Comparison of Automatic Control & Operational Research Techniques Applied to Large Systems Analysis & Control

Editors
M J Pelegrin
CERT, Toulouse, France
J H Delmas
ENSAE/DERA-CERT, Toulouse, France

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COMPARISON OF AUTOMATIC CONTROL AND OPERATIONAL RESEARCH TECHNIQUES APPLIED TO LARGE SYSTEMS ANALYSIS AND CONTROL

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Editors

M. J. PELEGRIN

Ingénieur Général,
Directeur du CERT,
Centre d'Etudes et de Recherches de Toulouse, Cedex, France

and

J. H. DELMAS

ENSAE - Ecole Nationale Supérieure
de l'Aéronautique et de l'Espace,
Chef du Groupe au CERT,
Centre d'Etudes et de Recherches de Toulouse, Cedex, France

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FOREWORD

AIMS OF THE SYMPOSIUM

The operational research techniques were designed during the last world war to solve military logistical problems : they can now be applied to large systems studies : industrial , economical, etc ... Automation techniques had the same evolution after the study of small size processes (fabrication units, small workshop, industrial machines, etc ...), they can now be applied to large systems (factories, technical and economical systems, etc...)

So, after important theoretical developments during the last two or three decades, we are presently concerned with applications to large systems ; we have obtained already good results but modelisation (reliability, accuracy) and model utilisation (numerical analysis) still require many studies.

Though these techniques are often the same, they have their own originality and are often complementary. Thus, it has been thought useful to organize a first common symposium which, after having reviewed the basic principles of those two sciences, will enable a direct comparison (advantages and disadvantages) of their applications in various fields (industrial, traffic, communication, economical ...)

The aim of this symposium is to bring together Automatic Control and Operational Research techniques at a time when so many changes occur in the economical field resulting from a limitation of resources (energy, raw materials) and environment pollution. These two sciences must certainly contribute to a better resources use and a diminution of pollution due to industrial processes.

This symposium will not only consist of a comparison of the techniques used in Operational Research and Automatic Control. The mutual and respective contributions in analysis and control of larger systems will be, of course, completely included in the general topic of this symposium.

Marc PELEGRIN

PREFACE

Les techniques de Recherche Opérationnelle ont été conçues, durant la dernière guerre mondiale, pour résoudre des problèmes logistiques militaires ; elles peuvent maintenant être appliquées à des études de systèmes complexes dans les domaines industriel, économique ... Les techniques de l'Automatique ont suivi la même évolution ; après des études d'asservissements de systèmes simples, elles sont maintenant également appliquées à la commande ou à la gestion des systèmes complexes (techniques, industriels, économiques, ...). Ainsi, après deux ou trois décennies de développements théoriques très féconds, les développements actuels concernent dans leur grande majorité, la préhension de systèmes de plus en plus vastes : de ce fait, les optima sont meilleurs mais les difficultés de modélisation (fiabilité, précision) puis de traitement du modèle (approximations numériques), nécessitent encore de longues études.

Bien que voisines, ces techniques ont leur propre originalité et sont souvent complémentaires. Il a donc paru intéressant d'organiser un premier symposium sur ce thème. Celui-ci doit permettre, après avoir rappelé les principes de base de ces deux sciences, de comparer directement leurs avantages et leurs inconvénients, dans différents domaines d'applications (industriel, transport, communication, économie ...).

Voilà pourquoi ce symposium a pour objectif de jeter un pont entre les techniques développées dans ces deux domaines, à un moment où des bouleversements économiques importants ont lieu par suite de la prise de conscience de la limitation des ressources (énergie et matières premières) et par suite des contraintes de plus en plus fortes que le respect de l'environnement impose.

Ces deux disciplines peuvent contribuer à la meilleure utilisation des ressources comme à la diminution des effets nocifs du développement industriel sur l'environnement.

Ce symposium ne consistera pas uniquement en la comparaison des techniques utilisées en Recherche Opérationnelle et en Automatique. Les contributions mutuelles et respectives de ces techniques dans l'analyse et la conduite des systèmes complexes font, bien évidemment, partie intégrante du thème général de ce symposium.

