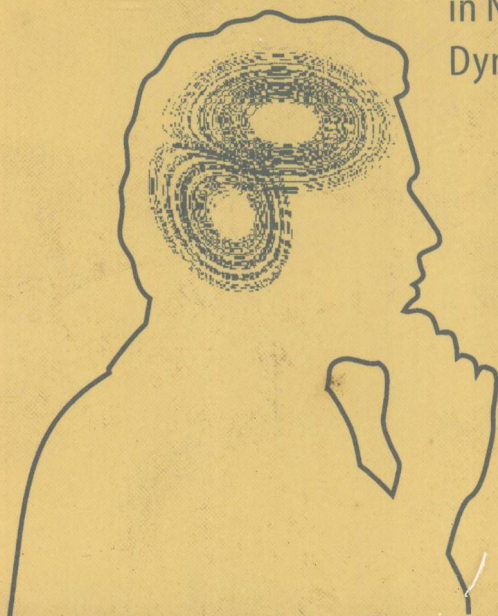


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Michail Zak Joseph P. Zbilut Ronald E. Meyers

From Instability to Intelligence

Complexity
and Predictability
in Nonlinear
Dynamics



Springer

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Michail Zak Joseph P. Zbilut

Ronald E. Meyers

From Instability to Intelligence

Complexity and Predictability
in Nonlinear Dynamics



Springer

Authors

Michail Zak
Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Drive, MS 525-3660
Pasadena, CA 91109, USA

Joseph P. Zbilut
Department of Molecular Biophysics and Physiology
Rush University, 1653 W. Congress
Chicago, IL 60612, USA

Ronald E. Meyers
US Army Research Laboratory
2800 Powder Mill Rd.
Adelphi, MD 20783, USA

Cataloging-in Publication Data applied for.

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Zak, Michail:

From instability to intelligence : complexity and predictability in nonlinear dynamics / Michail Zak ; Joseph P. Zbilut ; Ronald E. Meyers. - Berlin ; Heidelberg ; New York ; Barcelona ; Budapest ; Hong Kong ; London ; Milan ; Paris ; Santa Clara ; Singapore ; Tokyo : Springer, 1997
(Lecture notes in physics : N.s. M, Monographs ; 49)
ISBN 3-540-63055-4

ISSN 0940-7677 (Lecture Notes in Physics. New Series m: Monographs)
ISBN 3-540-63055-4 Edition Springer-Verlag Berlin Heidelberg New York

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Printed in Germany

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Typesetting: Camera-ready by authors
Cover design: *design & production* GmbH, Heidelberg
SPIN: 10550887 55/3144-543210 - Printed on acid-free paper

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Dedication

To the Memory of my Father

Fate, Time, Occasion, Chance and Change? -To this All things are
subject... -Prometheus Unbound

M. Zak

To Barbara, Anna, and Joe

Ein Blick von dir, ein Wort mehr unterhält Als alle Weisheit dieser Welt.

-Faust

J.P. Zbilut

To my wife Genevieve; children, Marc, Chris, Carole; and brothers Richard, and Sheridan

In the realm of Nature there is nothing purposeless, trivial,
or unnecessary. -Maimonides

R.E. Meyers

1

Preface

So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality. -A. Einstein

The word “instability” in day-to-day language is associated with something going wrong or being abnormal: exponential growth of cancer cells, irrational behavior of a patient, collapse of a structure, etc. This book, however, is about “good” instabilities, which lead to change, evolution, progress, creativity, and intelligence; they explain the paradox of irreversibility in thermodynamics, the phenomena of chaos and turbulence in classical mechanics, and non-deterministic (multi-choice) behavior in biological and social systems.

