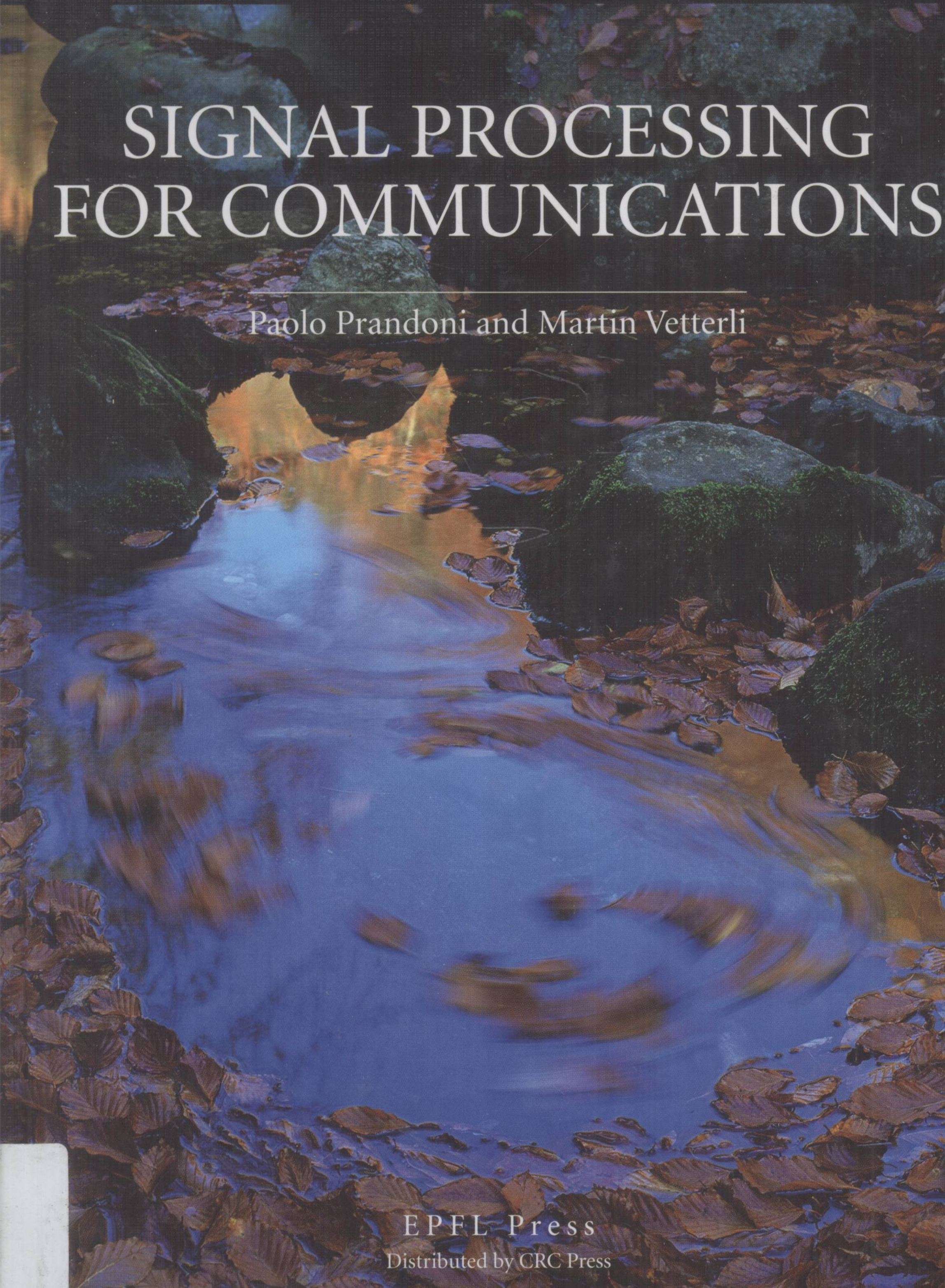
COMMUNICATION AND INFORMATION SCIENCES

SIGNAL PROCESSING FOR COMMUNICATIONS

Paolo Prandoni and Martin Vetterli



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About the cover photograph

Autumn leaves in the Gorges de l'Areuse, by Adrien Vetterli.

Besides being a beautiful picture, this photograph also illustrates a basic signal processing concept. The exposure time is on the order of a second, as can be seen from the fuzziness of the swirling leaves; in other words, the photograph is the average, over a one-second interval, of the light intensity and of the color at each point in the image. In more mathematical terms, the light on the camera's film is a three-dimensional process, with two spatial dimensions (the focal plane) and one time dimension. By taking a photograph we are sampling this process at a particular time, while at the same time integrating (i.e. lowpass filtering) the process over the exposure interval (which can range from a fraction of a second to several seconds).