Marc PELEGRIN

SESSION 1

Methodology: Complex Systems

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Session 3. Methodology: Decision Aid

Session 4. Planning

Session 5. Transportation Systems

PROJECT SCHEDULING AND CONTROL IN A COMPLEX - OF - OPERATIONS FRAMEWORK – COMPARATIVE REMARKS

J. Węglarz

Institute of Control Engineering, Technical University of Poznań, Poznań, Poland

Abstract. The paper presents a comparison between project scheduling and control in a complex-of-operations framework applied to the allocation of recoverable (or non-storable) resources. The main considerations are restricted to project duration as a project performance measure. Special attention is paid to the theoretical and practical interpretation of assumptions made within both approaches, especially those concerning resource requirements and mathematical models of activities. The interrelationships between both approaches are shown, as well as the possibilities of the joint application of the results obtained in each. On the basis of these considerations, certain desirable directions for further investigation in resource allocation theory are pointed out.

Keywords. Operations research, large-scale systems, optimisation, project scheduling, resource allocation.

INTRODUCTION

Problems of resource allocation among activities from a certain set, in particular task scheduling problems, are considered in many branches of operations research with economic, technical and computer applications. In the investigation of the mathematical nature of these problems as well as in approaches to their solution, theoretical and practical results from optimization theory, control theory, graph theory, and computational complexity theory, are used. In consequence, these problems constitute a typical example of integration between the systems sciences.

It is natural, at a certain point in the development of individual results, to attempt to compare and review these results, as well as to designate directions for further investigation. Some such attempts have been made within project scheduling and task scheduling, but these were however concerned with the same class of problems, from the viewpoint of certain general assumptions about such elements as the type of resource requirements and the mathematical model of an activity or task. The latter is taken as characterizing the performance of an activity in relation to the resources allotted, excluding information about its splittability (a splittable activity may be interrupted at an arbitrary unknown a priori moment, while a nonsplittable activity, cannot be interrupted during its performance).

In this paper we would like to draw another, in a certain sense deeper comparison of two classes of problems, differing in their types of resource requirements and their mathematical models of activities, while being concerned with the allocation of the same category of resources according to their recoverability (or storability). The first class of problems is connected with the project and task scheduling approach, whereas the second - with the control in a complex-of-operations approach. Let us notice that the term "control in a complex of operations" has here its classical meaning (Burkov,1969; Węglarz,1976) and not the generalized one (Słowiński and Węglarz,1976; Węglarz,1978) containing practically almost all resource allocation problems. Such an understanding follows, of course, from the purpose of this paper.

The comparison will be carried out for deterministic situations, and mainly, for the optimality criterion of project duration and for recoverable (or non-storable) resources, i.e. such for which we assume that the total amount is constrained at every moment. This choice follows from the fact that the results obtain -

ed for the above optimality criterion and resource category within the control in a complex-of-operations approach are the most penetrating and characteristic, thus they create a base for making out the desired directions for further investigation. Finally, let us stress that in the presented comparison we will not be interested in reviewing results obtained within the considered approaches, but rather in showing their specific points, as well as some interesting interrelationships among them, and the possibility of their joint application.

ASSUMPTIONS AND THEIR INTERPRETATION

In this Section we will present and interprete the most important assumptions made in typical problems from project and task scheduling theory, and in the control in a complex-of-operations framework.

In both approaches we will assume that there are given two sets: a set of resources $\mathcal{R} = (R_1, R_2, \ldots, R_p)$, where R_l, $l=1,2,\ldots,p$ denotes resource of type l (or resource l) and a set of activities (tasks, operations) $\mathcal{A} = (A_1, A_2, \ldots, A_n)$. Resources are classified into types taking into account the functions which they fulfil in the considered situation; resources (or rather resource units) of the same type do not have to be identical. We will call resources <u>recoverable</u> (or non-storable) if only their total amounts at every moment of project performance are given a priori (for example: machines, manpower, etc.); and <u>non-recoverable</u> (or storable) - those for which only the total consumption up to any given moment is determined (for example money, energy). It is easy to see that in practice resources do not strictly speaking exist which are only recoverable or non-recoverable in the sense described above, a point to which we will return later. However, till now, most results have been obtained under the assumption that resources in $\mathcal{R}$ are either recoverable or non-recoverable.

