Ladislav Ceniga

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图书馆藏书

E2010001118

Nova Science Publishers, Inc.
New York

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Library of Congress Cataloging-in-Publication Data
Available upon request.

ISBN 978-1-60456-085-5

Published by Nova Science Publishers, Inc. ✢ New York

ANALYTICAL MODELS OF THERMAL STRESSES IN COMPOSITE MATERIALS I

Preface

This book is dedicated with love to my dearest parents.

This book is the first volume of the trilogy *Analytical models of thermal stresses in composite materials I, II, III*, presenting, in each of the volumes, original results only, created by the author. The fact that the author proceeds from fundamental equations of Mechanics of Solid Continuum presented in Sections 3.1.3–3.1.6, 3.1.8 confirms originality of the results and accordingly establishment of new scientific school with an interdisciplinary character belonging to the scientific branch Applied Mechanics. As an imagination considered for the analytical models, an **elastic** solid continuum is represented by a multi-particle-(envelope)-matrix system consisting of components represented by **spherical** particles periodically distributed in an **infinite** matrix, without or with a **spherical** envelope on the surface of each of the spherical particles. The multi-particle-(envelope)-matrix system with different distribution of the spherical particles is considered as a model system for the determination of the thermal stresses in real composite materials with finite dimensions included in the categories as presented in Section 2.1 (see Items 1–4). Dependent on the particle and envelope radii, R_1 and $R_1 < R_2$, respectively, resulting in the envelope thickness $t = R_2 - R_1$, on the inter-particle distance d and on the particle volume fraction v, representing parameters of a real composite material, the thermal stresses originating during a cooling process are a consequence of the difference in thermal expansion coefficient of the components, as well as a consequence of the difference in dimensions of isotropic cubic crystalline lattices resulting from a phase transformation originating at least in one of the components.

As usual in Mechanics of Solid Continuum representing a fundamental basis of Applied Mechanics, the state of stress of a system can be determined using different

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mathematical techniques resulting in different differential equations for an investigated quantity, e.g. radial displacement, radial stress. Consequently, integration constants included in solutions of each of the different differential equations are derived regarding defined boundary conditions. With regard to the Castigliano's theorem (see Section 3.1.9) and to the different solutions describing the state of stress of the system, such solution is considered to exhibit minimal energy of the system.

With regard to Volume I, the thermal stresses acting in the isotropic multi-particle-(envelope)-matrix system, represented by the isotropic components, are determined using eight different mathematical techniques (see Sections 5.1–5.3, 6.1, 7.1, 7.2, 8.1) applied to the transformed equilibrium equations (3.35), (3.36) and consequently resulting in eight different linear differential equations with **non-zero right sides** related to the radial displacement of an arbitrary point in the isotropic elastic solid continuum, considering boundary conditions defined in Section 4.1.

With regard to specific conditions for the isotropic multi-particle-envelope-matrix system defined in Section 9.1, Equation (3.35) is not considered, and using ten different mathematical techniques presented in Section 9.2, Equation (3.36) is transformed to ten different linear differential equations with **zero right sides**, exhibiting solutions suitable for the determination of the thermal stresses in the spherical particle and envelope. Additionally, with regard to completeness of the presented topic, as presented in Sections 5.7, 6.5 and 9.5, the solutions to result from the eight and ten different linear differential equations with and without right sides, respectively, are used for the determination of the thermal stresses in an isotropic one-particle-(envelope)-matrix system containing, in contrast to the isotropic multi-particle-(envelope)-matrix, one spherical particle only, considering boundary conditions defined in Section 4.2. Finally, Section 9.6.1 presents the mathematical techniques resulting in fifteen different linear differential equations with **zero right sides** related to the radial stress acting in an arbitrary point in the isotropic elastic solid continuum, in contrast to Sections 5.1–5.3, 6.1, 7.1, 7.2, 8.1, 9.2 related to the radial displacement.

Along with two different mathematical techniques, applied to **both** Equations (3.35), (3.36) and resulting in two different differential equations with **zero right sides** presented in Volume II, the analytical models of the thermal stresses acting in the **isotropic** components of the isotropic multi-particle-(envelope)-matrix system, determined in Volumes I, II as extensive as possible regarding a physical point of view, exhibit accordingly permanent validity without a need of the updating in future.

