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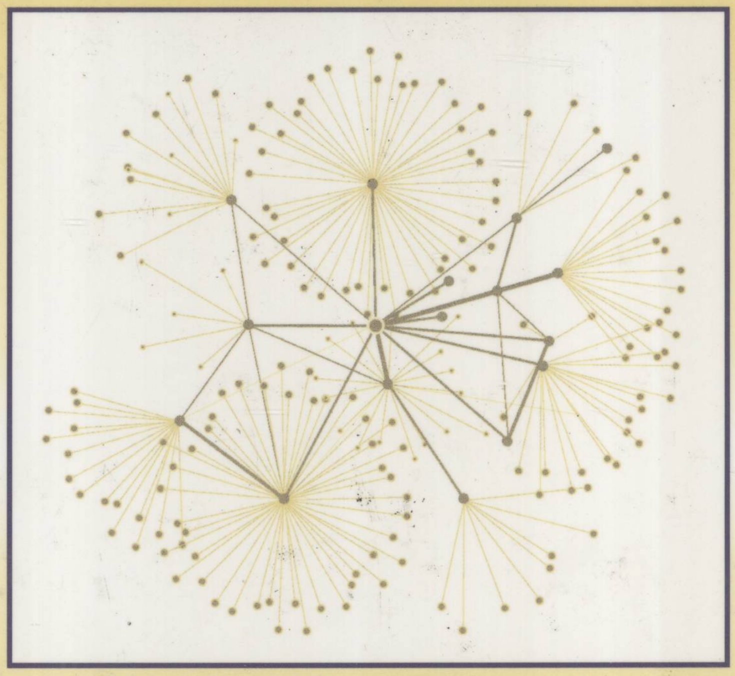
Weakly Coupled Systems and Applications



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Weakly Coupled Systems and Applications

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Preface

This book is intended for engineers, mathematicians, physicists, and computer scientists interested in control theory and its applications. It describes a special class of linear and bilinear control systems known as weakly coupled systems. These systems, characterized by the presence of weak coupling among subsystems, describe dynamics of many real physical systems such as chemical plants, power systems, aircraft, satellites, machines, cars, and computer/communication networks.

Weakly coupled control systems have become an extensive area of research since the end of the 1960s when the original papers of Professor Kokotović and his coworkers and graduate students were published. A relatively large number of journal papers on weakly coupled control systems were published from the 1970s through the 1990s. The approaches taken during the 1970s and 1980s were based on expansion methods (power series, asymptotic expansions, and Taylor series). These approaches were in most cases accurate only with an $O(\varepsilon^2)$ accuracy, where ε is a small, weak coupling parameter. Generating high-order expansions for these methods has been analytically cumbersome and numerically inefficient, especially for higher dimensional control systems. Moreover, for some applications it has been demonstrated in the control literature that $O(\varepsilon^2)$ accuracy is either not satisfactory or in some cases has not solved weakly coupled control problems.

The development of high-accuracy efficient techniques for weakly coupled control systems began at the end of the 1980s in the published papers of Professor Gajić and his graduate students and coworkers. The corresponding approach was recursive in nature and based on fixed-point iterations. In the early 1990s, the fixed-point recursive approach culminated in the so-called Hamiltonian approach for the exact decomposition of weakly coupled, linear-quadratic, deterministic and stochastic, optimal control, and filtering problems. In the new millennium, Professor Kecman developed the generalized Hamiltonian approach based on the eigenvector method. At the same time, Professor Mukaidani and his coworkers discovered a new approach for studying various formulations of optimal linear, weakly coupled control systems.

This book represents a comprehensive overview of the current state of knowledge of both the recursive approach and the Hamiltonian approach to weakly coupled linear and bilinear optimal control systems. It devises unique powerful methods whose core results are repeated and slightly modified over and over again, while the methods solve more and more challenging problems of linear and bilinear weakly coupled, optimal, continuous- and discrete-time systems. It should be pointed out that some related problems still remain unsolved, especially corresponding problems in the discrete-time domain, and the optimization problems over a finite horizon. Such problems are identified as open problems for future research.