The set $\mathcal{A}$ is partially ordered by an relation $\prec$, which specifies operational precedence constraints: $A_i \prec A_j$ denotes that A_i must be completed before A_j can start.

Differences at the stage of problem formulation between the two approaches lie in resource requirements and mathematical models of activities. Let us start with the resource requirements. In project and task scheduling problems concerning recoverable resources, the resource requirements of each activity usually involve a number of resource units of every type which may be chosen from a given finite set. In many cases these sets cantain only one number, especially in classical problems of scheduling tasks on machines without additional resources, number 1. On the other hand, in problems considered within the control in a complex-of-operations framework, the resource requirements of an activity involve an amount of resource of every type which is arbitrary within a given interval, in particular $<0,\infty)$. It should be stressed that these resource requirements may here concern resources treated as recoverable, which are continuously divisible, for example fuel flow, power, approximately manpower or primary memory pages in computer systems, whereas from a traditional point of view they concern non-recoverable resources (time/cost trade-off problems).

Of course, not only resource requirements determine the specificity of the approaches. To have a full picture we have to consider them together with the mathematical models of activities. These models, in project and task scheduling problems with recoverable resources, are given in the form of activity execution times for particular variants of resource allocations. However in the control in a complex-of-operations approach the mathematical model of activity A_i is generally given in the following form:

$$dx_i(t)/dt = \begin{cases} f_i[x_i(t), \bar{r}_i(t), t] & \text{for } t \in <0, T_i> \\ 0 & \text{for } t \notin <0, T_i> \end{cases} \qquad (1)$$

where: $x_i(t)$ is the state of activity A_i at moment t, $x_i(0)=0$; $\bar{r}_i(t) = (r_{i1}(t), r_{i2}(t), \ldots, r_{ip}(t))$ is the vector of amounts of resources allotted to A_i at moment t, p is the number of resource types; $f_i(\cdot)$ is a non-decreasing, continuous function, $f_i(0)=0$; T_i is the finishing time of A_i, i.e. the shortest time (in general unknown a priori) for which $x_i(t) \geq w_i$, w_i being the known value denoting the final state of A_i, also called the volume or the size of A_i.

Before passing to general remarks concerning model (1) let us comment on the value w_i which has to be known a priori. Note that because $x_i(0)=0$ and $x_i(T_i)=w_i$ we may write $\int_0^{T_i} f_i[\bar{r}_i(t)]dt = w_i$. Thus, w_i in fact denotes an objective measure of work related to the

performance of A_i. This may be for example the number of elementary operations needed to perform A_i, the volume, in cubic meters, of a constructed building, etc. Moreover, for linear $f_i(\cdot)$, w_i can be expressed in units "resource × time" for example "man-hours".
Now let us characterize generally the considered model. Three specific features may be pointed out. Firstly, as we have said, the model may concern continuously divisible, recoverable resources, which may be allotted to activities in amounts belonging to certain intervals: for activity A_i

$$r_{ik}(t) \in \langle a_{ik}, b_{ik} \rangle, \quad 0 \leq a_{ik} \leq b_{ik} \leq \infty, \quad k=1,2,\dots,p, \tag{2}$$

for every $t \in \langle 0, T_i \rangle$.
The total amounts of each resource is constrained for every $t \geq 0$:

$$\sum_{i \in X_t} r_{ik}(t) \leq N_k(t), \quad k=1,2,\dots,p,$$

where X_t is the set of activities which are being performed at moment t; $N_k(t)$ is a known function.
Secondly, this model is general enough for carrying but studies concerning the properties of optimal solutions, i.e. assignments, considered in time, of resources from the set $\mathcal{R}$ to activities from the set $\mathcal{A}$, fulfilling the imposed constraints and optimizing a given project performance measure. Because of the way in which time enters into this model, we may find, in the most natural way, solutions in which the amount of resources allotted to an activity changes during its performance.
Thirdly, the model shows interconnections between project scheduling theory and classical optimal control theory, a fact which is of great methodological importance.
Moreover, as we will see in Section 4, this model may be utilized for the formulation of new problems, which may be more adequate in certain practical situations. To end with, let us stress that in the control in a complex-of--operations approach it is assumed that all activities are splittable. From the theoretical point of view this assumption seriously restains the class of problems which may be optimally solved using this approach. In practice however, we often obtain optimal solutions in which no activity must be split. For certain cases, for example for independent activities and for $a_i=0$, $i=1,2,\dots,n$ in (2), we can even prove that in the optimal solution no activity will be split. Moreover, as we will see in Section 4, in certain important practical cases we may construct good heuristics ensuring the non-splittability of activities in the obtained solutions. We cannot also forget that in certain practical situations, for example those concerning resource allocation in computer systems, splittability of activities (tasks, programs) may be allowed.

INTERCONNECTIONS BETWEEN APPROACHES

In this Section we would like to examine the border between the approaches applied to the problems under consideration, through which certain interflows of ideas may be observed.
We will assume that we are dealing with splittable activities.
Let us start by remarking that the resource requirements considered in the project scheduling approach may be identified with those in the control in a complex-of-operations approach, if the set of possible amounts of resources of particular types, which may be allotted to specific activities is uncountable. As we have already mentioned, such a situation may arise in practice when we have to allocate continuously divisible resources like power, fuel flow, etc. However, it is often purposeful from a computational point of view, to make such an assumption to approximate a real situation for example for manpower or primary memory pages in computer systems,when the number of resource units is sufficiently great.
The precise determination of the class of problems for which the above approximation is useful is very difficult; we may,however, point out certain characteristic features of this class. Generally speaking, the class contains problems for which the application of the control in a complex-of-operations approach allows for the utilization of analytic results which simplify the finding of optimal solutions. This may take place especially for the one-resource case, with all functions $f_i(\cdot)$ in (1) being either convex or concave and with problems in which no restrictions on amounts of particular resource types allotted to activities, $r_{ik}(t)$, exist; i.e. in (2) $a_{ik}=0$, $b_{ik}=\infty$ for all i and k.
In the remaining cases, however, it is usefull to reverse this approximation and to treat resource requirements as discrete. Instead of intervals (2) we may consider corresponding finite sets containing discrete amounts of each resource which may be allotted to particular activities (we can assume that the resource allocation may vary over time, assuming one arbitrary feasible amount at every moment). Then, for activity A_i, $i=1,2,\dots,n$, we may calculate performing times τ_{ij}, $j=1,2,\dots,$

k_i where k_i denotes the number of all possible vectors of the resource amounts for activity A_i. Next, denoting these vectors by $\bar{r}_{ij}=(r_{ij1},r_{ij2},\ldots,r_{ijp})$, we have $\tau_{ij} = w_i/f_i(\bar{r}_{ij})$, $j=1,2,\ldots,k_i$. Knowing the performing times τ_{ij} of all activities we may find the optimal solution by solving one or more times the proper linear programming (LP) problem formulated analogously as in (Węglarz and others, 1977). Of course, from the computational point of view in the general case this is not so easy, because of the large number of variables and the number of times that the LP problem has to be solved (this number is equal to the number of possible orderings of nodes in an activity-on-arc graph representing operational precedence constraints among activities in $\mathcal{A}$). The consequences of the first difficulty may be reduced to a certain degree by the use of procedure ARSME (Węglarz and others, 1977) with a slightly modified subroutine GEN. The second difficulty however may be avoided by the use of the heuristic method for finding the ordering of nodes in the graph, which after solving only one LP problem leads to solutions differring from the optimum by 2.0% on average from the project duration point of view (Słowiński, 1978). The above described interconnections between the approaches are connected with natural approximations concerning the resource requirements of activities. Now we will describe a more fundamental interconnection connected with the mathematical models of activities. Let us consider the problem of allocating continuous resources when functions $f_i(\cdot)$, $i=1,2,\ldots,n$ in (1) are linear and resource requirements concern intervals (2) where $a_{ik}=0$ for all i and k (the last assumption may easily be omitted).

