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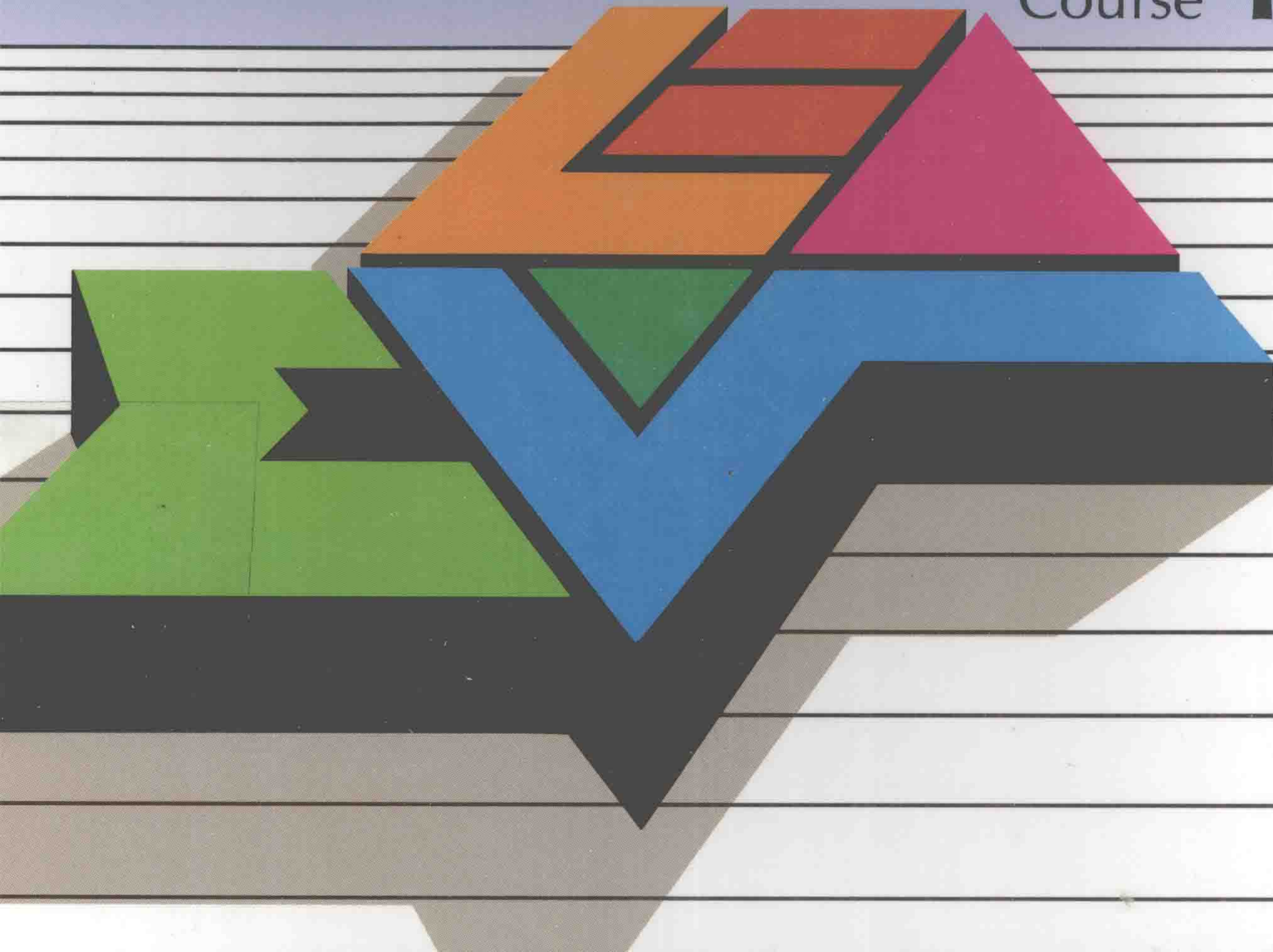
INTEGRATED MATHEMATICS