Devoted to an anisotropic elastic solid continuum, Volume III presents analytical models of the thermal stresses acting in the one-axial and triaxial anisotropic components of the anisotropic multi-particle-(envelope)-matrix system. Accordingly, combinations of the analytical models related to the isotropic, one-axial and triaxial anisotropic components presented in the three volumes result in analytical models related to the isotropic-one-axial-anisotropic, isotropic-triaxial-anisotropic and one-triaxial anisotropic multi-particle-(envelope)-matrix systems.

In addition to the thermal stresses acting in each of the components, all volumes of the trilogy present formulae for **corresponding quantities** represented by the elastic energy density w, the elastic energy W and the thermal stresses σ_1, σ_2, σ_3 acting in three mutually perpendicular directions and inducing the elastic energy density w_1, w_2, w_3, respectively, used for the determination of thermal-stress induced phenomena. Presented in Volume III in forms of general formulae to include the parameters w, σ_i, w_i $(i=1,2,3)$ related to each of the isotropic, one-axial and triaxial anisotropic components, surface and curve integrals of w, σ_i, w_i over a corresponding surface and along a corresponding curve in the isotropic, one-axial and triaxial anisotropic elastic solid continuum, respectively, result in the thermal-stress induced phenomena including

1. elastic energy fluctuations to represent energy barriers,
2. thermal-stress induced strengthening or weakening,
3. crack formation conditions along with critical values of the parameters R_1, R_2, v of composite materials,
4. analytical models of thermal-stress induced cracks to form in brittle composite materials.

Along with a transformation of the thermal stresses to stresses originating in a crystalline lattice as a consequence of the presence of a central substitutive atom presented in Chapter 11 of Volume I for cubic crystalline lattices and in Volume III, the trilogy *Analytical models of thermal stresses in composite materials I, II, III* represents **an integrated scientific work** with regard to **elasticity** of the multi-particle-(envelope)-matrix system, to the **spherical** particle and the **spherical** envelope, and to the **infinite** matrix.

With regard to the selected topics of Applied Mathematics in Sections 12.4.1–12.4.5 presenting the mathematical knowledge to be required for the study of this book, Volume I as well as the trilogy, exhibiting an interdisciplinary character with respect to Items 1–4, serve several categories of readers, including senior undergraduates and PhD. students in Mechanical Engineering, Applied Mechanics, Applied Physics, Materials Engineering, along with researchers and practicing engineers working at universities, scientific institutes and in industry. Readers of the categories are recommended to study Section *1.8.2 Categories of readers* in detail[1].

[1]This work was supported by the Slovak Research and Development Agency under the contract No. COST-0022-06.

About the author

Born in 1965 in Košice, Slovak Republic, graduated from the Mechanical Engineering Faculty of the Technical University in Košice (1988) (Department of Mechanical Engineering Technology), and from the Faculty of Sciences of the P. J. Šafárik University in Košice (1993) (Department of Physics of Solids), both with distinction, awarded the prize of the Chancellor of the Technical University in Košice for excellent study results, aimed at heat and chemical treatment and magnetic properties of amorphous alloys (1994–2000), defending a PhD. thesis in Physics of Condensed Matters and Acoustics at the Institute of Experimental Physics of the Slovak Academy of Sciences in Košice (1999), since 2000 employed at the Institute of Materials Research of the Slovak Academy of Sciences in Košice, Dr. Ladislav Ceniga currently works on analytical models of thermal stresses and related thermal-stress induced phenomena in composite materials[2].

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Contents

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Chapter 1

Outline of principles

1.1 Cell model

To derive thermal stresses in a multi-particle-(envelope)-matrix system with infinite dimensions replacing the types of real composite materials with finite dimensions as presented in Section 2.1 (see Items 1-4), an infinite matrix is **imaginarily** divided into identical cells, and a shape of the cell containing a central spherical particle with the particle centre O corresponds to particles distribution, and without or with a spherical envelope on the surface of each of the spherical particles (see Fig. 2.1a,b), where the spherical particles, the spherical envelopes and the infinite matrix represent components of the multi-particle-envelope-matrix system. The thermal stresses are consequently investigated within the cell, and additionally, such imaginary dividing of the infinite matrix is required regarding the particles distribution that the cells can fulfil the infinite matrix perfectly, and accordingly rectangular-based and hexagonal-based prismatic cells are considered in this book. Corresponding mathematical techniques similar to those presented in Sections 2.2, 2.3 can be applied to such cells to fulfil the infinite matrix perfectly, exhibiting a different shape than the rectangular-based and hexagonal-based prismatic cells.

