ONE MILLION

HENDRIK HERTZBERG

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FOR BIG TRINY
(KATRINA MCCORMICK BARNES)
AND LITTLE TRINY
(KATRINA HERTZBERG)

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You may have wondered: just how big is a million, really? Well, stop wondering. See pp. 1–200.

The plan of this book is simple. Five thousand dots to a page. Ten thousand on each double-page spread. Two hundred pages. Result: one million dots, not counting the period at the end of this sentence.

The progression is marked at the top of each page. Just for fun, notes are scattered here and there like Burma Shave signs on a two-lane highway. Each note corresponds to a number, and the dot signifying that number drops down like a prize in a gumball machine.

When notes refer to events in historical time, the scale is usually measured in days. Conveniently, most of recorded history has taken place within the past one million days. The first dot represents January 1, 2010, and from there we time-travel in reverse, through the world wars, the Industrial Revolution, the Middle Ages, the empires of Caesar and Alexander, all the way back to the misty, myth-laden times of Romulus and Remus and the Etruscans. The millionth dot drops us off on November 7, 728 BCE.

When notes refer to the present time, the figures are the latest available, or are drawn from a typical year in the last decade, whichever is appropriate.

One million is a pretty big number. But in a world fast approaching a human population of seven billion souls, a million can be a trifle. In some of the notes, therefore, each dot itself stands for a million; in others, more modestly, for a thousand.

The facts herein are, so far as I know, accurate as of 2008. They are drawn from many sources, all of which I believe to be reliable. However—and I must stress this point—*this book is not a reference work*. Unlike the plastic bags your dry cleaning comes back in, *this book is a toy*. But it is a toy with a didactic purpose, like a chemistry set or an anatomically correct doll.

Riffle slowly through these pages and you will see with some precision what is meant by one million. If you've just heard on the radio that a million people have been left homeless in a tsunami, a few moments with this book will more fully impress you with the dimensions of that disaster. If you're wondering what it would be like to have a million bucks in your checking account, look at the first few pages (representing, if you're lucky, your own hoard) and then flip thoughtfully through the rest. If you're curious about those seven billion souls, imagine (or, better, buy) seven thousand copies of this book. That would be enough to fill three walls of a fair-size room lined floor to ceiling with bookshelves.

A million is a wonderfully provocative quantity. In one direction, it points to the immensity of the Earth and its inhabitants. In the other, it exposes the pathetic puniness of our planet and its people next to the vastness of the universe.

NUMBERS AND THEIR NAMES

As a species, we human beings possess a talent for naming things that considerably exceeds our capacity to understand them. Everyone knows the names of infinity, eternity, truth, and God; few people, if any, know what they are. Naming a thing is often a substitute for knowing what it actually is—in some instances a pretty good substitute. This is certainly the case with numbers. We know the names of big numbers. We know how to work with the symbols we have made for them. We may not understand their amplitude, but we manipulate them with blithe insouciance. Ten million times ten million? A hundred trillion. Problem solved.

In the everyday English of the prescientific age, the available words for big numbers were not very...well, not very numerous: "dozen," "score," "gross," "hundred," "thousand," "myriad" (which technically means ten thousand but has always been more commonly used to mean simply a lot). That was about it. This vocabulary of quantity was serviceable enough for a time when the Earth was believed to be the center of the universe, which itself was thought to be only a few thousand years old.

"Million" is of comparatively recent coinage. The English word comes to us from the Romans, via the French and the Italians. It combines *mille*, which means "a thousand" in Latin and most of its daughter languages, with "-ion," which lexicographers call an "augmentative suffix." The editors of the *Oxford English Dictionary* found uses of the English word "hundred" as far back as the year 950 CE, but "million" doesn't make its first known written appearance until four centuries later, in a 1362 manuscript that speaks of "Milions more of Men and of Wymmen." The word pops up about a decade later, in a religious poem about "a Milioun Angeles." Chaucer wrote of angels, too—"many a Millioun" of them. In the King James Bible (1611), the brothers of Abraham's bride address her as follows: "And they blessed Rebekah, and said unto her, Thou art our sister, be thou the mother of thousands of millions, and let thy seed possess the gate of those which hate them" (Genesis 24:60). In *Hamlet*, the prince, telling the play-within-the-play actors about a theater piece that apparently went over big with the critics but was box-office poison, says, "The play, I remember, pleased not the million."

Shakespeare, Chaucer, and the Biblical translator were neither the first nor the last writers of English to use the word "million" hyperbolically. Its use in the precise numerical sense—a thousand thousand, ten to the sixth power—became common in the West only in the past couple of centuries. In the East, however, where ancient mythology, like modern paleontology, deals in ages of hundreds of millions of years, the business of number-naming made considerable progress at an early date. "The time and number sense of the ancient Indians was extraordinary," writes Jawaharlal Nehru, modern India's first prime minister, in *The Discovery of India*. "They had a long series of number names for very high numerals. In India there were eighteen specific denominations (10^{18}) and there are even longer lists. In the story of Buddha's early education he is reported to have named denominations up to 10^{50}"—that is, up to the number we would represent by a one followed by fifty zeroes.

The Greeks, Romans, and Persians apparently had no terminology for denominations above the myriad, i.e., ten thousand (10^4=10,000). This was a problem for antiquity's most accomplished mathematician, Archimedes, who tackled it in a short book (or long scroll), delightfully titled *The Sand-Reckoner*.

Addressing himself to his patron, the king of Syracuse, Archimedes begins *The Sand-Reckoner* briskly:

> Some people believe, King Gelon, that the number of [grains of] sand is infinite in multitude. I mean not only of the sand in Syracuse and the rest of Sicily, but also of the sand in the whole inhabited land as well as the uninhabited. There are some who do not suppose that it is infinite, and yet that there is no number that has been named which is so large as to exceed its multitude.

It is clear that if those who hold this opinion should conceive of a volume composed of the sand as large as would be the volume of the earth when all the seas in it and hollows of the earth were filled up in height equal to the highest mountains, they would not know, many times over, any number that can be expressed exceeding the number of it.

The great mathematician then does something astounding and audacious: he painstakingly calculates the number of grains of sand that would be required to fill up the world. As if that weren't enough, he then calmly figures out how many it would take to fill up the entire universe. To be sure, he assumed that the universe was smaller than we know it to be today. But he (and he alone, almost) knew it was plenty big. To express how big, and then to count the grains he would need to stuff it with sand, he improvised a new mathematical language, based on exponents of the myriad. Suffice it to say that a myriad raised to the myriadth power raised to the myriadth power makes a big sandbox.

The modern world obliges us to concern ourselves with the kinds of numbers that only people like Archimedes and the Buddha bandied about millennia ago. There is, of course, no such thing as "the biggest number in the world"—our number system is open-ended, and another zero can always be added. Or a one: if we let n stand for the biggest number in the world, it won't stay standing long, because $n + 1$ is bigger.

Similarly, names for very large numbers can be conjured by taking the suffix "-illion" and loading it up with Latin prefixes. This yields "words" like "quattuortrigintillion," which, it is safe to say, very few people, if any, have ever said out loud with their eyes closed. (A quattuortrigintillion, if you care, is ten to the 105th power.) But the biggest number that has a conversation-friendly name is the googolplex.

A googolplex is ten carried to the power of a googol, or the numeral 1 followed by a googol of zeroes. A googol is not to be confused with a Google—or, rather, it is to be confused with one, thanks to the most successful branding campaign in Internet history. (A Google search for "Google" yields 2.66 billion hits, whereas a Google search for "googol" yields only 362,000.)

Anyway, a googol, which stands in relation to a googolplex as a gnat to the galaxy, is itself (despite the cuteness of its name, which was coined in 1938 by a nine-year-old boy) a very large number. It is 10^{100}, pronounced "ten to the hundredth power"—ten multiplied by itself a hundred times, or, written out in the conventional manner, 10,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000. This number is, as the saying goes, big enough for all practical purposes. It is so big, in fact, that it is a million million billion times larger than the number of all the electrons in the universe (estimated at 10^{79} by Sir Arthur Eddington).

A googolplex, to repeat, is 10 to the googolth power, or 1 followed by a googol of zeroes. Thanks to the genius who hit upon the idea of writing large numbers as exponents of ten, a googolplex can be expressed quite concisely as 10^{googol} or $10^{10^{100}}$. This looks innocuous, but actually it is a number so colossally huge, so monstrously stupendous, so titanically gargantuan that it renders trivial anything in human or galactic experience. Not that the googolplex is entirely without its uses. Edward Kasner and James Newman, the authors of *Mathematics and the Imagination*, once asked themselves how long it would take a book to jump up into your hand if you held it patiently on the end of a string. This odd event would come to pass if all the air molecules on the underside of the book, in their random movements, happened to bump into it at the same instant. "The right answer," Kasner and Newman concluded, "is that it will almost certainly happen sometime in less than a googolplex of years—perhaps tomorrow." So even a googolplex can have an application, though it would be stretching a point to call it a practical application.