The presentation is based on the research work of the authors and their coworkers. The book presents a unified theme about the exact decoupling of the corresponding optimal control problems and decoupling of the nonlinear algebraic

Riccati equation into independent, reduced-order, subsystem-based algebraic Riccati equations.

Each chapter is organized to represent an independent entity so that readers interested in a particular class of linear and bilinear weakly completed control systems can find complete information within a particular chapter. The book demonstrates theoretical results on many practical applications using examples from aerospace, chemical, electrical, and automotive industries. To that end, we apply theoretical results obtained from optimal control and filtering problems represented by real mathematical models of aircraft, power systems, chemical reactors, and so on.

The authors are thankful for support and contributions from their colleagues, Professors S. Bingulac, H. Mukaidani, D. Petkovski, B. Petrović, N. Prljaca, and X. Shen, and Drs. D. Aganović, I. Borno, Y.-J. Kim, M. Qureshi, and V. Radisavljević.

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1 Introduction

This book is intended for engineers, mathematicians, physicists, and computer scientists interested in control theory and its applications. It studies special classes of linear and bilinear control dynamic systems known as weakly coupled systems. These systems, characterized by the presence of small parameters causing weak connections among subsystems, represent many real physical systems such as absorption columns, catalytic crackers, chemical plants, chemical reactors, helicopters, satellites, flexible beams, cold-rolling mills, power systems, electrical circuits, large space flexible structures, computer/communication networks, paper making machines, etc. The techniques presented show how to study independently, from the subsystem level perspective and with a high accuracy deterministic and stochastic, continuous- and discrete-time, optimal control and filtering problems for the considered class of systems.

Each chapter is organized to represent an independent entity so that readers interested in a particular class of weakly coupled control systems can find complete information within the particular chapter. The book demonstrates theoretical results on many practical applications using examples from aerospace, chemical, electrical, and automotive industries.

This book presents reduced-order (subsystem level) algorithms and techniques for optimal control of weakly coupled linear and bilinear dynamic systems composed, in general, of n subsystems. For the reason of simplicity, at many places we consider only two weakly coupled subsystems. The book is written in the spirit of parallel and distributed computations (Bertsekas and Tsitsiklis 1989, 1991) and parallel processing of information in terms of reduced-order controllers and filters (Gajić et al. 1990; Gajićc and Shen 1993; Aganović and Gajić 1995; Gajić and Lim 2001). It covers almost all important aspects of optimal control theory in the context of continuous and discrete, deterministic and stochastic weakly coupled linear systems, and major aspects of optimal control theory of bilinear weakly coupled systems.

The material considered in the book is mostly based on the authors' research accomplishments during the last 20 years, which resulted in many journal and conference papers and three monographs (Gajić et al. 1990; Gajić and Shen 1993; Aganović and Gajić 1995) on analysis and synthesis of optimal controllers and filters for weakly coupled control systems. Consequently, the material presented in this monograph in an integral part of all our previous publications. It also represents extensions, improvements, corrections, new ideas, and overviews of all our previous work on weakly coupled control systems.

