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Stanislav V. Emelyanov and Sergey K. Korovin

# Control of Complex and Uncertain Systems

**New Types of Feedback** 



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With 336 Figures







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### **Preface**

In this monograph we consider one of the main problems of the automatic control theory, namely, the stabilization problem and the methods of its solution in their evolution, i.e., beginning with the simplest statement of this problem and gradually making it more complicated, we analyze in detail the possibilities of different methods of its solution. The complication begins with an increase in the uncertainty factors in the statement of the problem, and the methods of solution become more complicated correspondingly. This approach makes it possible to consider the general trends in the development of the principles and methods of the automatic control theory. The last fact is obviously very important and urgent since, in the new situation, the mastering of the general mechanisms of formation of control is more important than the knowledge of specific methods of its synthesis.

It should be pointed out that we do not consider the proposed point of view concerning the development of the automatic control theory to be the only possible approach since the problem of the mechanism of generation of feedback is far from being trivial and different approaches to this mechanism are possible. The larger the number of these approaches the better since they bring us closer to the understanding of the fundamental mechanisms of functioning of feedback. This is very significant both theoretically and practically since the modern methods of stabilization are oriented, in the main, to an "intensive" solution of the problem whereas nature demonstrates remarkable examples of solving stabilization problems with the use of very limited means and in rather strained circumstances. This essential difference testifies that a genuine feedback theory has not yet been worked out, that many things are not yet clear, and that the principal discoveries in this sphere are yet to come.

Investigating this complicated and delicate problem, we are far from laying claim to grasping the crux of the matter, but we are sure that the theory that we propose directly concerns the matter and seems to be quite natural and convincing.

Some words are due about the structure of the monograph. As was already pointed out, we try to go from simple things to more complicated ones and begin, naturally, from linear objects and the methods of the theory of linear control systems. Since we place special emphasis on the principles of problem solving

and on the conceptual interpretation of the results, we avoided mathematically strict statements and proofs. It stands to reason that all facts and statements presented in the book can be strictly substantiated and many of them are well known from literature.

In the monograph we compare the applications of different methods of control for solving stabilization problems under different conditions, namely, external forces, parameters, the structure and order of an object. For this purpose, sample models of objects are, obviously, especially useful, and therefore only models of this kind are considered in this monograph. However, when carrying out the analysis, we use different forms of description of control objects, namely, structural, operator, and differential forms, since some facts seem to be more convincing in a certain description and other facts are more convincing in a different description.

It can be seen from the book that the nonlinearity becomes more and more important as the stabilization problem becomes more complicated. In addition, it becomes clear that there cannot be good stabilization without nonlinear feedback and it is precisely the nonlinear feedback that makes a control system capable of demonstrating the needed behavior in complicated and constantly varying external and internal conditions.

It turns out that beginning with a certain level of complexity of the problem, a "good" control is necessarily nonlinear. It is known that in the nonlinear world there are no regular ways or universal methods typical of local theories since the specific features of nonlinearity impose certain constraints. For the theory developed in this monograph the structural methods of analysis and synthesis of systems turn out to be very useful, and therefore we pay so much attention to the description of these methods.

The purposeful use of nonlinearities in control makes it possible to operate with principally new "nonintensive" or "compensational" mechanisms of suppressing uncertainty factors, in particular, the techniques which are based on the use of positive feedback and unstable motions and which allow the system to gather momentum by itself and work until conditions are created for suppressing the disturbances and uncertainty factors. It is precisely the positive feedback and the instability that play a key role in some problems.

It should finally be pointed out that the stabilization problem should not be considered in the restricted sense since many important problems of control theory can be reduced to the stabilization problem, say, the problems of differentiation and optimization. However, since problems of this class are important and rich in content, we devote special sections to their analysis.

The authors express their deepest gratitude to many people who made possible the appearance and development of the binary control theory: to some people for their benevolent reaction and mild criticism when we first appeared in public with our reports, to other people for their selfless and creative work on the topical problems of the theory, to our opponents for their severe, may be not always justified but, in the end, useful criticism.

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### Introduction

It is believed now that the elaboration of the basic themes of automatic control theory is almost completed and, correspondingly, the center of investigation is now in the domain of applications, working out efficient methods of analysis, and designing control systems. It is also believed that the appearance of new ideas and principles is possible only upon a transition to objects of a new nature.

It should be pointed out that there are, indeed, certain reasons for this view-point, and these reasons are rather weighty. The automatic control theory has made an impressive progress, and today it can propose a wide spectrum of solution methods for various problems of applied automatics. The field of practical applications of the automatic control theory is very wide, and one cannot think of the contemporary technology without means of automation and, hence, without the use of recommendations of control theory. This is one aspect. Another aspect is that the most modern mathematical apparatus is used more and more actively in control theory whereas the books and articles in scientific journals concerning control theory are, in the main, of generalizing, summing up character. It may seem that there is a clear evidence that control theory is close to perfection and completes its development.