The concept of instability is an attribute of dynamical models that describe change in time of physical parameters, biological or social events, etc. Each dynamical model has a certain sensitivity to small changes or “errors” in initial values of its variables. These errors may grow in time, and if such growth is of an exponential rate, the behavior of the variable is defined as unstable. However, the overall effect of an unstable variable upon the dynamical system is not necessarily destructive. Indeed, there always exists such a group of variables that do not contribute to the energy of the system. In mechanics such variables are called ignorable or cyclic. Usually, an ignorable variable characterizes orientations of a vector or a tensor with respect to a certain frame of reference. An exponential growth of such a variable does not violate the boundedness of the energy, so even if the instability persists, the system still continues to function. However,

its behavior can be significantly different from the pre-instability state in the same way in which a turbulent flow is different from a laminar one. If the original system is conservative (for instance, as a set of molecules in a potential field), its post-instability behavior may attain some dissipative features: the mean, or regular component characterizing macroscopic properties will lose some portion of its initial energy to irregular fluctuations, and this will lead to irreversibility of the motion, despite the fact that the original system was fully reversible. Hence, the instability of ignorable variables can "convert" a deterministic process into a stochastic one whose mean behavior is significantly different from the original one. Based upon this paradigm one can introduce a chain of irreversible processes of increasing complexity which can be interpreted as an evolutionary process.

When dynamical models simulate biological, or social behavior, they should include the concept of "discrete events", i.e., special critical states which give rise to branching solutions, or to bifurcations. To attain this property, such systems must contain a "clock" – a dynamical device that generates a global rhythm. During the first half of the "clock's" period, a critical point is stable, and therefore, it attracts the solution; during the second half of this period, the "clock" destabilizes the critical point, and the solution escapes it in one of several possible directions. Obviously, the condition of uniqueness of the solution at the critical points must be relaxed. Thus, driven by alternating stability and instability effects, such systems perform a random walk-like behavior whose complexity can match the complexity of biological and social worlds.

Finally, based upon these paradigms, one can develop a phenomenological approach to cognition to include "quantum-like" features.

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2

Introduction

Certainly no subject or field is making more progress on so many fronts at the present moment, than biology... -R. Feynman

The beginnings of the 20th century witnessed a curious development in the history of science. The applications of statistics to the diverse phenomena of the biological and social sciences were about to explode as a result of the work of such people as Pearson and Fisher. On the other hand, the world of physical sciences was still avoiding the use of stochastic models, although Boltzmann and Gibbs had supplied sufficient reason not to do so. Now, near the end of the 20th century, the state of affairs has changed considerably. The physical sciences have come to appreciate the significant insights into low dimensional systems which appear random, while at the same time, the biological and social sciences are increasingly interested in deterministic descriptions of what appear to be very complex phenomena. The intersection of these two approaches would appear to be the often neglected, sometimes unwanted phenomenon of "noise." Often relegated to status as nuisance, noise has become more appreciated really as "that which we cannot explain." And in this explanation noise has become recognized as a possible deterministic system itself, perhaps involved with quantum effects, with a complicated description that interacts with observables on a variety of length scales. At the juncture between the physical and biological this noise creates myriad effects which ultimately redound to the very basic ideas regarding the constitution of what we know as living matter.

That this should be so is not surprising. Although it was not that long ago that scientists felt that a understanding of classical Newtonian laws of

motion could provide the key to understanding all of existence, the current climate appreciates that with some modifications, this might still be true: the movements of ions through cellular channels are being investigated by biologists with a seriousness that would be the envy of an experimental physicist. Indeed some of the very time-honored models of the physical sciences such as spin lattices are being used to explore this area. And why not? At this level the very fundamental laws of physics control discrete molecular events which have profound importance for living tissues. Ultimately, the dynamics at this level govern the way neurons, and other humoral agents orchestrate the myriad events to maintain the human organism. "Neural nets" are once again being studied as true models of the nervous system, not only by biologists, but by physicists as well.

