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Linear Circuits, Systems and Signal Processing: Theory and Application

edited by
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LINEAR CIRCUITS, SYSTEMS AND SIGNAL PROCESSING:
THEORY AND APPLICATION

PREFACE

In part because of its universal role as a first approximation of more complicated behavior and in part because of the depth and breadth of its principle paradigms, the study of linear systems continues to play a central role in control theory and its applications. Enhancing more traditional applications to aerospace and electronics, application areas such as econometrics, finance, and speech and signal processing have contributed to a renaissance in areas such as realization theory and classical automatic feedback control. Thus, accompanying a general maturing and increased applicability in important fields such as parameter identification, the development of algebraic and geometric methods and in the control of distributed parameter systems, the last few years have witnessed a remarkable research effort expended in understanding both new algorithms and new paradigms for modeling and realization of linear processes and in the analysis and design of robust control strategies.

The papers in this volume quite strongly reflect these trends in both the theory and applications of linear systems and were selected from the invited and contributed papers presented at the 8th International Symposium on the Mathematical Theory of Networks and Systems held in Phoenix on June 15-19, 1987.

We would like to thank our co-organizer R. Saeks, for yeoman service and for freely sharing his advice in his inimitable way. Pam Newton has earned our gratitude for her efficient efforts in the compilation of this volume.

Christopher I. Byrnes and Clyde F. Martin
Lubbock, April 25, 1988

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Linear Circuits, Systems and Signal Processing:
Theory and Application
C.I. Byrnes, C.F. Martin and R.E. Saeks (editors)

THE CASCADE LIMIT, THE SHORTED OPERATOR AND QUADRATIC OPTIMAL CONTROL

by

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It is shown that the solution of a discrete quadratic optimal control problem can be written in terms of the shorted operator of a tridiagonal matrix. Additionally, the solution of the control problem may be expressed as the infinite cascade limit of an n-port resistive network. Since the cascade limit has a variational formulation in terms of partitioned matrices, the solution of the optimal control problem also has such a variational formulation.

1. INTRODUCTION

Our goal is to characterize the solution of a discrete time quadratic optimal control problem in terms of the shorted operator and the cascade limit of matrices. To this end, we first briefly review the shorted operator and the cascade sum. After that we show that the solution of the discrete control problem is the infinite cascade limit of the appropriate matrix. We will present an abbreviated version of this work. Complete details will appear elsewhere.

We consider n x n complex matrices, denoted A, B etc. The adjoint of A is denoted A^*. A is said to be Hermitian positive semidefinite (HSD) if $A = A^*$, and $\langle Ax,x\rangle \geq 0$ for all vectors x. $\langle\ ,\ \rangle$ is the inner product. We use the partial order $A \geq B$ whenever A−B is HSD. The Moore-Penrose generalized inverse of A is denoted A^+.

2. SHORTED OPERATOR AND CASCADE ADDITION

We consider HSD matrices partitioned as follows:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

A is HSD whenever A_{11} and $S(A) = A_{11} - A_{12}A_{22}^{+}A_{21}$ are HSD.

The matrix S(A) is termed the shorted operator (matrix) of A; it is also known as the generalized Schur complement. The reader should consult references 1 or 5 for additional material on this operator. In particular, we assume the reader is familiar with alternative characterizations of the shorted operator. In terms of electrical networks, if A is the impedance matrix of an n-port resistive network then S(A) is the impedance matrix of the network which has some of its ports short circuited. Hence the name shorted operator. Another network induced operation, which we use in this work, is the cascade sum. Let A and B be HSD matrices partitioned as A above, and with the additional constraint that A_{11} and A_{22} are the same size (similarly B_{11} and B_{22}). The cascade sum of A and B, denoted $A \circ B$, is defined as follows:

$$A \circ B = \begin{bmatrix} A_{11} - A_{12}DA_{21} & A_{12}DB_{12} \\ A_{21}DB_{21} & B_{22} - B_{21}DB_{12} \end{bmatrix}$$

where $D = (A_{22} + B_{11})^{+}$

The cascade connection of n-port networks provides the underpinning for the cascade sum. The reader can see [2] or [3] for additional information on the cascade sum. The following proposition is needed for what follows; the proof may be found in [3].

<u>Proposition 1</u>: Let A and B be HSD matrices, then the cascade sum of A and B may be represented by the following variational principle:

$$\langle A \circ B \begin{bmatrix} c \\ c \end{bmatrix}, \begin{bmatrix} d \\ d \end{bmatrix} \rangle = \inf_{x} \left(\langle A \begin{bmatrix} c \\ -x \end{bmatrix}, \begin{bmatrix} c \\ -x \end{bmatrix} \rangle + \langle B \begin{bmatrix} x \\ d \end{bmatrix}, \begin{bmatrix} x \\ d \end{bmatrix} \rangle \right)$$

Given the HSD matrix A, one can define a sequence of matrices by sequentially cascading A with itself: $A_0 = A$, and $A_{n+1} = A \circ A_n$. It is known that the A_n sequence has a HSD limit [2,6], termed the cascade limit of A. The limit has been characterized by Ando [6] as follows:

<u>Proposition 2</u>: The cascade limit of the matrix A is given by a block diagonal matrix, whose upper left block X_{11} satisfies the following variational principle: