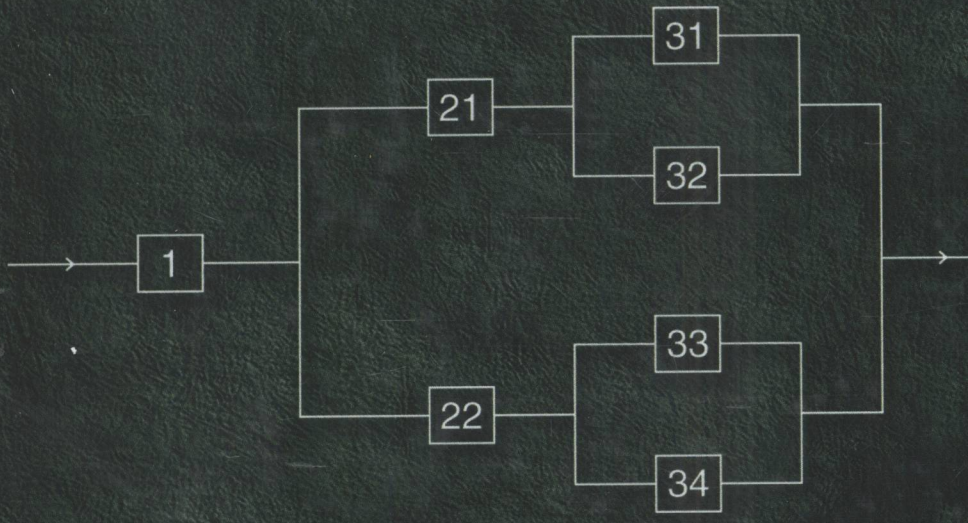
MATHEMATICAL MODELS FOR SYSTEMS RELIABILITY



Benjamin Epstein
Ishay Weissman

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

TB114.3
E64

MATHEMATICAL MODELS FOR SYSTEMS RELIABILITY

Benjamin Epstein
Ishay Weissman

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

A CHAPMAN & HALL BOOK

Chapman & Hall/CRC
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

© 2008 by Taylor & Francis Group, LLC
Chapman & Hall/CRC is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works
Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number-13: 978-1-4200-8082-7 (Hardcover)

This book contains information obtained from authentic and highly regarded sources Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The Authors and Publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access www.copyright.com (http://www.copyright.com/) or contact the Copyright Clearance Center, Inc. (CCC) 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

Library of Congress Cataloging-in-Publication Data

Epstein, Benjamin, 1918-
Mathematical models for systems reliability / Benjamin Epstein and Ishay Weissman.
p. cm.
"A CRC title."
Includes bibliographical references and index.
ISBN 978-1-4200-8082-7 (alk. paper)
1. Reliability (Engineering)--Mathematics. 2. System failures (Engineering)--Mathematical models. I. Weissman, Ishay, 1940- II. Title.

TA169.E67 2008
620'.00452015118--dc22 2008010874

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

MATHEMATICAL MODELS FOR SYSTEMS RELIABILITY

Dedicated to

my wife **Edna**

and my children **Shoham, Tsachy** and **Rom**

Ishay Weissman

此为试读,需要完整PDF请访问: www.ertongbook.com

Preface

This book has evolved from the lectures of Professor Benjamin (Ben) Epstein (1918–2004) at the Technion—Israel Institute of Technology. Throughout his tenure at the Technion, from 1968 until his retirement in 1986, he designed and taught two courses on Reliability Theory. One, which he considered to be fundamental in reliability considerations, was *Mathematical Models for Systems Reliability*. The second course was *Statistical Methods in Reliability*. As these titles indicate, although there was some overlapping, the first course concentrated on the mathematical probabilistic models while the second course concentrated on statistical data analysis and inference applied to systems reliability.

Epstein was one of the pioneers in developing the theory of reliability. He was the first to advocate the use of the exponential distribution in life-testing and developed the relevant statistical methodology. Later on, when Sobel Milton joined Epstein's Math Department in Wayne State, they published their joint work in a sequence of papers. Here is what Barlow and Proschan say in their now classical book [3]:

> In 1951 Epstein and Sobel began work in the field of life-testing which was to result in a long stream of important and extremely influential papers. This work marked the beginning of the widespread assumption of the exponential distribution in life-testing research.

Epstein's contributions were officially recognized in 1974, when the American Society for Quality decorated him with the prestigious Shewhart Medal.

