科技资料

Strings 90

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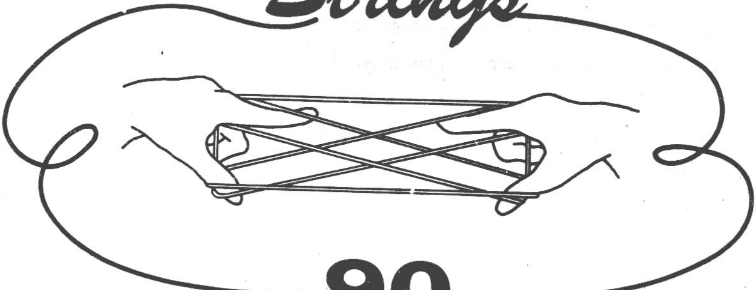

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Editors

R. Arnowitt
R. Bryan
M. J. Duff
D. Nanopoulos
C. N. Pope
E. Sezgin

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PREFACE

Superstrings continue to present the most ambitious and challenging problems in theoretical physics, both on the very small scales of elementary particles, and on the very large scales of astrophysics and cosmology. Yet it is becoming increasingly apparent that some of the most basic problems in string theory, such as the breaking of supersymmetry, and the choice of vacuum state, cannot be answered within the framework of a weak-coupling perturbative expansion. This need for a non-perturbative framework was reflected in the talks given at *Strings 90*, the fourth annual superstring workshop, held for the second consecutive year on the campus of Texas A&M University, from March 12th–17th 1990. Thus, in addition to the more familiar paths trod by string theorists, this year witnessed a new interest in exactly-soluble models of quantum gravity in two dimensions. Another new non-perturbative approach exploited a possible strong/weak coupling duality between the heterotic string and heterotic 5-brane in ten dimensions. Attempts to go beyond conventional string dynamics with new structures such as W algebras were also to the fore. Another new feature that surfaced this year was the first predictions from string theory of experimental signatures of new physics, both for the four dimensional and Calabi-Yau string vacua.

The meeting was attended by 150 physicists from the USA, Europe, the Soviet Union, Israel, India, China and Japan. *Strings 90*, as with *Strings 89*, not only provided a forum for the world's foremost physicists, but also presented a valuable opportunity for the younger generations of aspiring theorists to learn more of this fascinating subject. Once again, we should like to express our thanks to all the speakers for providing such a consistently high level of presentation and for contributing to these proceedings.

The workshop was funded by generous grants from the College of Science and the Physics Department of Texas A&M University, the National Science Foundation, the Department of Energy, and the Bryan/College Station Chamber of Commerce. We gratefully acknowledge their financial support.

It is a pleasure to thank the members of the International Advisory Committee for their time and wisdom; the other members of the local Organising Committee, Yunhai Cai, Chia-Chu Chen, Katsumi Itoh, Sunny Kalara, Jorge Lopez and George Siopsis; graduate students Hong Lu, Jian-Xin Lu, Vephuoc Nguyen, David Ring, Shawn Shen, Sinichi Urano, Xu-Jing Wang, Jizhi Wu and Kai-Wen Xu for their hard work and enthusiasm; and also Young In Arnowitt

Karen Carroll, Billie Posey and Barbara Sloan for their help and patience in making sure that all went smoothly on the day.

A special word of thanks is due once again this year to Lore Angele for providing more of her beautiful photographs of the Texan landscape.

Richard Arnowitt
Ronald Bryan
Michael Duff
Dimitri Nanopoulos
Christopher Pope
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NON-PERTURBATIVE STRING THEORY

DAVID J. GROSS
Joseph Henry Laboratories
Princeton University, Princeton, N.J.

ABSTRACT

The matrix model solution of c=1 matter coupled to two dimensional quantum gravity is reviewed, both in the case where the target space is the real line and a circle of finite radius. The role and physical significance of the nonsinglet states is analysed. The meaning of this theory as a two dimensional string theory is discussed and a fermionic field theory representation is constructed.

