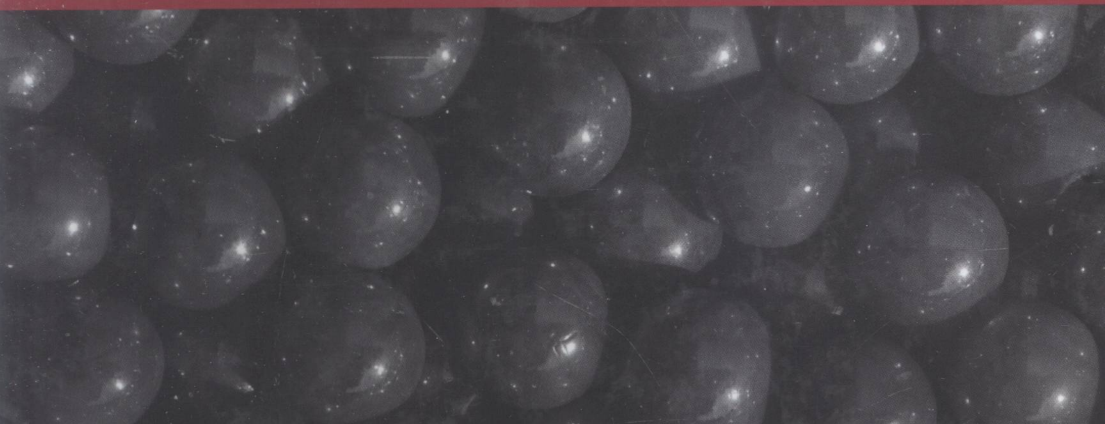
Constitutive Modeling of Soils and Rocks

Edited by

Pierre-Yves Hicher and Jian-Fu Shao



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Preface to the English Edition

The French version of this book appeared in 2002 as part of the "Material Mechanics and Engineering" series. The objective of this book was to create as complete as possible a corpus of knowledge and methods in this field.

In designing this book on the mechanical behavior of soils and rocks, we gathered together a number of internationally known specialists, who each brought a significant contribution to the knowledge of the experimental behavior of these materials, as well as their constitutive modeling. Our goal was to cover as far as possible the theories at the basis of the different approaches of modeling, and also to address the most recent advances in the field.

In translating this book into English, we hope to make available to a wider scientific and engineering public the approaches and school of thought which have dominated the field of geomaterial mechanics in France over the past few decades. We have put together present-day knowledge of mechanical behavior and their theoretical bases in order to construct an original, analytical framework which, we hope, will give readers a useful guide for their own research. Most of the chapters have been updated in order to include the most recent findings on the respective topics.

Finally, we wish to dedicate this book to the memory of Professor Jean Biarez, who not only played a ground-breaking role in the history of soil mechanics in France, but remains a source of inspiration to many of us today.

Pierre-Yves Hicher
Jian-Fu Shao

Preface to the French Edition

Soils and rocks possess a number of similar characteristics: both are highly heterogenous materials formed by natural grains. This alone gives them certain rheological features which distinguish them from other solid materials, such as a strongly non-linear character, a behavior which depends on the mean stress and shearing which induces volume variations, often dilatancy, which leads to unassociated plastic strains.

Soils and rocks can be studied at different scales. At the scale of one or several grains (from μm to cm), we can examine the discrete phenomena which govern the interactions between grains. They can be described using micro-mechanical models or analyzed in order to better understand the material behavior at a larger scale, typically the size of the material specimen: this approach corresponds to passing from a discontinuous to an equivalent continuous medium. Even though the size of the latter can vary, it has to be "sufficiently large" (typically from 1 cm to 1 dm) compared to the size of the material discontinuities in order to be representative of the equivalent continuous medium, whose behavior can be modeled by using certain concepts of continuous medium mechanics which ignore the notion of scaling in its basic equations.

However, some phenomena, such as the development of defects or cracks within the material specimen, are located at an intermediary scale, called the "meso" scale. It is thus necessary, in a constitutive model for continuous medium, to use scaling techniques in order to take into account these intermediary scales. This approach, still recent but potentially strong, can also be adapted to change the scale from the material specimen to the *in situ* soil or rock masses in geotechnical work modeling.

The constitutive models developed to describe the mechanical behaviors at the macroscopic scale can be roughly classified into two categories: those adapted to the behavior of "ductile" materials and those adapted to the behavior of "fragile" materials. The first category corresponds mainly to sandy or clayey soils, but also to soft rocks subjected to high confining stresses. The second category corresponds mainly to hard rocks, but also to certain soft rocks and highly overconsolidated clays subjected to small confining stresses. In ductile materials, the non-linear behavior is essentially due to irreversible grain displacements, which leads to a more or less significant hardening and to a pore volume change which induces volume changes at the scale of the specimen. In fragile materials, the non-linear behavior is due to the development of cracks, whose size may vary and whose direction depends on the principal stress directions.

In order to model ductile behaviors, plasticity (elastoplasticity or viscoplasticity) has shown to be an operational framework and the large majority of the constitutive models for soils and certain soft rocks belong to this category. However, for non-cohesive soils in particular, the difficulty of characterizing an elastic domain, determining the plastic mechanisms (potential and yield surface) experimentally, has led to the development of specific constitutive models, whose structure can be defined as incrementally non-linear.

In order to model fragile behaviors, the damage mechanics framework has been used to propose constitutive models adapted to describing irreversible phenomena linked to the deterioration of certain physical properties. In particular, they can take into account a large amount of rock properties: irreversible strains, dilatancy, induced anisotropy, hysteresis loop during loading-unloading due to opening and closing of mesocracks and frictional mechanisms along closed mesocracks.

In intermediary materials, the non-linear behavior can be due to microstructural changes, associating damage and hardening phenomena. Models coupling plasticity and damage have been developed to take into account this type of behavior.

After a general presentation of the constitutive models and their internal structures, each chapter will give a brief description of the different approaches mentioned above by focusing on a given class of materials. The first three chapters are devoted to the elastoplasticity theory applied to soils and soft sedimentary rocks. An alternative approach is then presented by means of the so-called incrementally non-linear models. The time-effect in clayey soils is analyzed in the framework of viscoplasticity. The behavior of hard rocks is then studied in Chapters 8 and 9, through the use of the damage theory at different scales. The modeling of the poromechanical behavior is also introduced in order to take into account the hydromechanical coupling in saturated porous rocks.

As the validity of any given model lies in its capacity to reproduce the observed material characteristics, the authors have placed the experimental data, obtained mainly from laboratory testing on intact soil and rock samples, under special consideration. The final chapter is devoted to parameter identification procedures. This is an important topic when dealing with natural materials because, each site being different from another, accurate parameter identification is essential for the quality of geotechnical work calculations, which is the final goal of this modeling approach.

Pierre-Yves Hicher
Jian-Fu Shao

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Chapter 1

The Main Classes of Constitutive Relations

1.1. Introduction

The study of the mechanical behavior of solid materials and its description by constitutive relations was for many years developed within the framework of isotropic linear elasticity characterized by Hooke's law, plasticity characterized by the Von Mises, Tresca and Mohr-Coulomb criteria, and viscosity characterized in the linear case by Newton's law. However, since the end of the 1960s, the development of more powerful numerical methods such as the finite element method and the use of high-performance computers has revived the study of material behavior, as it became possible to take into account a more realistic visco-elastoplastic modeling, albeit at the expense of much more complex formalisms.

Inside the three sets of equations defining a continuous medium mechanics problem, i.e. general equations (conservation equations), constitutive laws and boundary conditions, constitutive laws correspond to the more difficult part, particularly since the general framework in which the constitutive equations are inscribed remains often numerically imprecise. It is the comprehension of the absence of "physical laws" in this domain which gradually changed the designation of "constitutive laws" to "constitutive models". The latter corresponds better to the objective of giving a mathematical form to the mechanical properties of materials, whose complexity has been demonstrated by the diversity of the experimental results.

Chapter written by Félix DARVE.