Let us also assume that resources of particular types take part in the performance of particular activities in known proportions.

For activity A_i, $i=1,2,\ldots,n$:

$$\bar{r}_i(t)=\bar{\alpha}_i u_i(t) \qquad (3)$$

where $u_i(t)\in\langle 0,1\rangle$ for every $t\geqslant 0$,

$\bar{\alpha}_i=(\alpha_{i1},\alpha_{i2},\ldots,\alpha_{ip})$

is the known vector representing amounts of resources taking part in the performance of A_i for $u_i(t)=1$ and defined in such a way that its modulus $|\alpha_i|=$ max (i.e. for at least one k, $\alpha_{ik}=b_{ik}$). Thus, in (1) $f_i[\bar{r}_i(t)]=g_i[u_i(t)]=\beta_i u_i(t)$, where $\beta_i=f_i(\bar{\alpha}_i)$, $i=1,2,\ldots,n$. Of course, knowledge of w_i in this case does not introduce any additional information in comparison with the mathematical models of activities commonly used in the project scheduling approach, because knowing the activity performing time τ_i for an arbitrary $u_i\in\langle 0,1\rangle$ we have $w_i=\tau_i g_i(u_i)$.

Applying the classical project scheduling approach to our case, we could formulate the optimization problem analogously with the case of fixed resource requirements and one resource type. Here, we consider all subsets of the sets of activities which may be performed between the occurrance of consecutive nodes for a given ordering of nodes in an activity-on-arc graph. We should also add the obvious resource constraints following from this, that in fact resource requirements are of type (2) and amounts of resources allotted to particular activities in corresponding time intervals are in fact variables of the optimization problem. It is however easy to note that in this way we would have products of two variables, i.e. time intervals connected with particular subsets of activities and amounts of resources. Of course, the problem should be solved for all possible orderings of nodes in the considered graph. Let us mention that this natural approach to the formulation of the optimization problem may be generalized to the multi-resource case without assumption (3) and also for nonlinear functions $\tau_i(\bar{r}_i)$ as mathematical models of activities, in the way described by Węglarz (1978a). In all the cases however, finding the optimal solution of the obtained programming problem is practically impossible except for very simple situations.

Let us return to our problem under assumption (3), for which the application of the control in a complex-of-operations approach may lead to a much easier method for finding an optimal solution. Let us consider one of the orderings of nodes in the activity-on-arc graph, and denote by F_j, $j=1,2,\ldots,s-1$ (s is the number of nodes in the graph), the set of activities which may be performed between the occurence of node j and j+1. Let Q_i denote the set of indices of sets F containing activity A_i and x_{ij} the part of w_i performed during the time interval corresponding to the set F_j. Then, using the results obtained by Węglarz (1976) and Słowiński and Węglarz (1977) it may be proven that every feasible solution of the following set of linear inequalities:

$$T_{j\min}=\sum_{i\in F_j} x_{ij}/\beta_i \min_k\{N_k/\alpha_{ik}\}\geqslant x_{ij}/\beta_i$$

for every $i\in F_j$, $j=1,2,\ldots,s-1$

subject to:

$$\sum_{j\in Q_i} x_{ij}=w_i,\ i=1,2,\ldots,n,$$

$x_{ij} \geq 0$ for all i and j,
leads to the optimal solution of our problem for the considered ordering of nodes.
The likelyhood of obtaining the optimal solution in this way is quite large with very little computational effort.