Naturally, there can be no question of accumulating a googolplex of objects, since that many objects do not, according to the latest findings, exist. Even with a king-size cut in the capital gains tax, no one is going to become a googolplexaire. Nor, come to think of it, would it be possible to write out a googolplex in full as we wrote out a googol above, for if every book ever printed—not only here on Earth but on all the 100 trillion planets in the universe that many astronomers believe harbor higher civilizations—were filled with zeroes from cover to cover, the resulting number, while undeniably large, would still add up to only a few million trillionths of a googolplex. (Compared to the infinite, of course, even a googolplex is infinitely small.)

We owe our ability to write big numbers on small pieces of paper to an unknown but very smart resident of India, who, at about the time of the birth of Christ, invented the zero and the place-value system. These advances simplified number-writing in the same way that the phonetic alphabet simplified word-writing. Roman numerals, with which teachers have tormented their pupils for centuries, are clumsy not because they use M's and X's instead of 5's and 6's, but because they do not know how to multiply their value by changing their position.

Our own number system happens to be based on ten, which means that each time a numeral gets nudged one place to the left, its value increases ten times. The number 9,482, for instance, is just a shorthand way of writing 9,000 plus 400 plus 80 plus 2, or to put it another way, 2 plus (8 x 10) plus (4 x 10^2) plus (9 x 10^3). The fact that this system is decimal (that is, based on ten) is a biological coincidence. If we had six fingers on each hand, we would undoubtedly do our counting by twelves. In that case, the numeral one-zero (10) would have the value of twelve, and we would have to invent new single-place symbols for the values of ten and eleven (Ω and Δ would do nicely). We would then write the numbers one through twenty-four this way: 1, 2, 3, 4, 5, 6, 7, 8, 9, Ω, Δ, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1Ω, 1Δ, 20. This system would have distinct advantages, since twelve is evenly divisible by one, two, three, four, six, and itself, while ten is divisible only by one, two, five, and itself.

There is nothing holy about ten, but all of us are so conditioned to thinking in tens that it is difficult to think about numbers in any other terms. We regard ten and its multiples (hundreds, thousands, millions) as special—as rest stops on the turnpikes of quantity. And so they are, but only because we, not nature, put them there.

Too much shifting back and forth between number systems can get vexing, like trying to switch a full-size train onto a narrow-gauge track. To avoid mental derailments, it pays to stick to just one system. But the process of switching around is really not so different from switching between the Celsius and Fahrenheit temperature scales, say, or the English and metric systems of measuring distance. The boiling point of water stays the same whether you call it 100 degrees or 212 degrees. The distance from New York to California stays the same whether you call it 3,000 (miles) or 5,000 (kilometers). The number of days in a year stays the same whether you write it as 365 or (in the twelve-based system) 265.

Another little foray into the twelve-based number system will show just how arbitrary our ten-minded system is. In the twelve-based system, the figure 1,000,000 would have the value (and here we must suddenly shift back to "regular" numbers) of 12^6, or 2,985,584. The figure 1,000,000,000 would have the value of 5,159,780,352. Conversely, "our" one million would be expressed, in twelve-based numbers, as 412,160; "our" one billion would be expressed as 23Ω,Ω93,4Ω8. If the medieval Church had wanted to avoid all the hysteria surrounding the dawning of the first millennium, it could simply have decreed that henceforth the number system would be based on twelve; then the year 1000 CE would not have rolled around until 1728, by which time the Enlightenment had things well in hand. A bit later, we wouldn't have had to worry about our computers going haywire on account of the millennium bug, a.k.a. the Y2K problem, until 3456.

Speaking of computers, they have to use a number system based on two. Their system uses only two symbols, 0 and 1, and each time a digit moves to the left, it gets multiplied by two instead of by ten. So a computer counts to ten this way: 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010. We would find this system tiresome; the numbers, especially the big ones, run on and on. In computer numbers, there are 101,101,101 days in a year, and "one million" looks like this: 11,110,100,001,000,000. But computers seem to like it that way, because they only know how to count to two anyway, the poor dears (a switch is either on or off), and because, despite the proliferating digits, these "binary" numbers actually require fewer choices. We have to make seventy choices to write the number one million—seven digits, ten possible symbols per digit. Our computers only have to make forty—twenty digits, two symbols. Of course, that doesn't mean that our computers are smarter than we are and are therefore destined to overthrow us and take over the world. I think.

BIG, YES, BUT HOW BIG?