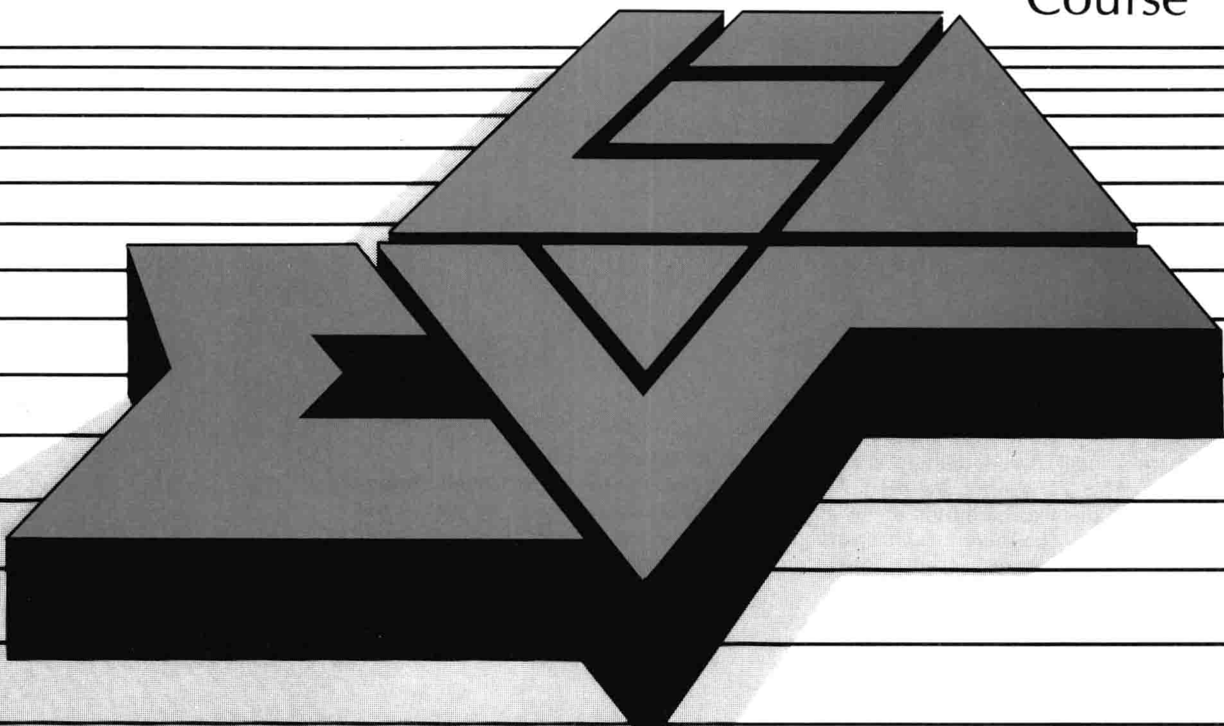
Course 1



MERRILL INTEGRATED MATHEMATICS

Klutch • Bumby • Collins • Egbers

Course 1



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The logo for *Merrill Integrated Mathematics* is emblematic of the blend of topics presented.

The equals sign (=) at the top indicates the inclusion of equations and algebra. The radical symbol ($\sqrt{}$) indicates that all numbers through the real numbers are treated. The angle symbol ($\angle$) and triangle ($\triangle$) symbolize geometry. The summation symbol (Σ) stands for probability and statistics.

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Preface

Merrill Integrated Mathematics, Course I, integrates the study of algebra and geometry. It also introduces the useful topics of logic, statistics, and probability. The goals of the text are to develop proficiency with mathematical skills, to expand understanding of mathematical concepts, to improve logical thinking, and to promote success. To achieve these goals the following strategies are used.

Build upon a Solid Foundation. The spiraled nature of the topics in *Merrill Integrated Mathematics* provides an on-going review of previously learned mathematical concepts throughout the text.

Utilize Sound Pedagogy. Concepts are introduced when they are needed and in a logical sequence. Each concept presented is then used both within that lesson and in later lessons.

Provide a Variety of Topics. A wide variety of topics increases student interest and motivation. Students are exposed to a more representative selection of mathematical ideas than would be possible in a traditional curriculum.