JOINT UTILIZATION OF THE APPROACHES

It may be observed that till now most of the results obtained in resource allocation theory have been obtained for situations which could be called "pure". For example, only splittable or only nonsplittable activities, recoverable or non-recoverable and discrete or consinuous resources have been considered. By "continuous" we understand resources which may be allotted to activities in arbitrary amounts belonging to certain intervals, as opposed to "discrete" resources. Of course, this classification depends on the allocation problem under consideration.
In this Section we would like to point out certain possibilities of studying "mixed" situations based on the joint utilization of the project scheduling and control in a complex-of-operations approaches and their results. These also form suggestions concerning some desired directions for further research
Let us consider a situation, where every activity from $\mathcal{A}$ may simultaneously need for its performance certain known amounts of discrete resources and amounts of continuous resources which may be chosen from given intervals. Then mathematical model of activity A_i could have the form (1) where $\dot{x}_i(t) = f_i[\bar{r}_i(t)]$ if $t \in \langle 0, T_i \rangle$ and all resource requirements of A_i are fulfilled at moment t, and $\dot{x}_i(t)=0$, otherwise. Leaving out the question of obtaining optimal solutions in this case, we will describe the general idea of a heuristic approach which seems to yield good results and certainly is very simple and elastic, consisting of two general steps. In the first step we find certain amounts of continuous resources, $\hat{\bar{r}}_i$, calculate activity performance times $\tau_i = w_i / f_i(\hat{\bar{r}}_i)$, i=1,2,...,n for these amounts, and on this basis find the schedule which optimizes a given performance measure, by solving an appropriate classical scheduling problem. The principle of chosing $\hat{\bar{r}}_i$ has to be elaborated and verified experimentally for particular problems, taking into account mainly the type of functions $f_i(\cdot)$ and the optimality criterion.
Till now, these principles have been elaborated for certain types of $f_i(\cdot)$ for the one-resource case or multiresource case under assumption (3) and project duration as an optimality criterion (Węglarz, 1978b).
In the second step we find the parts of activities performed in parallel in the obtained schedule and then we allocate continuous resources among particular sets of these parts. This is rather simple in many practical cases, because we are dealing with independent activities parts of activities .
Let us note that in particular steps of the algorithm we can fully utilize results from project scheduling and from control in a complex-of-operations framework. Especially interesting are problems in which the resource requirements of an activity in addition to continuous resources concern a machine from a set of parallel, identical machines. Then, in the first step of the algorithm, classical task scheduling algorithms can be utilized, which are polynomial in time for a number of cases. To conclude, let us discuss several points cennected with the above presented idea. Firstly, in the first step of the algorithm, by applying scheduling algorithms for non-splittable activities, we may ensure that no activity in the final schedule will be split for all cases in which in (2) $a_{ik}=0$ for all i and k. Secondly, in both steps of the algorithm different optimality criteria may be considered. Thirdly, further generalizations are possible, for example resource requirements for discrete resources may concern amounts from finite sets - then for every allocation variant of these resources, different functions $f_i[\bar{r}_i(t)]$ may be considered for each activity A_i, i=1,2,...,n. We may also take into consideration non-recoverable resources which are neither recoverable nor non-recoverable , for example power, when energy consumption is constrained. Then the resource constraints have to be completed by constraints of the type

$$\sum_{i=1}^{n} \int_{0}^{T_i} f_i(\cdot)\,dt \leq S(t) \text{ or } \sum_{i=1}^{n} \int_{0}^{T} f_i(\cdot)\,dt \leq M,$$

where T is the project duration. Probability studies can also be developed for example for random parameters in mathematical models of activities (Bubnicki, 1971).

FINAL REMARKS

To sum up the considerations of this paper, we may conclude that the control in a complex-of-operations approach is especially valuable, for dealing with continuous resources of both categories. Thanks to its generality, this approach allows the general properties of optimal solutions to be examined which may simplify the methods used for obtaining them. Results obtained within this approach also allow for the finding approximate solutions in cases when discrete resources may be treated as continuous ones and when the natural approach leads to very complicated optimization problems. Greater utilization of the results obtained in both approaches is also possible in more general models with different types of activity resource requirements.

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