1. Introduction

The conventional approach to two dimensional gravity and to string theory is perturbative with respect to fluctuations of the topology. One sums over two dimensional geometries by first performing the functional integral for fixed topology (genus= number of handles) and then summing over genus. However, this sum is very badly behaved. The higher terms grow as factorials of the genus, and the positivity of these terms renders the series non Borel summable.[1] This situation is made worse by the lack of an adequate nonperturbative framework for the theory. Such a framework, (for example, a useful formulation of second quantized string theory) should be capable of reproducing the topological series as an asymptotic expansion, valid in the perturbative domain; but it should also provide a physical picture and a mathematical framework valid for strong coupling. It is essential that we develop nonperturbative methods if we are to relate unified string theories to the real world. At the perturbative level of string theory there are many too many possible worlds, *i.e.* classical vacua about which consistent perturbative expansions can be made. All of them have undesired features, such as unbroken supersymmetry and massless dilatons. One must hope that nonperturbative physics will lift the degeneracy and break the unwanted symmetries. This is strongly suggested by the divergence and non Borel summability of perturbation theory which can be taken as an indication of the nonperturbative instability of the classical vacua [1].

One of the main motivations for studying two dimensional gravity coupled to simple matter is that this provides a toy model for string theory, in which these nonperturbative issues might be explored with greater ease. There are other motivations. These theories are the simplest examples of quantum gravity, and might be used to probe issues of topology change in a finite and tame gravitational theory. They might also be used in the study to study of critical behavior in three dimensions, where the phase boundaries are two dimensional random surfaces. In addition, it has long been a goal to construct a string theory representation of QCD. For all of these purposes it is necessary to enlarge our understanding of string theories and to go beyond the special critical dimensions. In doing so we lose some of the simplicity of the critical string (the decoupling of the two dimensional metric from the matter) that gives rise to the enhanced symmetry of unified string theory. What we gain is the opportunity to consider simple models with a few degrees of freedom. The simplest of all toy models are those where we couple two dimensional gravity to the simplest two-dimensional field theories– the minimal models of matter with central charge less than one, which have only a finite number of degrees of freedom. Indeed the reduction in the number of degrees of freedom has enabled us recently to *find explicit analytic solutions to all orders in perturbation theory* for many such theories.

The study of these simple examples will hopefully teach us lessons that can be applied to critical string theory. Even more–it might be the case that there is no real distinction between critical and noncritical string theory. It is well known that the critical theory, say the 26-dimensional bosonic string, can be identified with a theory of $c = 25$ noncritical matter coupled to two dimensional gravity, wherein the Liouville field supplies the time coordinate[11]. The most radical point of view is that there is only one (or three including super and heterotic strings) string theory, the critical string representing being an expansion of the theory about a particularly symmetric classical solution. Thus in studying the toy models with central charge c we are exploring a $D = c + 1$ dimensional corner of critical string theory.

The recent progress is based on matrix model methods for generating cutoff, discrete representations of the perturbative expansion of string amplitudes. Here the geometry of the world sheet of the string (or two dimensional space in the case of pure gravity) is approximated by a dense Feynman graph, in the limit where the number of vertices becomes infinite. The topology is selected by means of the $\frac{1}{N}$ expansion of a SU_N invariant matrix model. The sum over all Feynman graphs of given genus and given number of vertices is a discrete version of the functional integral over metric tensors[2]. Remarkably, in many cases these discrete models can be handled with greater ease than their continuous analogs.

The great advance of last year was made by realizing that these matrix model representations simplify drastically in the *double scaling limit*. This is the limit where one adjusts the coupling constant, λ, to equal a critical value at which the loop expansion of the matrix model begins to diverge and sends N to infinity in just such a way that the string coupling, $g_s^2 \equiv \frac{1}{N^2(\lambda-\lambda_{cr})^{2+\gamma_s}}$, remains constant. This is precisely the limit which allows one to sum over all continuum surfaces of arbitrary topology. The double scaling limit of the matrix model representation of sums over discretized random surfaces has been used to calculate sums over all continuum surfaces to all orders in the topological expansion [3–6]. The remarkably rich structure that emerged has also illuminated the connection of two dimensional quantum gravity with topological field theories [7] and KdV hierarchies [4,8,9].