Epstein's lecture notes for *Mathematical Models for Systems Reliability* have never been published. However, in 1969 they were typed, duplicated and sold to Technion students by the Technion Student Association. Soon enough, they were out of print, but luckily, five copies remained in the library, so students could still use (or copy) them. After Epstein's retirement and over the last two decades, I taught the course, using Epstein's notes. During the years, I added some more topics, examples and problems, gave alternative proofs to some results, but the general framework remained Epstein's. In view of the fact that the *Statistical Methods in Reliability* course was no longer offered, I added a

brief introduction to Statistical Estimation Theory, so that the students could relate to estimation aspects in reliability problems (mainly in Chapters 1–3 and 6).

It is my conviction, that the material presented in this book provides a rigorous treatment of the required probability background for understanding reliability theory. There are many contemporary texts available in the market, which emphasize other aspects of reliability, as statistical methods, life-testing, engineering, reliability of electronic devices, mechanical devices, software reliability, etc. The interested reader is advised to Google the proper keywords to find the relevant literature.

The book can serve as a text for a one-semester course. It is assumed that the readers of the book have taken courses in Calculus, Linear Algebra and Probability Theory. Knowledge of Statistical Estimation, Differential Equations and Laplace Transform Methods are advantageous, though not necessary, since the basic facts needed are included in the book.

The Poisson process and its associated probability laws are important in reliability considerations and so it is only natural that Chapter 1 is devoted to this topic. In Chapter 2, a number of stochastic models are considered as a framework for discussing life length distributions. The fundamental concept of the hazard or force of mortality function is also introduced in this chapter. Formal rules for computing the reliability of non-repairable systems possessing commonly occurring structures are given in Chapter 3. In Chapter 4 we discuss the stochastic behavior over time of one-unit repairable systems and such measures of system effectiveness as point-availability, interval and long-run availability and interval reliability are introduced. The considerations of Chapter 4 are extended to two-unit repairable systems in Chapter 5. In Chapter 6 we introduce the general continuous-time Markov chains, pure birth and death processes and apply the results to n-unit repairable systems. We introduce the transitions and rates diagrams and present several methods for computing the transition probabilities matrix, including the use of computer software. First passage time problems are considered in Chapter 7 in the context of systems reliability. In Chapters 8 and 9 we show how techniques involving the use of embedded Markov chains, semi-Markov processes, renewal processes, points of regeneration and integral equations can be applied to a variety of reliability problems, including preventive maintenance.

I am extremely grateful to Malka Epstein for her constant encouragement and unflagging faith in this project. I would also like to thank the Technion at large, and the Faculty of Industrial Engineering and Management in particular, for having provided such a supportive and intellectually stimulating environment over the last three decades. Lillian Bluestein did a a superb job in typing the book, remaining constantly cheerful throughout.

Finally, and most importantly, I want to thank my wife and children for their continual love and support, without which I doubt that I would have found the strength to complete the book.

Ishay Weissman
Haifa, Israel
April 2008

Contents

此为试读,需要完整PDF请访问: www.ertongbook.com

CHAPTER 1

Preliminaries

1.1 The Poisson process and distribution

The Poisson process and its generalizations play a fundamental role in the kinds of reliability models that we shall be considering. In discussing what we mean by a Poisson process, it is very helpful to think of random phenomena such as the emission of α-particles by a radioactive substance, telephone calls coming into an exchange, customers arriving for service, machine breakdowns over time, etc. To be specific, let us imagine that we start observing some radioactive material with a Geiger counter at time $t = 0$, and record each emission as it occurs (we assume an idealized counter capable of recording each emission). Of special interest is $N(t)$, the number of emissions recorded on or before time t. Any particular realization of $N(t)$ is clearly a step function with unit jumps occurring at the times when an emission occurs. Thus, if emissions occur at $0 < t_1 < t_2 < t_3 < \cdots < t_k \ldots$, the associated $N(t)$ is

$$\begin{aligned} N(t) &= 0, \qquad 0 \le t < t_1, \\ N(t) &= 1, \qquad t_1 \le t < t_2, \\ N(t) &= 2, \qquad t_2 \le t < t_3, \\ &\vdots \\ N(t) &= k, \qquad t_k \le t < t_{k+1}, \\ &\quad \text{etc.} \end{aligned}$$

Each act of observing the counter for a length of time t_0 produces a possible realization of the random function $N(t)$ in a time interval of length t_0. One such realization is shown in Figure 1.1. $N(t_0)$, the height of the random function $N(t)$ at a fixed time t_0 is a discrete random variable, which can take on the values $0, 1, 2, \ldots$. It is of interest to find the probability that $N(t_0) = n$, which we denote as $P[N(t_0) = n]$ or more succinctly as $P_n(t_0)$, for $n = 0, 1, 2, \ldots$.