During the last 30 years, a large variety of constitutive models have been developed and many workshops organized all over the world have shown that it is important for developers as well as users of models to be able to obtain guiding ideas and a general framework of analysis. The objective of this chapter is to try to formulate both of these.

This general framework will be more useable if it can be unified, and we intend to show that it can be applied to elastoplasticity as well to viscoplasticity or damage theory. We thus invite the reader to a wide presentation of constitutive relations for solid materials.

Two preliminary comments need to be made. First, we should explain why the chapter covers rheology in an incremental form. Two main reasons have made such an incremental presentation indispensable. The first is physical and is linked to the fact that, as soon as some plastic irreversibility is mobilized within the material, the global constitutive functional, which relates the stress state $\sigma(t)$ at a given time t to the strain state $\varepsilon(t)$ history up to this time, is in principle very difficult to formulate explicitly as this functional is singular at all stress-strain states (or more precisely non-differentiable, as will be shown). An incremental formulation enables us to avoid this fundamental difficulty. The second reason is numerical and stems from the fact that material behavior, and usually also the modeling of engineering works, exhibits many non-linearity sources which imply that the associated boundary value problem must be solved by successive steps linked to increments of loading at the boundary. Therefore, such finite element codes need to express the constitutive relations incrementally.

Our second comment concerns the use of incremental stress and strain rather than the stress and strain rates. Here also, it is the physical nature of the phenomena which determines our choice: in elastoplasticity, and more generally for all non-viscous behaviors, physical time does not play any role and, as a consequence, the derivatives with the physical time have no real meaning. Therefore, the incremental form appears to be intrinsically significant and can in fact be attached straightforwardly to the rate: the incremental strain is the product of the strain rate with the time increment, while the incremental stress is the product of Jaumann's derivative of the stress tensor with the time increment. It is, however, incorrect to speak of stress and strain increments, since the incremental strain (for example) corresponds to a small strain variation only in the case of a sufficiently small strain.

This chapter begins with a traditional presentation of the rheological functional. We will show the limits of the functional expression and overcome this limitation by establishing the incremental rheological formalism. First, we will cover the case of non-viscous materials. The notion of "tensorial zones" will allow us to present the different classes of non-viscous models. Then, we will come back to the general case by considering models which take into account any kind of irreversibility.

1.2. The rheological functional

The basic concepts of continuous medium mechanics are taken for granted. The tangent linear transformation, characterized by the matrix of the gradient of the material particle positions, is assumed to describe correctly the material geometric deformation, even if some theories, called "second gradient theories", consider that this first order approximation by the tangent linear transformation from the positions at a given time to the actual positions is not sufficient, and subsequently introduce second order terms [MUH 91]. We also assume that the constitutive law of a material element does not depend on the neighboring elements (some theories called "non-local theories" consider that the behavior of a basic material particle depends on a finite deformation field around that particle [PIJ 87]). These two hypotheses define a specific class of materials called "simple media" [TRU 74] for which we will develop a theoretical analysis.

The starting point of rheology is thus based upon a principle of determinism, which can be expressed as follows: if a given loading path is applied to a material sample, the material response is determined and unique, i.e., the principle of determinism applies only in conditions where there is uniqueness of the rheological response. Passing through a bifurcation point gives several possible responses. The choice of one of these responses is guided by existing imperfections which are not taken into account in the description of the material mechanical state or in the mode of loading application (control in force or in displacement, for example).

The first expression of the principle of determinism is obtained by stating that stress state $\sigma(t)$ at a given time t is a functional of the history of the tangent linear transformation up to this time t. This implies that it is necessary to know the entire loading path in order to deduce the associated response path.

From a mathematical point of view, this is stated by the existence of a rheological functional F:

$$\sigma(t) = F[\varepsilon(\tau)] \tag{1.1}$$

$$-\infty < \tau \leq t$$

where E(t) is the strain part of the tangent linear transformation E at time t, also called the deformation gradient. Deformation gradient E is the Jacobian matrix of position f(X,t) of material point X at time t. The existence of such a functional, and