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Operations Research Support Methodology

edited by Albert G. Holzman

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Operations Research Support Methodology

INDUSTRIAL ENGINEERING

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PREFACE

Many introductory books in the area of Operations Research have been published during the past decade. The objective of these books has been primarily to acquaint students with quantitative methods in the decision-making process, focusing primarily on management functions. The techniques and methodologies covered in these introductory books are usually quite standard or conventional, including topics such as linear programming, game theory, queuing theory, dynamic programming, inventory theory, and simulation.

As is obvious from the Table of Contents of this book, the major thrust of topics is quite different from a typical book on Operations Research. However, a prime factor in the selection of articles to be included in this book was its present and potential support to Operations Research activity. Some of the articles are directly related to Operations Research, while others are more fundamental and have utility in many other areas. For example, articles such as barrier methods and bivalent programming are readily recognized as OR topics, whereas the subject areas of fuzzy sets and artificial intelligence may not be so closely associated with the more classical interpretation of OR.

It is felt by the editor that the material included has relevance to a broad spectrum of people involved with many different kinds of OR related activities. For example, the articles on Complementarity Problems and Fixed Point Computer Methods are expositions of more advanced considerations of mathematical programming; on the other hand, the articles on Abstract Algebra and Functional Analysis are basic mathematical concepts which are not directly associated with OR, but are supportive of some OR studies.

Most of the articles are in-depth and self-contained. All of the authors are very distinguished in their respective areas, and do have national and international reputations. The criteria submitted to the authors before preparation of their articles are enumerated as follows: "The type of article sought is of simple exposition but of scholarly and exhaustive treatment including whatever history and background that seems appropriate, a review of the state of the art thinking relative to the topic, as well as a discussion of unsolved problems and any interesting research approaches to problem solutions; although the article is to be primarily aimed at the user who is relatively uninformed on the topic, it should be able to withstand review by the informed specialists on that topic as a sound statement; on the other hand, even this informed person may find some material that is stimulating to him."

These criteria were presented as guidelines to the authors, and it is obviously impossible for an author to equally satisfy all of these objectives. In fact, some topics do not lend themselves to the attainment of certain stated criteria. The prime purpose of the cited goals was to give each author the tenor of presentation for the article requested. For example, the term "simple exposition" will have a different interpretation dependent upon one's maturity and background in the area being covered.

The collection of articles included is such that the novice as well as the more advanced operations researcher will find at least some of the articles helpful. However, the material is oriented more towards the person who already has some background in OR and wishes to expand his knowledge in related areas.

It is important to note that these articles were written originally for a 15-volume Encyclopedia of Computer Science and Technology, edited by Jack Belzer, Albert G. Holzman, and Allen Kent. For this Encyclopedia, we considered "computer science and technology" in a broad sense, including areas such as mathematics, probability, management science, and operations research. These methodologies have been used extensively by people in the computer field.

In the review of manuscripts for the Encyclopedia, I identified some excellent articles which I termed as support methodologies in the OR field. Unfortunately, these articles are scattered throughout many volumes of the Encyclopedia, and as a result, it is not realistic for an individual interested in the selected OR related topics to have these readily available to him. I felt that a single book containing these selected articles would be of interest to a wide range of people in OR and the related areas of management science, industrial engineering, and applied mathematics.

While most of the articles are approximately the same number of pages, there is a significant variance in the length of several articles. As an example, the first article on abstract algebra greatly exceeds the length of any of the others. I had seriously considered reducing the number of pages for this article, but concluded that the complete article was developed so well with illustrations and examples that it would be inappropriate to cut up the article to meet some arbitrary page limit. In fact, it could be used as an introductory "book" to Abstract Algebra.

Certainly it is recognized that there is some imbalance among the articles, and this can be expected when each article is written by an individual who reflects his own background and experiences.

The 20 articles selected have been categorized under the following four major headings to give structure to the book.