In order to compute $P_n(t_0)$, one must make some assumptions about the way

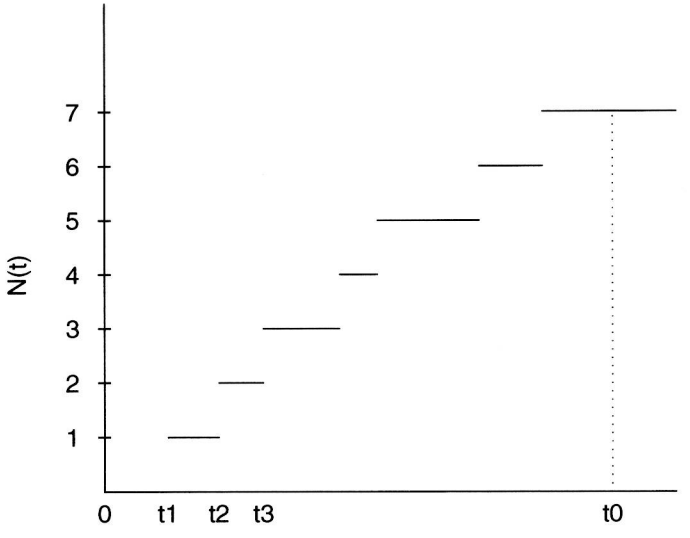


Figure 1.1 *A particular realization of* $N(t)$ $(0 \leq t \leq t_0)$*, the number of emissions up to time* t_0.

in which emissions occur over time. The simplest set of assumptions, which seems to be reasonable not only for emissions by a radioactive substance but in many other contexts, is the following:

(i) The number of emissions observed in two (or more) nonoverlapping time intervals are mutually independent random variables.

(ii) The probability of an emission in the time interval $(t, t+h]$, where $h > 0$ is small, is "approximately" λh, or more precisely $\lambda h + o(h)$. Here λ can be thought of physically as the emission rate per unit time.

(iii) The probability of two or more emissions in the time interval of length h is $o(h)$.

The term $o(h)$ in (ii) and (iii) means a quantity which goes to zero faster than h, namely, $o(h)/h \to 0$ as $h \to 0$.

A random process developing over time and meeting these three conditions (where the word *emission* would be replaced by whatever is appropriate) is said to be a *(temporally) homogeneous Poisson process*. The term *homogeneous* refers to our assuming in (ii) that λ is the same for all time. Assumptions (i) and (ii) taken together state that the probability of an emission in $(t, t+h]$ is approximately λh independent of how many emissions have occurred in $(0, t]$. Assumption (iii) states that, for small h, the probability of observing a

clustering of two or more emissions in $(t, t+h]$ is "negligible" when compared with the probability of occurrence of a single emission.

The reader is reminded that a discrete random variable X, which takes on the integer values $0, 1, 2, \ldots, k, \ldots$ with probabilities

$$P[X = k] = e^{-\alpha}\frac{\alpha^k}{k!} \quad (\alpha > 0),$$

is said to be a *Poisson random variable* with parameter α. We now prove that, for the homogeneous Poisson process, for each fixed $t > 0$, $N(t)$ is a Poisson random variable with parameter λt, i.e.,

$$P_n(t) = P[N(t) = n] = e^{-\lambda t}\frac{(\lambda t)^n}{n!}, \quad n = 0, 1, 2, \ldots. \tag{1.1}$$

