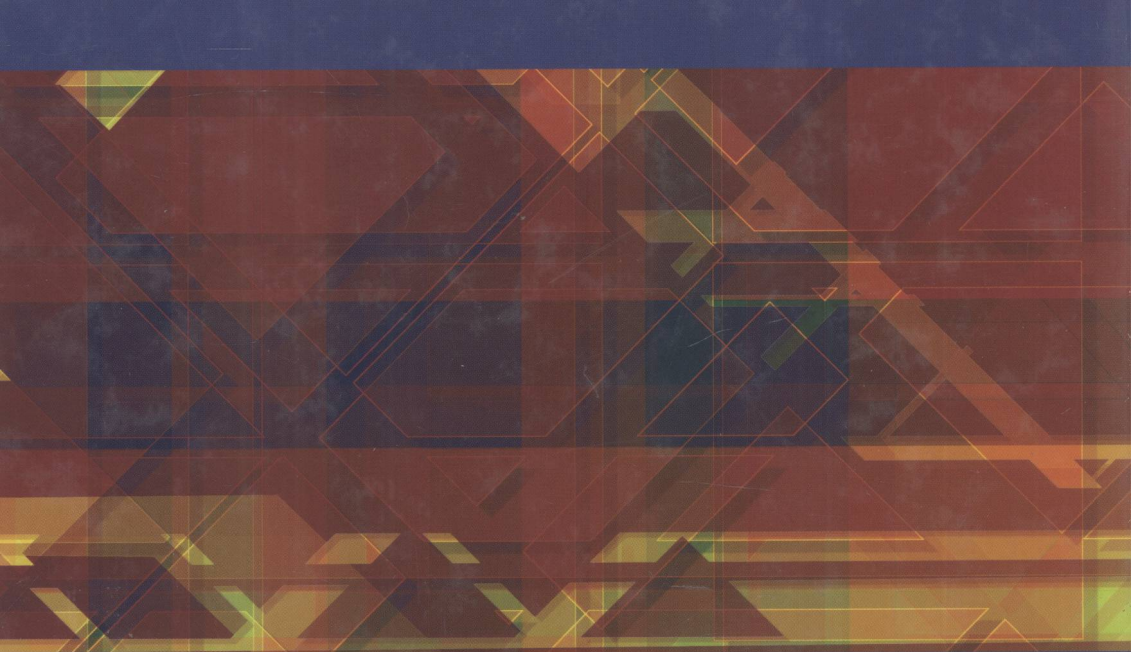




Homogenization of Coupled Phenomena in Heterogenous Media

Jean-Louis Auriault

Claude Boutin

Christian Geindreau

ISTE

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