1. Mathematical Foundations
2. Mathematical Methods
3. Operations Research Related Concepts and Solution Methodologies
4. Linguistics and Behavioral Concepts

The first three categories comprise the greatest portion of the book and are probably the most directly related support functions to Operations Research. The articles listed under Linguistics and Behavioral Concepts are not typically found in an OR book, and are included to suggest possible new concentration areas for OR people. While Fuzzy Sets is relatively new, Artificial Intelligence, and Adaptive and Learning Systems have been of concern to psychologists and computer science people for a much longer period of time.

The well-known Professor Herbert A. Simon in his Distinguished Lecture at the ORSA/TIMS meeting in Chicago in 1975, expressed the need for operations researchers to become acquainted with artificial intelligence for heuristic problem solving.

One of the major contributions of this book is the collection in a single volume of many in-depth articles by experts in their fields for a broad array of topics that presently are supportive of Operations Research or have the potential to contribute to this area. It is not meant to imply that this is an all inclusive coverage; instead, it should be considered merely as a sample of many possible subjects that could have been included.

Albert G. Holzman

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CONTENTS

MATHEMATICAL FOUNDATIONS

The chapter on Abstract Algebra introduces the reader to the basic concepts, with emphasis in the article on automata theory and finite-state machine models. The first three major sections, which discuss the fundamentals of abstract algebra, are (1) algebra of sets, (2) relations, partitions and functions, and (3) Boolean algebra. With this foundation the author then develops in the next two sections switching (Boolean) foundations, and mathematical description of sequential networks. Finite automata are introduced in the first part of the latter section. The final two sections focus on the elements of number theory and algebraic systems. As stated in the Preface of this book, this chapter on Abstract Algebra greatly exceeds the length of any of the other articles, but the editor concluded that because the article was so well developed, replete with illustrations and examples, it was considered completely inappropriate to dissect the article to meet an arbitrary page limit. In fact, this article could serve as an introductory "book" on Abstract Algebra.

As a result of the early introduction of Linear Programming to the OR field, and the continuing strong interest in research and applications in LP over the past 25 years, Linear and Matrix Algebra is a mathematics prerequisite for an understanding of much of the literature in Operations Research. The chapter on Linear and Matrix Algebra begins with the concept of a vector, and then develops with the aid of many examples the general notions of vector spaces and subspaces, linear independence and dimension, basis of vector spaces and simultaneous linear equations. Since efficient methods for solving simultaneous linear equations involves matrices, the concept of a matrix is then introduced, concentrating on inner and outer product of vectors, bases and coordinate systems, and concluding with a discussion of linear transformations and matrix algebra.

In contrast to Abstract Algebra, and Linear and Matrix Algebra, which already have been found to be very useful in OR and computer science, Functional Analysis is more abstract and its utility has been more in research papers. The chapter on Functional Analysis addresses itself in the introduction to the transition from classical analysis, which is concerned with the real line (or the complex plane or euclidean n-space) and functions on it, to functional analysis which considers more general "spaces" and mappings between them. Abstract concepts and methods are introduced as a means of simplification and a way towards greater generality, which often suggests a unified treatment of problems of different nature. Since metric spaces may be likened to the real line in elementary analysis, they are fundamental in functional analysis. The concepts of vector spaces, Banach spaces, and Hilbert spaces are then discussed. This is followed by a review of linear operators and linear functionals which are of great importance in considering vector spaces in functional analysis. The properties of certain inverse operations which arise quite naturally in connection with the problem of solving systems of linear algebraic equations, differential equations, and integral equations are reviewed in the section on spectral theory.

ABSTRACT ALGEBRA

INTRODUCTION

A digital system is comprised of networks of two-state devices which are capable of storing and processing representations of information. Progress in technology has resulted in the availability of highly reliable "digital building blocks" for synthesizing discrete-parameter information systems.

Abstract algebra provides the computer scientist with the means for mathematically modeling the behavior of discrete-parameter information systems. From a mathematical description, the computer scientist is able to determine the capability of a theoretical system. Structures are then classified and synthesis procedures developed. This article is devoted to topics of abstract algebra which play a central role in the development of automata theory and finite-state machine models.