The initial idea of weak coupling dealing with eigenvalues and eigenvectors of a weakly coupled system matrix can be found in the work of Milne (1965). The linear weakly coupled control systems were introduced to the control audience by Professor Petar Kokotović in 1969 (Kokotović et al. 1969; see also Kokotović 1972), and since then they have been studied in different setups by many well-respected control engineering researchers, for example (to name a few), Sundararajan and Cruz (1970), Haddad and Cruz (1970), Kokotović and Singh (1971), Medanić and Avramović (1975), Ishimatsu et al. (1975), Ozguner and Perkins (1977), Delacour et al. (1975), Ozguner and Perkins (1977), Delacour et al. (1978), Mahmoud (1978), Khalil and Kokotović (1978), Petkovski and Rakić (1979), Washburn and Mendel (1980), Kokotović (1981), Looze and Sandell (1982), Peponides and Kokotović (1983), Tzafestas and Anagnostou (1984), Sezer and Siljak (1986, 1991), Calvet and Title (1989), Kaszkurewicz et al. (1990), Siljak (1991), Srikant and Basar (1991, 1992a,b), Basar and Srikant (1991), Su and Gajić (1991, 1992), Al-Saggaf (1992), Aganović and Gajić (1993), Riedel (1993), Geray and Looze (1996), Finney and Heck (1996), Hoppensteadt and Izhikevich (1997), Derbel (1999), Lim and Gajić (1999), Gajić and Borno (2000), Mukaidani (2006a,b, 2007a–c), Kecman (2006), Huang et al. (2005), and Kim and Lim (2006, 2007).

Traditionally, solutions of the main equations of analysis and synthesis of *linear* optimal controllers and filters (Anderson and Moore 1990), Riccati-type (Lancaster and Rodman 1995) and Lyapunov-type equations (Gajić and Qureshi 1995) were obtained for weakly coupled systems in terms of Taylor series and power-series expansions with respect to a small weak coupling parameter ε. Approximate feedback control laws were derived by truncating expansions of the feedback coefficients of the optimal control law (Kokotović et al. 1969; Haddad and Cruz 1970; Ozguner and Perkins 1977; Delacour 1978; Petkovski and Rakić 1979). Such approximations have been shown to be near-optimal with performance that can made as close to the optimal performance as desired by including enough terms in the truncated expansions.

In this book, we will study linear weakly coupled control systems by using two new approaches developed by the authors during the last 20 years: the recursive approach (based on fixed point iterations) and the so-called Hamiltonian approach (based on block diagonalization of the Hamiltonian matrix of optimal control theory of linear systems). Consistently, the book is divided into three parts: Part I—Recursive approach for linear weakly coupled control systems, Part II—Hamiltonian approach for linear weakly coupled control systems, and Part III—Bilinear weakly coupled control systems.

The *recursive approach* to weakly coupled control systems (based on fixed point iterations) originated in the late 1980s and at the beginning of the 1990s in the papers by Gajić and his coworkers (Petrović and Gajić 1988; Harkara et al. 1989; Shen and Gajić 1990a–c; Shen 1990; and Qureshi 1992). It has been shown that the recursive methods are particularly useful when the coupling parameter ε is not extremely small and/or when any desired order of accuracy is required, namely, $O(\varepsilon^k)$,* where $k = 2, 3, 4, \ldots$. In some applications a very good approximation is required, such as for a plant-filter augmented system (Shen and Gajić 1990a), where the accuracy of $O(\varepsilon^k)$, $k \geq 6$ was needed to stabilize considered real world

* $O(\varepsilon^k)$ stands for $C\varepsilon^k$, where C is a bounded constant and k is any arbitrary constant.

closed-loop electric power system. The recursive methods are particularly important for optimal output feedback control problems, where the solution of highly nonlinear algebraic equations is required. The effectiveness of the corresponding reduced-order algorithm and its advantages over the global full-order algorithm are demonstrated in Harkara et al. (1989) on a 12-plate chemical absorption column example. Obtained results strongly support the necessity for the existence of reduced-order recursive numerical techniques for solving corresponding nonlinear algebraic equations. In addition to the reduction in required computations, it can be easier to find a good initial guess and to handle the problem of nonuniqueness of the solution of corresponding nonlinear equations. The recursive approach to continuous and discrete, deterministic and stochastic, linear weakly coupled control systems was further advanced in the papers by Skatarić et al. (1991, 1993), Skatarić (1993), Hogan and Gajić (1994), Borno (1995), Borno and Gajić (1995), Gajić and Borno (2000), and Skatarić (2005). The recursive approach to bilinear weakly coupled control systems was considered in Aganović and Gajić (1995).