For I have known them all already,
known them all—
Have known the evenings, mornings,
afternoons,
I have measured out my life with
coffee spoons;
I know the voices dying with a dying fall
Beneath the music from a farther room.
So how should I presume?

—T. S. Eliot, "The Love Song of J. Alfred Prufrock"

As a unit of measurement, the coffee spoon happens to lie midway between the very small and the very large. The length of a coffee spoon is about one thousand times greater than that of a grain of sand, and about one-thousandth the height of a skyscraper. It is a hundred million times longer than a sugar molecule, and a hundred million times shorter than the diameter of the Earth. It is a trillion times as long as an electron, and a trillion times smaller than the distance from the Earth to the Sun.

Technically, I realize, it is man, not his coffee spoons, who is the measure of all things. And he is happily situated to do the measuring. (As is she, of course.) He is not so big as to ignore the little things, like the bacteria living in his intestines (to whom he is the size of the Earth), the atoms of which he is made (in comparison to which he is as large as the solar system), and the protons in the nuclei of those atoms (next to which his height is roughly equal to the distance to Alpha Centauri). Yet he is not so small that his sense organs, and their optical and electronic extensions, cannot perceive something of the enormity of the cosmos. And his brain is so flexible and powerful that it can map the molecules of its own genetic code and plot the chemistry of stars in distant galaxies.

Lest we grow too impressed with ourselves, however, it is well to recall the physical insignificance of ourselves and our little planet in the larger scheme of things. If, for example, the Earth is imagined to be the size of a golf ball, the Sun would be a sphere twenty feet in diameter, and it would be nearly a half-mile away. If the solar system were the size of a golf ball, the Sun would be visible only under a microscope. The nearest star would be six hundred feet away, and the breadth of our galaxy, the Milky Way, would be some 2,600 miles, about the distance between New York City and Mexico City. If the galaxy were the size of a golf ball, the solar system would be about as big as a molecule. Other galaxies—peas, marbles, softballs, and soccer balls—would lie here and there, from a yard to several miles away. And here the analogy must stop, for the next question ("What if the whole universe were the size of a golf ball?") is, from any normal person's perspective, absurd. A Zen master, however, would not have a problem with it. Neither would a cosmologist, for if, as some research suggests, the universe is closed (meaning it has enough matter in it so that eventually it will stop expanding and will collapse back in on itself), or if our universe is merely one among a multitude in a multiverse, then from the "outside," wherever that is, the universe might actually be the size of a golf ball.

The star we are pleased to call the Sun is huge only from our provincial perspective. If a really big star, say Epsilon Aurigal B, were suddenly to take the place of the Sun, it would do more than merely engulf the nearer planets; the entire solar system would fit into a tiny sphere at its center, with a diameter less than two-tenths of 1 percent of that of the whole star. Our Sun is a little less than half again as dense as water; by comparison, the density of a neutron star can approach a theoretical limit of 1,800,000,000,000 tons per cubic inch. At that density, Earth would collapse into a little ball less than seven miles from surface to center.

Time, like space, is immense, and our perception of it tends to be distorted by the extreme youth of our species, just as our spatial perception is often distorted by our tiny size. The universe is now thought to be some fifteen billion years old, and the Earth has existed for a bit less than a third of that time. Human beings of one kind or another have been walking around for only a couple of million years, and people in their present (though not necessarily final) form for a couple of hundred thousand. Recorded history has been incredibly brief—less than a million weeks. If the dots in this book are allowed to represent the age of the Earth, then life forms with more than one cell do not emerge until around page 164, dinosaurs not till the top of page 194. People more or less like us turn up around halfway across the last line of the last page, and all recorded history is represented—overrepresented—by the last two dots. Even if we zoom in to let each dot be one year (and the first dot the first campfire kindled by our distant *Homo erectus* ancestors), the history of civilization happens on the last two pages, the Visigoths sack Rome two-thirds of the way down the last page, the history of the United States begins four lines from the bottom, the Japanese bomb Pearl Harbor at the end of the second to last line, and Barack Obama's presidency begins at the second to last dot.

The steady, ticking time of clocks and calendars is one of those convenient lies by which we make life smoother for ourselves; or to put it another way, that kind of time, like Euclidian geometry and Newtonian physics, is one of those models that is true for many places and purposes but not for reality as a whole. All of us, not just the mystics and the stoners, are familiar with the experience of time flowing more slowly or more quickly than the clock tells us; and in science, simple, unitary conceptions of time have been laid to rest by modern physics. The best known of Einstein's equations established that as matter accelerates, it gains in mass; and at speeds approaching that of light—the universal limit, at least in this universe—the gain is enormous. There is another effect: as speed increases, time begins to stretch. Indeed, at the speed of light—a speed unattainable except by light itself—time stops. The starlight you see on a clear night may have started its journey to the pupil of your eye thousands of years ago—millions, if your eye is pressed to the eyepiece of a telescope. Yet that light has not aged at all. It has not deteriorated or lost any of its detail. It is exactly as it was when the pharaohs ruled or when dinosaurs roamed the Earth.

