. This led to a substantial research effort to develop a theory that could explicitly address the robustness issue in feedback design. The pioneering work in the development of the forthcoming theory, now known as the $\mathcal{H}_\infty$ optimal control theory, was conducted in the early 1980s by Zames [170] and Zames and Francis [171]. In the $\mathcal{H}_\infty$ approach, the designer from the outset specifies a model of system uncertainty, such as additive perturbation