Facilitate Learning. A clear, concise format aids the student in understanding the mathematical concepts. Furthermore, many photographs, illustrations, graphs, and tables provide help for the student in visualizing the ideas presented.

Use Relevant Real-Life Applications. Applications provide a practical approach to mathematics, relating it to other disciplines and to everyday life.

The text offers a variety of special features to aid the student.

Student Annotations	Help students identify important concepts as they study.
Selected Answers	Allows students to check their progress as they work. These answers are provided at the back of the text.
Mixed Review	Provides students with a quick review of skills and concepts previously taught.
Chapter Review	Permits students to review each chapter by working sample problems from each section.
Chapter Test	Enables students to check their own progress.
Mathematical Excursions	Enliven and help maintain student interest by providing interesting side trips. Topics are varied and include glimpses into the development and uses of mathematics.
Problem Solving Application	Instructs students in the uses of different problem-solving techniques and strategies as tools for solving problems.

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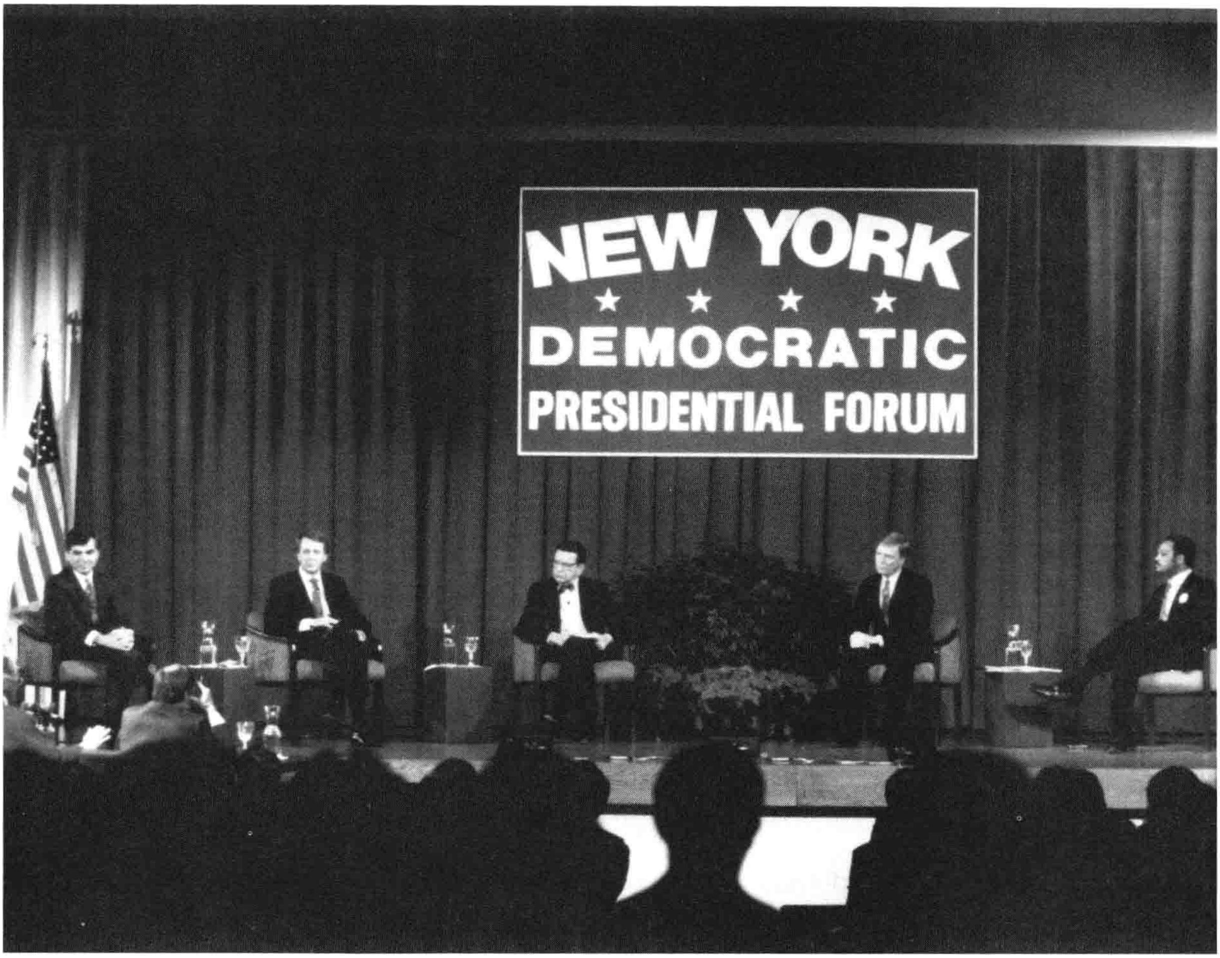
Introduction to Logic

Chapter 1

Candidates for political office often appear to have differing ideas of what is logical. They can use the same information to arrive at very different conclusions. The purpose of your work in logic is to learn some of the ground rules in reaching conclusions.



1.1 Statements and Sentences

Most discussions, or arguments, which use logic begin with a statement. Consider the following statements.

1. Lima is the capital of Delaware.
2. Lima is not the capital of Delaware.
3. The average snowflake weighs 20 pounds.
4. $30 \div 2 = 15$
5. The Twelfth Amendment to the Constitution of the United States was ratified in 1842.

Statements may be true, such as 2 and 4, or they may be false such as 1, 3, and 5.

Definition of Statement and Truth Value

A **statement** is any sentence which is true or false, but *not* both. The truth or falsity of a statement is called its **truth value.**

A statement also may be called an <u>*assertion*</u> *or* <u>*closed sentence*</u>*.*

The truth value of statements 2 and 4 is true, and the truth value of statements 1, 3, and 5 is false.

Questions are *not* statements. Consider the following.

Was President James Buchanan ever married?

The question has an answer which is true or false, but the question itself *cannot* be described as true or false.

Similarly, truth values *cannot* be assigned to a command such as "Clean your room" or an exclamation such as "Right on!" Therefore, commands and exclamations are *not* statements.

Consider the following.

He was the fifteenth President of the United States.

$$x + 3 = 12$$

You *cannot* determine the truth value of such sentences unless you know what the placeholders *he* and *x* represent. For example, if $x = 6$, the sentence $x + 3 = 12$ is false; if $x = 9$, the sentence is true.

The placeholders in mathematical sentences are called **variables.** An **open sentence** contains one or more placeholders or variables. A variable may be replaced by any member of the **domain** or **replacement set.** The set of all replacements from the domain which make an open sentence true is called the **solution set** of the open sentence.

A set of numbers is indicated by braces. {1, 2, 3} is the set of numbers 1, 2, 3.

Example

1 Find the solution set for the open sentence $8 - 5 = x$ if the domain is $\{0, 1, 2, 3\}$.

The only replacement which makes the sentence $8 - 5 = x$ true is 3. Thus, the solution set is $\{3\}$.

In this book we will use special symbols for some familiar sets of numbers.

$\mathcal{N} = \{1, 2, 3, 4, \ldots\}$ or the set of natural numbers
$\mathcal{W} = \{0, 1, 2, 3, \ldots\}$ or the set of whole numbers

Examples

2 Find the solution set for each open sentence if the domain is $\mathcal{W}$.

a. $8 - 3 = x$ Replacing x by 5 produces the true statement $8 - 3 = 5$. Therefore, the solution set is $\{5\}$.

b. $2y > 4$
For $y = 0$, is $2 \times 0 > 4$? *No*
For $y = 1$, is $2 \times 1 > 4$? *No*
For $y = 2$, is $2 \times 2 > 4$? *No*
For $y = 3$, is $2 \times 3 > 4$? *Yes*
For $y = 4$, is $2 \times 4 > 4$? *Yes*
The sentence $2y > 4$ is true when y is replaced by any number greater than 2. Thus, the solution set is $\{3, 4, 5, \ldots\}$.