The linear weakly coupled system composed of two subsystems is defined by

$$\begin{aligned} \frac{dx_1(t)}{dt} &= A_1x_1(t) + \varepsilon A_2x_2(t) \\ \frac{dx_2(t)}{dt} &= \varepsilon A_3x_1(t) + A_4x_2(t) \end{aligned} \tag{1.1}$$

where ε is a small weak coupling parameter and $x_1(t) \in R^{n_1}$ and $x_2(t) \in R^{n_2}$ are state space variables ($n_1 + n_2 = n$, n is the system order). Matrices A_i, $i = 1, 2, 3, 4$, are constant and $O(1)$. It is assumed that magnitudes of all the system eigenvalues are $O(1)$, that is $|\lambda_j| = O(1)$, $j = 1, 2, \ldots, n$, implying that matrices A_1 and A_4 are nonsingular with $\det\{A_1\} = O(1)$ and $\det\{A_4\} = O(1)$. This is the standard assumption for weakly coupled linear systems, which also corresponds to the block diagonal dominance of the system matrix A (Chow and Kokotović 1983). Hence, the main results presented in this book are valid under the following weak coupling assumption.

Assumption 1.1 Matrices A_i, $i = 1, 2, 3, 4$, are constant and $O(1)$. In addition, magnitudes of all system eigenvalues are $O(1)$, that is, $|\lambda_j| = O(1)$, $j = 1, 2, \ldots, n$, which implies that the matrices A_1, A_4 are nonsingular with $\det\{A_1\} = O(1)$ and $\det\{A_4\} = O(1)$.

This assumption in fact indicates block diagonal dominance of the system matrix. It states the condition which guarantees that weak connections among the subsystems will indeed imply weak dynamic coupling. Note that when this assumption is not satisfied, the system defined in Equation 1.1, in addition of weak coupling can also display multiple timescale phenomena (singular perturbations), as considered in Phillips and Kokotović (1981), Delebeque and Quadrant (1981), and Chow (1982), for large-scale Markov chains and power systems. In the case when Assumption 1.1 is not satisfied, the slow coherency method (Chow 1982) can be used to form a reduced-order slow aggregate model that represents a long-term equivalent of the original system. Using the slow coherency method, the system (Equation 1.1) will be decoupled into three subsystems. The slow coherency method will not be covered in this book. However, in Chapter 3, we will present a class of systems that display both weak coupling and singular perturbations phenomena. The reader interested in coherency based decomposition methods is referred also to Kokotović et al. (1982).

The following simple example demonstrated importance of Assumption 1.1 for the definition of weakly coupled linear systems.

Example 1.1

Consider two "weakly" coupled linear systems. The first one satisfies the weak coupling Assumption 1.1, that is, both its eigenvalues are $O(1)$, and the second system has one eigenvalue of $O(\varepsilon)$ and two eigenvalues of $O(1)$

$$\frac{dx(t)}{dt} = \begin{bmatrix} -1 & 2\varepsilon \\ -1.5\varepsilon & -2 \end{bmatrix} x(t)$$

$$\frac{dz(t)}{dt} = \begin{bmatrix} -1 & 2\varepsilon & \varepsilon \\ 1.5\varepsilon & -2 & \varepsilon \\ \varepsilon & \varepsilon & -2\varepsilon \end{bmatrix} z(t)$$

The decoupled, reduced-order, state models of these systems can be obtained by neglecting $O(\varepsilon)$ terms, that is

$$\frac{dx(t)}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x(t)$$

$$\frac{dz(t)}{dt} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} z(t)$$

Assuming that initial conditions for these two systems are given by $x(0) = [1\ \ 1]^T$ and $z(0) = [1\ \ 1\ \ 1]^T$, we have presented in Figures 1.1 and 1.2, respectively for the second- and third-order systems, the system state responses due to initial conditions (zero-input responses) for both the original and decoupled systems. The responses for the decoupled subsystems (obtained by setting $\varepsilon = 0$) are denoted by the dashed lines. It can be seen from Figure 1.1 that the response of the original and decoupled systems are close to each other, $O(\varepsilon)$ apart for all times, which is expected from a weak coupling system that satisfies Assumption 1.1. However, Figure 1.2 indicates, that the third-order system state space response for one of the state variables, corresponding to $O(\varepsilon)$ eigenvalue, is not close to the corresponding response of the decoupled subsystem. Even more, it can be seen from the same figure that this state variable is much slower than the remaining two state space variables indicating the presence of two timescales in this system.