To prove Equation (1.1) let us consider two adjacent intervals $(0, t]$ and $(t, t + h]$. We first show how we can relate $P_0(t + h)$ to $P_0(t)$. To do this we note that the event $\{N(t + h) = 0\}$ (no emissions in $(0, t + h]$) takes place if and only if the events $\{N(t) = 0\}$ (no emission in $(0, t]$) and $\{N(t + h) - N(t) = 0\}$ (no emission in $(t, t + h]$) both occur. Hence, it follows from the independence assumption (i) that

$$P[N(t + h) = 0] = P[N(t) = 0] \cdot P[N(t + h) - N(t) = 0]. \tag{1.2}$$

It is an immediate consequence of assumption (ii) that

$$P[N(t + h) - N(t) = 0] = 1 - \lambda h + o(h). \tag{1.3}$$

Combining Equations (1.2) and (1.3) we get

$$P_0(t + h) = P_0(t)(1 - \lambda h) + o(h), \tag{1.4}$$

or

$$\frac{P_0(t + h) - P_0(t)}{h} = -\lambda P_0(t) + \frac{o(h)}{h}. \tag{1.5}$$

Letting $h \to 0$, we are led to the differential equation

$$P_0'(t) = -\lambda P_0(t). \tag{1.6}$$

Similarly we can relate $P_n(t + h)$ to $P_n(t)$ and $P_{n-1}(t)$ for $n \geq 1$ and by going to the limit as $h \to 0$ to obtain a differential equation expressing $P_n'(t)$ in terms of $P_n(t)$ and $P_{n-1}(t)$. For the case where $n = 1$ $(n \geq 2)$, there are two (three) mutually exclusive ways of observing the event, $\{N(t + h) = n\}$ (exactly n emissions occur in $(0, t + h]$). These are:

(a) $\{N(t) = n\}$ and $\{N(t + h) - N(t) = 0\}$ occur
or

(b) $\{N(t) = n - 1\}$ and $\{N(t + h) - N(t) = 1\}$ occur
or

(c) $\{N(t) = n - k\}$ and $\{N(t+h) - N(t) = k\}$ occur, $2 \le k \le n$.
(If $n = 1$, only (a) or (b) can occur.)

It follows from the assumptions for a homogeneous Poisson process that the probabilities associated with (a), (b), and (c), respectively, are:

$$P_n(t)[1 - \lambda h + o(h)], \qquad P_{n-1}(t)[\lambda h + o(h)], \qquad o(h). \tag{1.7}$$

Hence, using the theorem of total probability, and combining terms of $o(h)$, we get

$$P_n(t+h) = P_n(t)[1 - \lambda h] + P_{n-1}(t)\lambda h + o(h) \tag{1.8}$$

or

$$\frac{P_n(t+h) - P_n(t)}{h} = -\lambda P_n(t) + \lambda P_{n-1}(t) + \frac{o(h)}{h}. \tag{1.9}$$

Letting $h \to 0$, Equation (1.9) becomes

$$P_n'(t) = -\lambda P_n(t) + \lambda P_{n-1}(t), \qquad n \ge 1. \tag{1.10}$$

We thus get the system of differential Equations (1.6) and (1.10) satisfied by $P_n(t)$. We also have the initial conditions $P_0(0) = 1$ and $P_n(0) = 0$, $n \ge 1$ (i.e., we assume that $N(t) = 0$ at $t = 0$).

The solution to Equation (1.6) subject to the initial condition $P_0(0) = 1$ is well known (can be easily verified by differentiation) and is given by

$$P_0(t) = e^{-\lambda t}. \tag{1.11}$$

Once we know $P_0(t)$, Equation (1.10), for $n = 1$, is equivalent to

$$\{e^{\lambda t} P_1(t)\}' = \lambda.$$

Integrating both sides gives

$$P_1(t) = e^{-\lambda t}(\lambda t + c)$$

for some constant c. Since $P_1(0) = 0$, we must have $c = 0$.

Proceeding inductively, we assume that

$$P_k(t) = e^{-\lambda t}\frac{(\lambda t)^k}{k!} \tag{1.12}$$

holds for $0 \le k \le n$ and we shall prove that it holds for $k = n + 1$. Under the induction assumption, Equation (1.10) is equivalent to

$$e^{\lambda t}(P_{n+1}'(t) + \lambda P_{n+1}(t)) = \{e^{\lambda t} P_{n+1}(t)\}' = \frac{\lambda(\lambda t)^n}{n!}. \tag{1.13}$$

Again, integrating both sides of Equation (1.13) gives

$$e^{\lambda t} P_{n+1}(t) = \frac{\lambda^{n+1}}{n!}\left(\frac{t^{n+1}}{n+1} + c\right),$$

此为试读,需要完整PDF请访问: www.ertongbook.com