The late Carl Sagan, a teacher and friend of mine, loved to talk about the astonishing effects of acceleration on time. A spaceship moving at a steady acceleration of one gravity (enabling the passengers to feel their normal weight, and their ship to attain a velocity of well over 99 percent of the speed of light) would, he said, reach the Andromeda Galaxy, a million light-years distant, in thirty years. On their return, the passengers would be sixty years older; but the Earth and its civilization, if either still existed, would have aged two million years in their absence. Such a voyage, in which the travelers would travel in time (one way) as well as space (round trip), is within the realm of theoretical possibility. Unfortunately, though, getting Congress to fund an experiment whose result would not be known for more than a hundred thousand generations would not be easy. *There might not be any people around to* greet the returning travelers, or, if there were some, they might not be recognizable, or they might not have a corporeal existence, or they might have decamped for more hospitable planetary climes.

Times change. For example, if we let this book represent the population of the world today—that would be some 6,800 souls per dot—then the population in prehistoric times would cover only about a third of one page. The population in the time of Caesar and Christ would cover only six pages; at the time of the Declaration of Independence, twenty-two; in 1950, just seventy-five. By a conservative estimate, 7 *percent of all the people who have ever lived are alive right now*, and it could be as much as 15 percent. Sooner or later, something's got to give.

With increases in population and technology, economic activity, too, has grown hugely. For sheer inflation, Weimar Germany, where a total of 496,585,346,000,000,000 marks found their way into circulation after the First World War, is still thought to hold the record, although in 2008, ATMs in Zimbabwe were dispensing hundred-billion-dollar bills. It remains the case, *however, that no single human conglomeration* disposes of bigger sums than the United States government—at this writing, about three and a half trillion, or 3,500 billion, dollars a year. To let this book stand for that budget, each dot must represent $3.5 million. Defense gobbles up about forty pages, health and social security around eighty-nine, interest on the national debt around sixteen, foreign aid and the space program about a page each, ditto all diplomatic and foreign-aid activities combined. The National Endowments for the Arts and the Humanities get thirty dots apiece. And if the very last dot were a pizza, the president's salary would be one slice.

We need not look to the Office of Management and Budget, or to the heavens, to find a multitude of things. Even in daily life, vastness is everywhere. An average sugar bowl contains about 400,000 grains of sugar. A hundred-mile railway trip will take the rider over 316,800 railway ties. A sheep's coat may contain as many as 126,000,000 wool fibers. A visit to the beach discloses Archimedean quantities of sand; at Coney Island, there are said to be a hundred billion billion grains. A talk on one of the world's six billion telephones will add a few dozen words to the thirty million billion or so uttered by humankind since the beginning of time. Big numbers are as much a part of our lives as our own names, which, indeed, they threaten to replace. We're all on speed dial now.

Notwithstanding the nonchalance with which such figures are dropped in all sorts of contexts, can we really understand them? Not very well, except in the most abstract, non-experiential way. The largest number of discrete objects of which most of us can form a mental picture is five or six, maybe a couple more—and even people whose minds are thought to be exceptional can't do much better. "Nine is about my limit," wrote Arthur C. Clarke, whose portraits of the future fascinated...well, millions of readers. "I can just visualize it by imagining three rows of three objects."

Our visual grasp of quantity is not, of course, limited absolutely by what we can picture with our eyes shut; through memory and association, we can extend it considerably further. Think of a starry night, or the experience of being in a big crowd. There were tens of thousands of people in the stadium (an average NFL game draws 68,661, while Barack Obama looked out on 75,000 when he accepted his party's 2008 nomination in Denver); and the stars in the sky, one might naturally suppose, were far more numerous. Not so: even on the clearest night, far away from city lights, only about 3,000 stars are visible to the naked eye.

At this level, ordinary human comprehension begins to break down. To know something intellectually and to experience it concretely are two very different things. All of us have had direct concrete experience with the smaller numbers, the ones and tens and hundreds, even the thousands. A million is harder to come by.

See pp. 1-200.

—HENDRIK HERTZBERG