Matrix model methods have so far been employed mostly to study models where the matter content has central charge less or equal to one. In fact, $c = 1$, the model of one scalar matter field coupled to quantum gravity, is a transition point for this approach, where a phase transition probably takes place due to the vanishing mass of the tachyon degree of freedom, which becomes truly tachyonic for $c > 1$. $c = 1$ also represents a calculational barrier for matrix model technology. One of the main reasons that the matrix models are soluble for $c \leq 1$ is that, of the N^2 degrees of freedom of the matrices, only the N eigenvalues matter. The other $N(N-1)$ angular degrees of freedom decouple[10]. This procedure breaks down when $c > 1$, where one finds that all N^2 degrees of freedom of the matrices become relevant. This is consistent with the emerging view of the string interpretation of these models as describing $D = c + 1$ string theory, in which the Liouville mode, which is identified with the space of eigenvalues of the matrix model, yields a (euclidean) time dimension[11–13]. The $D = 2$ string theory, described by the $c = 1$ matrix model, has only a limited number of degrees of freedom, since in two dimensions there are no transverse excitations and all we have are the center of mass of the string (the two-dimensional tachyon field), and a discrete infinity of quantum mechanical variables which occur at quantized momenta[14]. On the other hand, for $D > 2$ we find an infinite number of D-dimensional fields. Thus, it is not surprising that the number of relevant degrees of freedom suddenly increases as we pass the point $c = 1$.

The $c = 1$ theory is then, in many ways, the most complex of the soluble models to date, the one that is closest to realistic string theories in higher dimensions. In other ways it is it is the simplest of the soluble models where many of the results can be explicitly exhibited in terms of elementary functions. In this lecture I shall review the solution of this model, discuss the role of nonsinglet states and their relation to vortices and construct a two dimensionalfermionic field representation of the model.

2. The $c = 1$ Matrix Model

Let me first review the matrix model representation of the sum over random surfaces embedded in one dimension—$c = 1$ matter coupled to two dimensional quantum gravity[15]. The partition function can be generated by the Feynman diagrams of the large N limit of the euclidean quantum mechanics of a hermitean $N \times N$ matrix Φ,[16]

$$Z_N(\beta) = \int D^{N^2}\Phi e^{-\beta \int dt \, \mathrm{Tr}\left[\frac{1}{2}\dot{\Phi}^2 + U(\Phi)\right]} \quad , \tag{2.1}$$

where t runs from $-\infty$ to ∞ (from 0 to $2\pi R$) if the target space is the real line (a circle of radius R.)

The connection with $D = 1$ string theory is established by considering the perturbative expansion of the partition function. Each given order in $\frac{1}{N^2}$ corresponds to a sum over the surfaces of definite genus, generated by the Feynman graphs of that genus. Each graph is weighted by a factor $\left(\frac{N}{\beta}\right)^{\mathrm{Area}}$, and contains an integral over a product of propagators, $e^{-m|t_i - t_j|}$, connecting adjacent $\Phi(t)$'s on the graph. In the continuum limit, when we take $\frac{N}{\beta} \equiv g \to g_{critical}$ so that the perturbation series diverges and is dominated by infinite area terms, this should coincide with the continuum definition of a Gaussian variable coupled to two dimensional gravity.