For linear weakly coupled systems, the development of the decoupling transformation of Gajić and Shen (1989) is particularly important. With this nonsingular

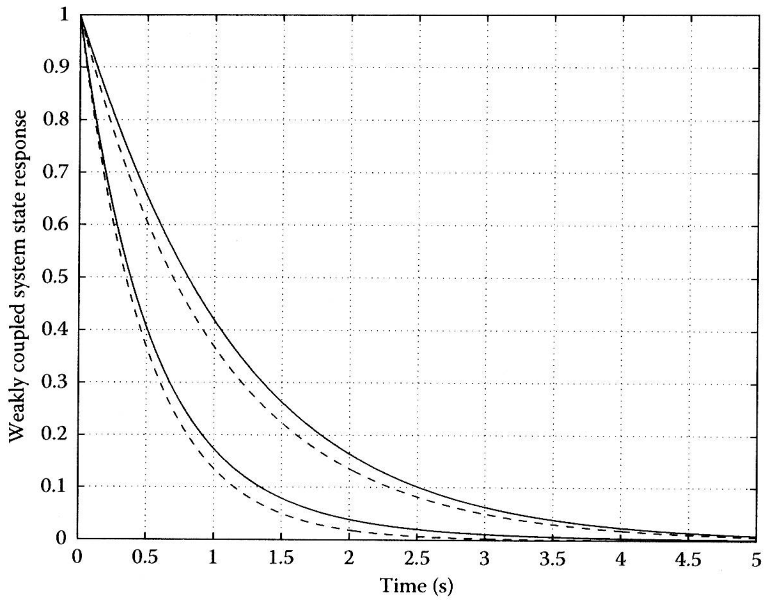

FIGURE 1.1 Zero-input response of the linear weakly coupled system (dashed lines denote the decoupled system response).

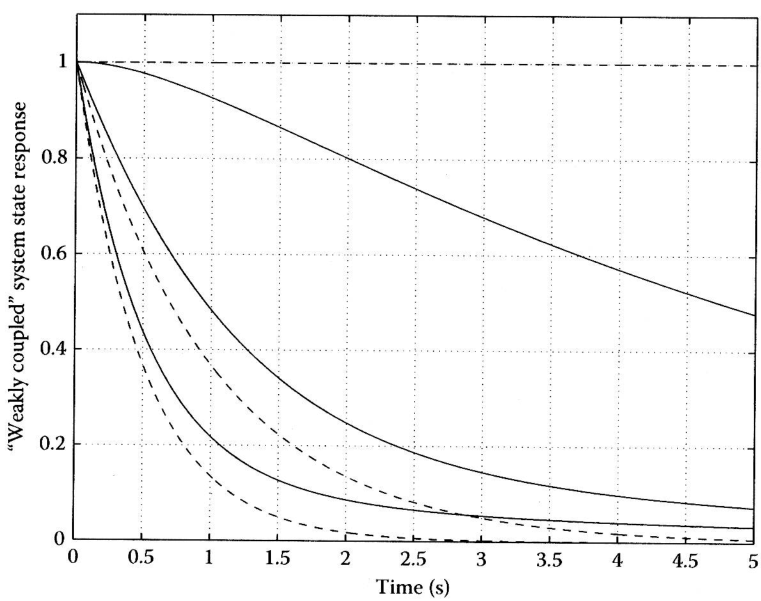

FIGURE 1.2 Zero-input response of a linear weakly coupled system (dashed lines denote the decoupled system response).