THE ALGEBRA OF SETS

Definitions

A *set* may be defined intuitively as a collection of *elements* or objects with some *common property*. The development of abstract algebra begins with the concept of a set.

The notation $a \in A$ denotes that "a is an element of the set A." If "s is not an element of A," it is denoted by $s \notin A$.

Two methods are commonly used to specify a set mathematically. One is to list all the elements within braces such as $\{-1, 0, 1\}$ or $\{1, 3, 5, \ldots\}$ where the dots indicate the omission of particular elements. When a listing is impractical or impossible, it is convenient to use a mathematical description as follows:

$$A = \{a \mid a \text{ has the property } W\}$$

The set A consists of *all elements a* having the *property W*.

EXAMPLE 1

$$S = \{s \mid 0 \leq s \leq 1\}$$

The set S in Example 1 is the collection of all real numbers between 0 and 1 inclusive.

A set with a finite number of elements is called a *finite set*. An *infinite set* such as S in Example 1 contains an infinite number of elements. When the elements of an infinite set can be placed into a one-to-one correspondence with the natural numbers $1, 2, 3, 4, \ldots$ (also called the positive integers), the set is said to be countable or denumerable. The infinite set of even integers is denumerable by virtue of the one-to-one correspondence

$$\begin{array}{ccccc} 1 & 2 & 3 & \ldots & n \\ \updownarrow & \updownarrow & \updownarrow & & \updownarrow \\ 2 & 4 & 6 & \ldots & 2n \end{array}$$

whereas the infinite set S in Example 1 is nondenumerable.

Subsets

If every element contained in set A is also contained in set B, then set A is a subset of set B. This is symbolized by

$$A \subseteq B$$

meaning "A is a subset of B." If set B contains at least one element that is not contained in A, then A is a *proper subset* of B. This is denoted by

$$A \subset B$$

Two sets are equal if they contain the same elements. That is, $A = B$ if and only if

$$A \subseteq B \quad \text{and} \quad B \subseteq A$$

Note that if two elements a and b are equal, they are identical. Hence

$$\{1, 2, 2, 3\} = \{1, 2, 3\}.$$

When considering sets, it is convenient to regard them as subsets of the *universe* or *universal set* $\mathscr{S}$. Once defined, $\mathscr{S}$ remains fixed throughout a given discussion.

The set containing no elements is called the *empty* or *null set* $\emptyset$. Since all null sets contain the same elements, namely, no elements, they are equal. There is, therefore, only one null set, and the term "*the* null set" is justified. The null set is defined as an *improper* subset of every set, and every set is a subset of the universe. Thus

$$\emptyset \subseteq A \subseteq \mathscr{S}$$

The null set may be characterized by

$$\emptyset = \{a \mid a \neq a\}$$

A finite set $\mathscr{S}$ containing n elements has 2^n subsets. These include the two improper subsets $\emptyset$ and $\mathscr{S}$. Enumerating the number of subsets involves counting the number of ways elements can be selected from $\mathscr{S}$ to form a subset.

EXAMPLE 2

The subsets that can be formed from $\mathscr{S} = \{0, 1, 2, 3\}$ are

$\emptyset$	$\{0\}$	$\{0, 1\}$	$\{0, 1, 2\}$	$\{0, 1, 2, 3\}$
	$\{1\}$	$\{0, 2\}$	$\{0, 1, 3\}$	
	$\{2\}$	$\{0, 3\}$	$\{0, 2, 3\}$	
	$\{3\}$	$\{1, 2\}$	$\{1, 2, 3\}$	
		$\{1, 3\}$		
		$\{2, 3\}$		

$\mathscr{S}$ contains $n = 4$ elements and has 2^4 or 16 distinct subsets. Subsets containing the same number of elements appear in the same column. The number of subsets containing i elements where $0 \leq i \leq n$ is the combination of n elements taken i at a time or

$$\binom{n}{i} = \frac{n!}{i!\,(n-i)!}$$

The total number of subsets is

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

The foregoing equation can be derived from the binomial expansion

$$(a + b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

by letting $a = b = 1$. Note that $0! = 1$ from

$$(r - 1)! = \frac{r!}{r}$$

where $r = 1$. There is only *one* way in which n elements can be taken none at a time, i.e.,

$$\binom{n}{0} = \frac{n!}{0!\,n!} = 1$$

The subset with no elements is $\emptyset$.