The standard method for dealing with this problem as $N \to \infty$ is to reducethe number of degrees of freedom from the N^2 matrix elements of Φ to its N eigenvalues, λ_i, by writing the matrix as $\Phi(t) = \Omega(t)^\dagger \Lambda(t) \Omega(t)$, where Ω is unitary and Λ diagonal. In the case of the $c = 0$ model, in which Φ has no t dependence, the action does not depend on Ω, which can be integrated out. This is not the case for $c = 1$, due to the kinetic term $\mathrm{Tr}[\dot{\Phi}(t)^2]$, which can be written as $\mathrm{Tr}[\dot{\Phi}^2] = \mathrm{Tr}\left(\dot{\Lambda}^2 + [\Lambda, A][\Lambda, A]\right)$, where we have introduced the pure gauge field $A(t) \equiv \dot{\Omega}\Omega^\dagger(t)$. Now, even though $A(t)$ does not decouple, the integral over it can be performed, following the classic work of Itzykson and Zuber and of Mehta [17]. The essential point is that the semi-classical evaluation of the integral over the Ω matrices is exact, at least if the Ω's are unconstrained. This is the content of the formula,

$$\int \mathcal{D}\Omega e^{\mathrm{Tr}\left[\Omega A \Omega^{\dagger} B\right]} = \sum_{p(i)} \frac{e^{\sum_i a_i b_{p(i)}}}{\Delta(a_i)\Delta(b_{p(i)})} = \frac{\mathrm{Det}[e^{a_i b_j}]}{\Delta(a_i)\Delta(b_i)} \quad , \tag{2.2}$$

where the a_i (b_i) are the eigenvalues of A (B), $p(i)$ is a permutation of the i's, and $\Delta(\bullet)$ is the Vandermonde determinant $\Delta(a_i) = \prod_{i<j}(a_i - a_j)$. Using this one can then perform the Ω integral in (2.1) (most precisely by discretizing the t interval), and show that the effect is to precisely cancel the factors of $\Delta(\lambda)$ that come from the measure of integration, $\mathcal{D}\Phi = \mathcal{D}\Omega \prod_i d\lambda_i \Delta^2(\lambda)$. The result is that the partition function on a finite interval, $t_1 < t < t_2$, is given by

$$Z_N(\beta) = \int \prod_i d\lambda_i(t)\Delta(\lambda_i(t_2))e^{-\beta \int_{t_1}^{t_2} dt \sum_i [\frac{1}{2}\dot{\lambda}_i^2 + U(\lambda_i)]}\Delta(\lambda_i(t_1)) \quad . \tag{2.3}$$

The antisymmetric factors of $\Delta(\lambda_i)$, project out of any intermediate state the totally antisymmetric component. In other words, the λ_i are fermionic variables!

Alternatively, we may carry out canonical quantization of the $SU(N)$ symmetric matrix quantum mechanics[16,19]. The hamiltonian is

$$H = -\frac{1}{2\beta^2 \Delta(\lambda)} \sum_i \frac{d^2}{d\lambda_i{}^2}\Delta(\lambda) + \sum_i U(\lambda_i) + \sum_{i<j} \frac{\Pi_{ij}^2 + \tilde{\Pi}_{ij}^2}{(\lambda_i - \lambda_j)^2} \quad , \tag{2.4}$$

where the Π_{ij} and $\tilde{\Pi}_{ij}$ are generators of $left$ rotations on Ω, $\Omega \to A\Omega$. When the target space is the infinite real line the only state that matters in the calculation of the matrix element of the time evolution operator e^{-HT}, as $T \to \infty$, is the ground state of the Hamiltonian, which is in the trivial representation of $SU(N)$, given by an $SU(N)$ singlet wave functions, $\psi(\lambda_i)$, which does not depend on the angular matrices Ω, and is a symmetric function of the eigenvalues λ_i. The hamiltonian of eq. (2.4), when acting on $\chi(\lambda_i)$, a fully antisymmetric wave function $\chi(\lambda_i) = \Delta(\lambda_i)\psi(\lambda_i)$, reduces to the sum of single particle hamiltonians,

$$h_i = -\frac{1}{2\beta^2}\frac{d^2}{d\lambda_i^2} + U(\lambda_i) \quad , \tag{2.5}$$

thus reducing the problem to the physics of N non-interacting fermions moving in the potential $U(\lambda)$, with the Planck constant $\hbar$ set equal to $\frac{1}{\beta} \sim \frac{1}{N}$. The free energy,