The flurry of activity in this broad area is not unremarkable given that biological systems are often poorly defined. Until the present, most of our understanding of biological systems has been defined by phenomenological descriptions guided by statistical results. Linear models with little consideration of underlying processes have tended to inform such processes. What is more frustrating has been the failure of such models to explain transitional, and apparently aperiodic changes of observed records. The resurgence of nonlinear dynamics has provided an opportunity to explain these processes more systematically, and with a formal explanation of transitional phenomena.

Certainly, nonlinear dynamics is not a panacea. Linear descriptions do, in fact, account for many biological and social processes. Additionally, there is the danger to assume that chaotic correspondence with experimental data "explains" the system. Scientists are all too familiar with the pitfalls of model-making. Mathematics is the language of science, but the language is not the science. Physics itself is replete with examples of this tension between mathematics and reality. Consider for example the debates regarding delta functions, and "infinitesimals." It was Einstein himself who cautioned about the interface between mathematics and the physical sciences (Einstein 1983).

At the same time there is the ever present concern that by learning about the intricacies of the processes, we neglect the global kinetics of a system. Continuing evidence suggests that there is a constant interplay between microscopic and macroscopic length scales, as well as randomness to create enormous variety and patterns in biology. And perhaps this is the important point that has emerged in this last decade of the century: we have traditionally maintained a perspective of looking for order, and disregarding randomness and instability as a nuisance; whereas the correct perspective may be to see this nuisance as an active process which informs order and vice versa.

The perspective we take here is to attempt to understand biological systems in a unique way, and this unique way involves the admittance of singularities both mathematically and biologically. In this endeavor we refer

to the comments made by James Clerk Maxwell over a century ago when he pointed out (Campbell and Garnett 1884): "Every existence above a certain rank has its singular points: the higher the rank the more of them. At these points, influences whose physical magnitude is too small to be taken account of by a finite being, may produce results of the greatest importance. All great results produced by human endeavor depend on taking advantage of these singular states when they occur."

Certainly, biological organisms are of a high rank, and indeed, many of these singularities have already been uncovered. From a topological perspective Winfree (1987) has demonstrated time and again that biological oscillators admit singularities. Other work has argued from first principles and experimentation that physiological singularities must exist in order for the organisms to maintain adaptability (Zbilut et al. 1996, Zbilut et al. 1995). What has not been adequately appreciated is the reconciliation between classical Newtonian dynamics and these biological phenomena. This monograph represents a modest attempt in this direction. In order to proceed, certain problems in classical dynamics need to be highlighted.

Classical dynamics describes processes in which the future can be derived from the past, and past can be traced from future by time inversion. Because of such determinism, classical dynamics becomes fully predictable, and therefore it cannot explain the emergence of new dynamical patterns in nature, in biological, and in social systems. This major flaw in classical dynamics has attracted attention of many outstanding scientists (Gibbs, Planck, Prigogine, etc.). Recent progress in understanding the phenomenology of nonlinear dynamical systems was stressed by the discovery and intensive studies of chaos which, in addition to a fundamental theoretical impact, has become a useful tool for several applied methodologies. However, the actual theory of chaos has raised more questions than answers. Indeed, how fully deterministic dynamical equations with small uncertainties in initial conditions can produce random solutions with a stable probabilistic structure? And how this structure can be predicted? What role does chaos play in information processing performed by biological systems? Does it contribute into elements of creativity, or irrationality (or both!) in the activity of a human brain? All these questions, and many others which are related to them, will be discussed in this monograph.

The monograph treats unpredictability in nonlinear dynamics, and its applications to information processing. The main emphasis is on intrinsic stochasticity caused by the instability of governing dynamical equations. This approach is based upon a revision of the mathematical formalism of Newtonian dynamics, and, in particular, upon elimination of requirements concerning differentiability, which in some cases lead to unrealistic solutions.

This new mathematical formalism allows us to reevaluate our view on the origin of chaos and turbulence, on prediction of their probabilistic structures, and on their role in information processing in biological systems.