To wine, women and song.
Paolo Prandoni

To my children, Thomas and Noémie, who might one day learn from this book the magical inner-workings of their mp3 player, mobile phone and other objects of the digital age.
Martin Vetterli

Preface

The present text evolved from course notes developed over a period of a dozen years teaching undergraduates the basics of signal processing for communications. The students had mostly a background in electrical engineering, computer science or mathematics, and were typically in their third year of studies at Ecole Polytechnique Fédérale de Lausanne (EPFL), with an interest in communication systems. Thus, they had been exposed to signals and systems, linear algebra, elements of analysis (e.g. Fourier series) and some complex analysis, all of this being fairly standard in an undergraduate program in engineering sciences.

The notes having reached a certain maturity, including examples, solved problems and exercises, we decided to turn them into an easy-to-use text on signal processing, with a look at communications as an application. But rather than writing one more book on signal processing, of which many good ones already exist, we deployed the following variations, which we think will make the book appealing as an undergraduate text.

1. Less formal: Both authors came to signal processing by way of an interest in music and think that signal processing is fun, and should be taught to be fun! Thus, choosing between the intricacies of z-transform inversion through contour integration (how many of us have ever done this after having taken a class in signal processing?) or showing the Karplus-Strong algorithm for synthesizing guitar sounds (which also intuitively illustrates issues of stability along the way), you can guess where our choice fell.

   While mathematical rigor is not the emphasis, we made sure to be precise, and thus the text is not approximate in its use of mathematics. Remember, we think signal processing to be mathematics applied to a fun topic, and not mathematics for its own sake, nor a set of applications without foundations.

2. More conceptual: We could have said “more abstract”, but this sounds scary (and may seem in contradiction with point 1 above, which of course it is not). Thus, the level of mathematical abstraction is probably higher than in several other texts on signal processing, but it allows to think at a higher conceptual level, and also to build foundations for more advanced topics. Therefore we introduce vector spaces, Hilbert spaces, signals as vectors, orthonormal bases, projection theorem, to name a few, which are powerful concepts not usually emphasized in standard texts. Because these are geometrical concepts, they foster understanding without making the text any more complex. Further, this constitutes the foundation of modern signal processing, techniques such as time-frequency analysis, filter banks and wavelets, which makes the present text an easy primer for more advanced signal processing books. Of course, we must admit, for the sake of full transparency, that we have been influenced by our research work, but again, this has been fun too!

3. More application driven: This is an engineering text, which should help the student solve real problems. Both authors are engineers by training and by trade, and while we love mathematics, we like to see their “operational value”. That is, does the result make a difference in an engineering application?

   Certainly, the masterpiece in this regard is C. Shannon’s 1948 foundational paper on “The Mathematical Theory of Communication”. It completely revolutionized the way communication systems are designed and built, and, still today, we mostly live in its legacy. Not surprisingly, one of the key results of signal processing is the sampling theorem for bandlimited functions (often attributed to Shannon, since it appears in the above-mentioned paper), the theorem which single-handedly enabled the digital revolution. To a mathematician, this is a simple corollary to Fourier series, and he/she might suggest many other ways to represent such particular functions. However, the strength of the sampling theorem and its variations (e.g. oversampling or quantization) is that it is an operational theorem, robust, and applicable to actual signal acquisition and reconstruction problems.

   In order to showcase such powerful applications, the last chapter is entirely devoted to developing an end-to-end communication system, namely a modem for communicating digital information (or bits) over an analog channel. This real-world application (which is present in all modern communication devices, from mobile phones to ADSL boxes) nicely brings together many of the concepts and designs studied in the previous chapters.

Being less formal, more abstract and application-driven seems almost like moving simultaneously in several and possibly opposite directions, but we believe we came up with the right balancing act. Ultimately, of course, the readers and students are the judges!

A last and very important issue is the online access to the text and supplementary material. A full html version together with the unavoidable errata and other complementary material is available at www.sp4comm.org. A solution manual is available to teachers upon request.

As a closing word, we hope you will enjoy the text, and we welcome your feedback. Let signal processing begin, and be fun!