Operations on Sets

If sets A and B are subsets of the same universe $\mathscr{S}$, there are several ways in which A and B can be combined to form new sets. Rules for forming new sets from A and B define *operations* on A and B. Operations of *union*, *intersection*, (absolute) *complementation*, and *relative complementation* (also called *difference*) are defined as follows:

1. Union $\cup$
   $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
2. Intersection $\cap$
   $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
3. Complement of A
   $\tilde{A} = \{x \mid x \in \mathscr{S} \text{ but } x \notin A\}$
4. Relative complement (difference between A and B) of B in A
   $A - B = \{x \mid x \in A \text{ but } x \notin B\}$

Note that $-B = \tilde{B}$, and $A - B$ may be expressed as $A \cap \tilde{B}$.

Venn diagrams, which appear in Fig. 1, are graphic descriptions of sets and of sets resulting from set operations. Elements of the universe are represented by the interior of a rectangle whereas elements of sets are represented by interiors of circles. The

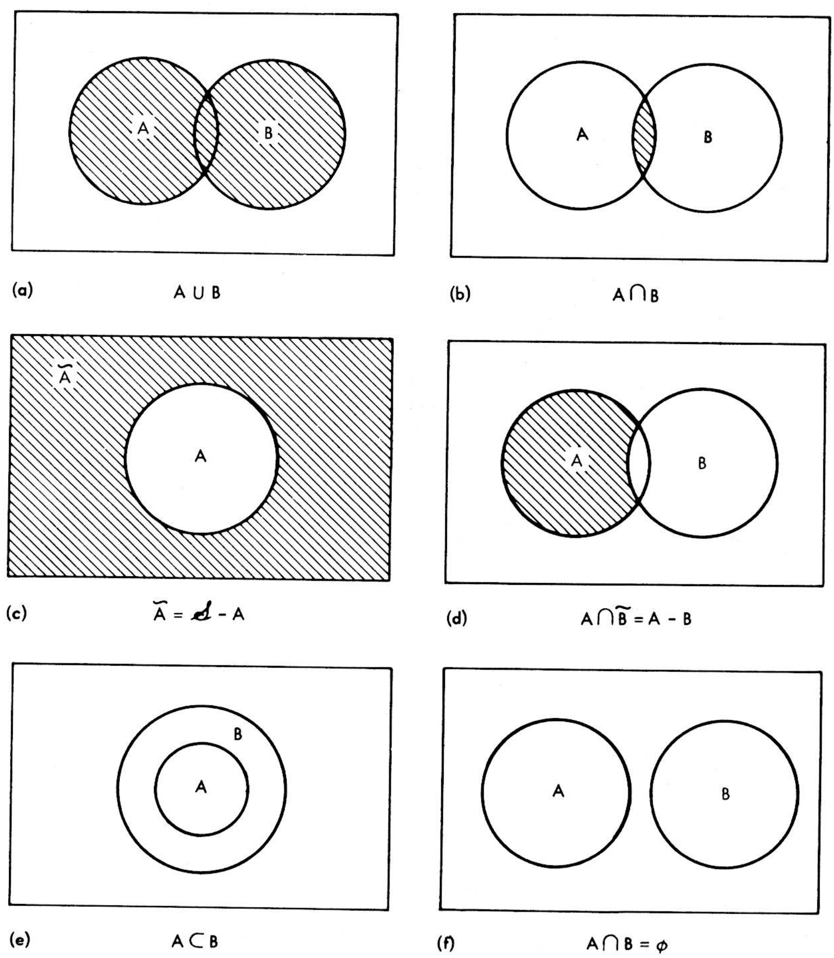

Fig. 1. Venn diagrams.