3 Find the solution set for $8 - x = 3$ if the domain is the set of even natural numbers, $\{2, 4, 6, 8, \ldots\}$.

For $x = 2$, is $8 - 2 = 3$? *No* For $x = 4$, is $8 - 4 = 3$? *No*
If the pattern is continued you can see that there are *no* replacements from the domain which make the sentence true. Therefore, the solution set is the empty set.

The **empty set,** or **null set,** is symbolized by { } or Ø.

Exercises

Exploratory **State whether each of the following sentences is a statement. If it is a statement, determine its truth value. If it is *not* a statement, state why.**

1. Shut the door!
2. Mars has two moons.
3. Is today Monday?
4. $x + 4 = 10$
5. $2x = 6$ is an equation.
6. Enough of this!
7. Is 3^2 equal to 6?
8. $0.05 = 5\%$
9. Monte Carlo is the capital of Nevada.
10. He was President of the United States during the War of 1812.
11. Zero is a natural number.
12. Are all squares also rectangles?
13. There is a whole number x such that $x + 4 = 12$.
14. She is a member of the Society of Actuaries.
15. Nigeria is a country in East Africa.
16. *Mano* is the Spanish word for *man*.

Written **Find the solution set for each of the following open sentences. The domain is {California, Georgia, Idaho, Massachusetts, Nebraska, New York, Pennsylvania}.**

1. Its capital is Harrisburg.
2. It is a landlocked state.
3. It is a New England state.
4. It is located east of the Mississippi River.
5. It is a country in South America.
6. It is located in the United States.

Find the solution set for each of the following open sentences. The domain is $\mathcal{W}$.

7. $x + 2 = 14$
8. $x + 15 = 25$
9. $x - 4 = 11$
10. $h - 11 = 14$
11. $8 - y = 2$
12. $12 - y = 7$
13. $x + x = 10$
14. $\frac{n}{2} = 8$
15. $\frac{y}{4} = \frac{1}{2}$
16. $x < 10$
17. $x + x + x = 3x$
18. $2x < 8$
19. $2n = 6$
20. $3x > 1$
21. $n + n = 8$
22. $8 - y < 6$
23. $y - 5 > 1$
24. $m + 0 = m$
25. $8 + x = 15$
26. $y > 9$
27. $x + 3 = 1$
28. $3x = 12$
29. $2x + 1 = 5$
30. $x^2 > 5$

Challenge **The set of integers, $\mathcal{Z}$, is $\{\ldots, {}^-3, {}^-2, {}^-1, 0, 1, 2, 3, \ldots\}$. Find the solution set for each of the following open sentences if the domain is $\mathcal{Z}$.**

1. $x + 3 = 1$
2. $y + 5 = 6$
3. $8 + x = {}^-2$
4. ${}^-3 + y = 2$
5. ${}^-7 + x = {}^-6$
6. $2y + 1 = {}^-7$
7. $3x - 2 = 10$
8. $4 + 2x = 8$
9. $x^2 = 4$

1.2 Negations

Statements are the building blocks in a logical system. A convenient way of referring to a specific statement is to represent the statement with a letter such as p or q.

Let p represent the following statement.

Arizona was admitted to the Union in 1892.

Suppose we want to say

"It is false that Arizona was admitted to the Union in 1892"

or, more simply,

"Arizona was not admitted to the Union in 1892."

We can write *not p*. The statement represented by *not p* is called the **negation** of p.

Definition of Negation

If a statement is represented by p, then *not p* is the negation of that statement. Likewise, p is the negation of *not p*. The symbol ~ is used for *not*. Thus, $\sim p$ means *not p*.

Example

1 Let p represent "it is snowing." Let q represent "$9 - 7 = 2$." Write the statements represented by each of the following.

a. $\sim p$ — "It is false that it is snowing" or "It is not snowing."

b. $\sim q$ — "$9 - 7 \neq 2$"

c. $\sim(\sim p)$ — This is the negation of $\sim p$. Thus, $\sim(\sim p)$ is "It is false that it is not snowing" or "It is snowing."

Is $\sim(\sim p)$ always the same as p?