Martin Vetterli and Paolo Prandoni

Spring 2008, Paris and Grandvaux

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The current book is the result of several iterations of a yearly signal processing undergraduate class and the authors would like to thank the students in Communication Systems at EPFL who survived the early versions of the manuscript and who greatly contributed with their feedback to improve and refine the text along the years. Invaluable help was also provided by the numerous teaching assistants who not only volunteered constructive criticism but came up with a lot of the exercices which appear at the end of each chapter (and their relative solutions). In no particular order: Andrea Ridolfi provided insightful mathematical remarks and also introduced us to the wonders of PsTricks while designing figures. Olivier Roy and Guillermo Barrenetxea have been indefatigable ambassadors between teaching and student bodies, helping shape exercices in a (hopefully) more user-friendly form. Ivana Jovanovic, Florence Bénézit and Patrick Vandewalle gave us a set of beautiful ideas and pointers thanks to their recitations on choice signal processing topics. Luciano Sbaiz always lent an indulgent ear and an insightful answer to all the doubts and worries which plague scientific writers. We would also like to express our personal gratitude to our families and friends for their patience and their constant support; unfortunately, to do so in a proper manner, we should resort to a lyricism which is sternly frowned upon in technical textbooks and therefore we must confine ourselves to a simple "thank you".

Contents

Chapter 1

What Is Digital Signal Processing?

A *signal*, technically yet generally speaking, is a a formal description of a phenomenon evolving over time or space; by *signal processing* we denote any manual or "mechanical" operation which modifies, analyzes or otherwise manipulates the information contained in a signal. Consider the simple example of ambient temperature: once we have agreed upon a formal model for this physical variable – Celsius degrees, for instance – we can record the evolution of temperature over time in a variety of ways and the resulting data set represents a temperature "signal". Simple processing operations can then be carried out even just by hand: for example, we can plot the signal on graph paper as in Figure 1.1, or we can compute derived parameters such as the average temperature in a month.

Conceptually, it is important to note that signal processing operates on *an abstract representation* of a physical quantity and not on the quantity itself. At the same time, the *type* of abstract representation we choose for the physical phenomenon of interest determines the nature of a signal processing unit. A temperature regulation device, for instance, is not a signal processing system as a whole. The device does however contain a signal processing core in the feedback control unit which converts the instantaneous *measure* of the temperature into an ON/OFF trigger for the heating element. The physical nature of this unit depends on the temperature model: a simple design is that of a mechanical device based on the dilation of a metal sensor; more likely, the temperature signal is a voltage generated by a thermocouple and in this case the matched signal processing unit is an operational amplifier.

Finally, the adjective "digital" derives from *digitus*, the Latin word for finger: it concisely describes a world view where everything can be ultimately represented as an integer number. Counting, first on one's fingers and then

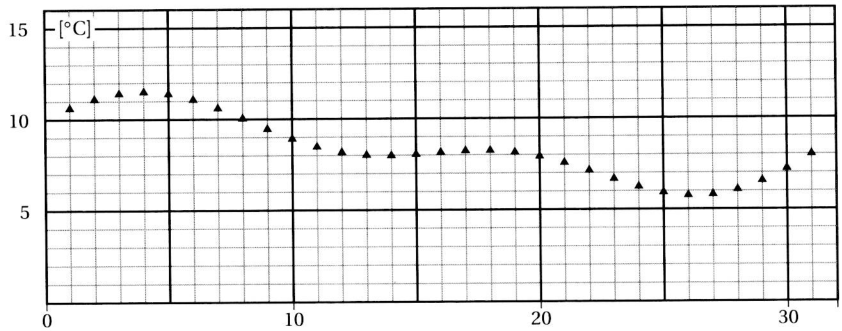

Figure 1.1 Temperature measurements over a month.

in one's head, is the earliest and most fundamental form of abstraction; as children we quickly learn that counting does indeed bring disparate objects (the proverbial "apples and oranges") into a common modeling paradigm, i.e. their cardinality. Digital signal processing is a flavor of signal processing in which everything *including time* is described in terms of integer numbers; in other words, the abstract representation of choice is a one-size-fit-all countability. Note that our earlier "thought experiment" about ambient temperature fits this paradigm very naturally: the measuring instants form a countable set (the days in a month) and so do the measures themselves (imagine a finite number of ticks on the thermometer's scale). In digital signal processing the underlying abstract representation is always the set of natural numbers regardless of the signal's origins; as a consequence, the physical nature of the processing device will also always remain the same, that is, a general digital (micro)processor. The extraordinary power and success of digital signal processing derives from the inherent universality of its associated "world view".

1.1 Some History and Philosophy

1.1.1 Digital Signal Processing under the Pyramids

Probably the earliest recorded example of digital signal processing dates back to the 25th century BC. At the time, Egypt was a powerful kingdom reaching over a thousand kilometers south of the Nile's delta. For all its latitude, the kingdom's populated area did not extend for more than a few kilometers on either side of the Nile; indeed, the only inhabitable areas in an otherwise desert expanse were the river banks, which were made fertile