The cell dimension along the axis x_i of the Cartesian system $(Ox_1x_2x_3)$ is a function of the inter-particle distance d_i $(i=1,2,3)$ of the particle volume fraction $v \in (0, v_{max})$ and the particle radius R_1, representing, along with radii of the spherical envelope $R_1 < R_2$, material parameters of a composite system, where v_{max} depends on the particles distribution (see Section 2.2). Resulting from the matrix infinity, analytical models of the thermal stresses in a certain cell are identical with those in any cell with the same shape.

1.2 Mathematical techniques

With regard to an arbitrary point with a position determined by a system of suitable coordinates, an infinitesimal spherical cap in the point P with a position determined by the spatial polar coordinates $[r, \varphi, \nu]$ (see Fig. 3.1) represents an infinitesimal part of a solid continuum within which the state of deformation and stress is investigated, as usual in Mechanics of Solid Continuum, where $r = |OP|$. Considering elasticity and isotropy of the solid continuum, the determination of the state of deformation and stress results from the Cauchy's, Saint-Venant's and equilibrium equations, and from the Hooke's laws, representing fundamental equations of Mechanics of Solid Continuum as a basis of Applied Mechanics presented in Section 3.1.

With regard to an experience of the author, the thermal stresses in the **isotropic** multi-particle-(envelope)-matrix system consisted of **isotropic components** can be derived using such mathematical techniques to lead to differential equations for **the radial displacement** u_{1q} of the infinitesimal spherical cap in each of the isotropic components [1]. Accordingly, the Cauchy's equations (see Eqs. (3.1)–(3.5)), representing relationships between the strain ε_{ijq} $(i, j = 1,2,3)$ and the radial displacement u_{1q} along with the derivations $\partial u_{1q}/\partial \varphi$, $\partial u_{1q}/\partial \nu$, are substituted to the Hooke's laws (see Eqs. (3.26)–(3.30)), and the stress σ_{ijq} is consequently derived as a function of u_{1q}, $\partial u_{1q}/\partial \varphi$, $\partial u_{1q}/\partial \nu$ (see Eqs. (3.31)–(3.34)). The dependence $\sigma_{ijq} = \sigma_{ijq}\left(u_{1q}, \partial u_{1q}/\partial \varphi, \partial u_{1q}/\partial \nu\right)$ is substituted to the equilibrium equations (3.15)–(3.17), representing relationships between the radial and tangential and shear stresses, σ_{11q} and σ_{22q}, σ_{33q} and σ_{12q}, σ_{13q}, respectively. Finally, the equilibrium equations are accordingly transformed to a system of differential equations, representing a relationship between the derivations $\partial^3 u_{1q}/\partial r \partial \varphi^2$, $\partial^3 u_{1q}/\partial r \partial \nu^2$, $\partial^2 u_{1q}/\partial \varphi^2$, $\partial^2 u_{1q}/\partial \nu^2$ (see Eq. (3.35)), and a relationship between u_{1q}, $\partial u_{1q}/\partial r$, $\partial^2 u_{1q}/\partial r^2$, $\partial^2 u_{1q}/\partial \varphi^2$, $\partial^2 u_{1q}/\partial \nu^2$ (see Eq. (3.36)).

As presented in Sections *Mathematical techniques* (see Sections 5.1–5.3, 6.1, 7.1, 7.2, 8.1), different mathematical techniques applied to the differential equations (3.35), (3.36) lead to different linear differential equations with non-zero right sides (see Eqs. (5.5), (5.13), (5.18), (6.3), (7.2), (7.11), (8.2), (8.3)) exhibiting solutions suitable for the determination of the thermal stresses in the spherical particle (see Sections 5.1–5.3), the spherical envelope (see Sections 5.1–5.3, 6.1, 7.1, 7.2, 8.1) and the cell matrix (see Sections 5.1–5.3, 6.1, 7.1, 7.2), considering the boundary conditions defined in Section 4.1.