Is this really so and are there ready solutions in control theory for every specific case? In certain situations this is really the fact, but most often control theory does not give recipes but only gives recommendations which must be subjected to experimental verification as to the adequacy to the situation under consideration. Therefore it is no accident that there exists a generally accepted sequence of stages in the development of automatic control systems, namely, the elaboration of a mathematical model of an object, the investigation and identification of the model, the formulation of requirements to the properties of the system, the choice of the law of control and performance of immitation experiment, the technological realization of the system and the conduction of a natural or seminatural experiment, and the adjustment of the system. In this sequence, the whole chain constituting the elaboration of a system or some of its links may be used repeatedly. If we also take into account that the realization of each stage requires certain creative effort, it becomes clear that the creation and exploitation of control system is a complicated process which requires the enlisting of services of highly qualified experts. This requirement obviously contradicts the mass character of automation.

Consequently, the elaboration, designing, and keeping a control system in working order is a "bottleneck" which retards the progress in the technology of

2 INTRODUCTION

automation. This is a challenge to control theory which must produce methods and tools that would make it possible to work out and exploit control systems with the use of small effort and without resort to highly qualified specialists.

To a certain degree, this problem can be solved with the use of systems of computer aid design (CAD), but not only of these systems. The CAD-system is a tool which is efficient only when it is well "equipped" theoretically. Otherwise the CAD-system can cope with the routine phase of the development but will not promote the solution of the creative problems of automation, and, strictly speaking, it is precisely these problems that require high qualification. Only a developed theory that not only gives strict recommendations for a certain class of situations but also suggests rules for reasonable actions in nonstandard situations and the techniques of obtaining an adequate solution for every specific case can become a foundation which will allow a further qualitative progress in automatic control. The CAD-system with elements of "intellect" in the theoretical basis is what we need today.

Does the modern control theory satisfy this requirement (the requirement of high "intellect")? We must state that it does not. There are many reasons for this.

The designer who works out a control system has to resolve an objective contradiction between the elaboration of an object in detail and the possibility of a further analytic investigation of the system, namely, the identification of its parameters and the synthesis of the controller. Very likely, this is the most difficult stage whose formalization is hardly possible. Although a certain real process lies, as a rule, at the basis of elaboration of a system, the researchers who work in automatics strive to design not an exact model but only an immitating model of the process which reflects "the most important properties" with respect to the preassigned input and output variables. This is the main thing that distinguishes the models of control theory from the models that are exploited in such fundamental disciplines as physics, chemistry, etc. And it should be pointed out that the concept of "the most important properties" often has an intuitive sense which poorly gives in to formalization. It is, perhaps, the reason why, when designing a control system, we have to return iteratively to this stage and make necessary corrections.

Because of this circumstance, the elaboration of "as simple models as possible" seemed to be the most natural way. This way led to the formation of a collection of standard models which are, in the main, exploited in control theory. At present, this arsenal is very poor and is based on linear models or models close to them. In this way, often to the detriment of the real circumstances, but in order to oblige the theory, a bank of simplified models has been formed with which we deal, in the main, in control theory and which is, in essence, one of the obstacles against which control theory "stumbles" in practice.

Thus, the priority in the development of the theory was given to analytics, and this, in turn, led to a hypertrophied development of analytical methods which are often similar as concerns their final results but differ in the means of their achievement and the conditions of their application, namely, in transfer functions, differential equations, input-output representations, frequency and

INTRODUCTION 3

time characteristics, etc. However, we cannot wring out much from simple models even when we have a powerful apparatus. This especially concerns automatic control systems (ACS) since the attention given to the solution of problems of synthesis of controllers is inadequate. In fact, this branch of control theory remains almost virginal. There exists a rather restricted collection of techniques of synthesis for a small number of standard situations.

We can say without exaggeration that today the processes of origination of controlling mechanisms are not clear. In all cases, the appearance of a new method of synthesis is due to invention rather than to theory. Therefore, in our view, the problem of finding the general principles of synthesis which would make it possible to obtain the required law of control in concrete circumstances as if automatically is very attractive. The elaboration of these general principles will predetermine, in our opinion, the development of control theory in the near future.

We can try to guess certain features of their development. It is clear, first of all, that nonlinearity must become an inalienable element of the theory. In the first place, this is the requirement of practice: constraints, nonlinearity of elements, etc. But this is not the only reason, examples from other branches of science (and control theory as well) clearly demonstrate that due account of nonlinear phenomena substantially enriches the theory many times over: the nonlinear "world" is incommensurably richer than the linear one, and precisely on this way new phenomena, principles and laws originate.

As an illustration, we can cite an example when the automatic control theory was considerably enriched due to the solution of problems on absolute stability, to the investigation of self-excited oscillation processes, and adaptive control. Examples from other branches of science, say, physics and chemistry, are even more expressive. However, this statement is almost obvious, and it is much more difficult to indicate some constructive way that would lead to the nonlinear "world". Does it exist?

In our opinion, it does, and this way lies in the direction of systematic use of the most important principle of cybernetics, namely, the principle of feedback. We have only to learn how to use it correctly in nonstandard situations. It is clear today that this principle is the basis of self-control and the development of all living things. However, only the negative feedback and, correspondingly, stable processes "work" now in full measure in automatic control theory. The "press" of linearity hinders the use of positive feedback and unstable processes. Only the transition to principally nonlinear systems will allow us to involve new effects connected with the employment of positive and alternating feedback. Our monograph serves as an illustration of this scientific paradigm.