With regard to the multi-particle-envelope-matrix system provided that $\beta_p \neq \beta_e = \beta_m$ and $\varepsilon_{11te} - \varepsilon_{11tp} \neq f(\varphi, \nu)$ (Eqs. (3.48)–(3.50), (12.23), (12.24)), Equation (3.35) is not considered, and Equation (3.36) is transformed to the linear differential equations (9.2), (9.6), (9.10), (9.13), (9.17), (9.21), (9.24), (9.28), (9.32), (9.36) with zero right side exhibiting solutions suitable for the determination of the thermal stresses in the spherical particle and envelope (see Section 9.2), where the coefficient β_q includes the thermal expansion coefficient α_q and the phase-transformation induced radial strain ε_{11tq}, the latter dependent or independent on the angles φ, ν of a system of the spatial polar coordinates $[r, \varphi, \nu]$. Additionally, the thermal stresses can be determined from the linear differential equations (9.126), (9.130), (9.134), (9.137), (9.141), (9.144), (9.147), (9.151), (9.155), (9.159), (9.162), (9.166), (9.174), (9.179) for the radial stress σ_{11q} with zero right sides, where

[1] With regard to a component consisted of anisotropic crystalline lattices with mutually different orientation of the crystalline lattice axes, the poly-crystalline component is considered to be isotropic [1, p 5–15].

the initial linear differential equation (9.126) is derived by the substitution of the Saint-Venant's equation (3.6) and the Hooke's laws (see Eqs. (3.18)–(3.20)) to the equilibrium equation (3.15) (see Section 9.6.1). Finally, to determine the thermal stresses in the multi-particle-envelope-matrix system provided that $\beta_p \neq \beta_e = \beta_m$ for $\varepsilon_{11te} - \varepsilon_{11tp} \neq f(\varphi, \nu)$, solutions for the spherical particle, for the spherical envelope and for the cell matrix are related to one of the linear differential equations with zero right side (see Eqs. (9.2), (9.6), (9.10), (9.13), (9.17), (9.21), (9.24), (9.28), (9.32), (9.36)) and to one of the different linear differential equations with non-zero right sides (see Eqs. (5.5), (5.13), (5.18), (6.3), (7.2), (7.11)), respectively.

The state of deformation and stress in one of the components is determined from a solution related to one of the different linear differential equations. Accordingly, solutions related to one or two, and to one, two or three different linear differential equations are considered for the multi-particle-matrix and multi-particle-envelope-matrix systems, respectively. With regard to the Castigliano's theorem (see Section 3.1.9), such combination of solutions resulting from the different linear differential equations is considered to result in minimal thermal-stress induced elastic energy of the cell, $W_c = W_p + W_m$ and $W_c = W_p + W_e + W_m$ (see Eqs. (3.44), (3.45)), related to the multi-particle-matrix and multi-particle-envelope-matrix systems, respectively, where W_p, W_e and W_m, as corresponding quantities, represent thermal-stress induced elastic energy of the spherical particle, the spherical envelope and the cell matrix (see Eqs.(3.42)–(3.45)), respectively.

Additionally, with regard to completeness of the presented topic and to the analysis in Section 2.2.2, provided that $\varepsilon_{11tm} - \varepsilon_{11tp} = f(\varphi, \nu)$, $\varepsilon_{11te} - \varepsilon_{11tp} = f(\varphi, \nu)$, $\varepsilon_{11tm} - \varepsilon_{11te} = f(\varphi, \nu)$ and $\varepsilon_{11tm} - \varepsilon_{11tp} \neq f(\varphi, \nu)$, $\varepsilon_{11te} - \varepsilon_{11tp} \neq f(\varphi, \nu)$, $\varepsilon_{11tm} - \varepsilon_{11te} \neq f(\varphi, \nu)$, Sections 5.7, 6.5 and 9.5 present analytical models of the thermal stresses in an **isotropic** one-particle-(envelope)-matrix system containing, in contrast to the multi-particle-(envelope)-matrix, one spherical particle only, respectively, considering the boundary conditions defined in Section 4.2.

Representing corresponding quantities, the thermal stress σ_{iq} along the axis x_i ($i = 1,2,3$; $q = p,e,m$) inducing the elastic energy density w_{iq}, and the thermal-stress induced elastic energy density w_q for the multi- and one-particle-(envelope)-matrix systems (see Section 3.1.8) are used for the determination of related phenomena (see Items $1, \ldots, 4$ in *Preface*), presented along with the thermal stresses in anisotropic multi-particle-(envelope)-matrix systems in Volume III.

Finally, with regard to the transformations presented in Chapter 11, formulae for the thermal stresses in the multi-particle-matrix system for the particles distribution with the inter-particle distance $d_i = d$ ($i = 1,2,3$) (see Fig. 2.1a) are transformable to stresses originating in a crystalline lattice as a consequence of presence of a central substitutive atom.

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