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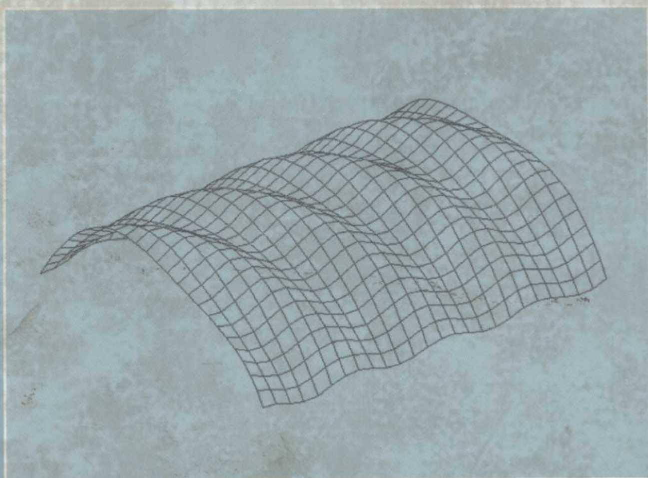

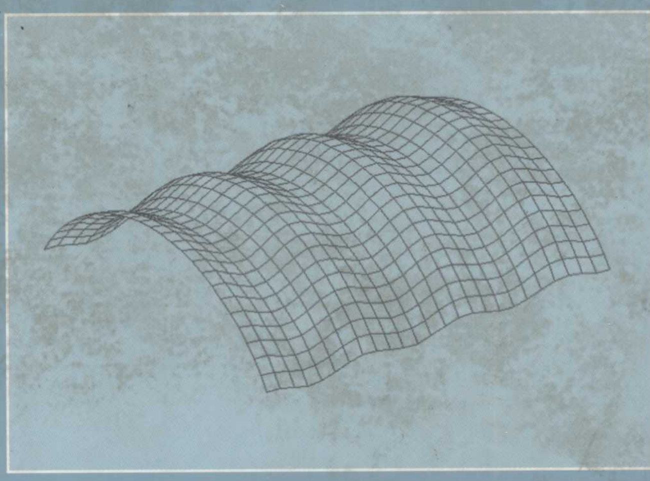

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MECHANICS OF COMPOSITE STRUCTURES

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Preface

The increased use of composites in aerospace, land, and marine applications has resulted in a growing demand for engineers versed in the design of structures made of fiber-reinforced composite materials. To satisfy this demand, and to introduce engineers to the subject of composites, numerous excellent texts have been published dealing with the mechanics of composites. These texts deal with those fundamental aspects needed by engineers new to the subject. Our book addresses topics not generally covered by existing texts but that are necessary for designing practical structures. Among the topics in this book of special interest to the designer, but that usually are not included in standard texts, are stress–strain relationships for a wide range of anisotropic materials; bending, buckling, and vibration of plates; bending, torsion, buckling, and vibration of solid as well as thin-walled beams; shells; hygrothermal stresses and strains; and finite element formulation. The material is presented in sufficient detail to enable the reader to follow the developments leading to the final results. The expressions resulting from the analyses are either readily usable or can be translated into a computer algorithm. Thus, the book should be useful to students and researchers wishing to acquire knowledge of some of the advanced concepts of the mechanics of composites as well as to engineers engaged in the design of structures made of composite materials.

The emphasis is on analyses built on fundamental concepts that are applicable to a variety of structural design problems. In presenting the material we have strived to follow the outline commonly used in texts dealing with the analysis of structures made of isotropic materials. We have consciously omitted empirical approaches. Test results are certainly of value to the engineer. However, for composites, these mostly apply only under specific circumstances and cannot readily be generalized to different materials and different applications. We have included material properties data to help the designer perform calculations without the need to search the literature.

The book is self-contained. Nevertheless, the reader will find it helpful to have a background in mechanics and in composites and some knowledge of differential equations and matrix algebra. We have made an effort to keep the notation as uniform as practicable and reasonably consistent with accepted usage. The principal symbols are summarized in a list of symbols.

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List of Symbols

We have used, wherever possible, notation standard in elasticity, structural analysis, and composite materials. We tried to avoid duplication, although there is some repetition of those symbols that are used only locally. In the following list we have not included those symbols that pertain only to the local discussion. Below, we give a verbal description of each symbol and, when appropriate, the number of the equation in which the symbol is first used.

Latin letters

A	area
A^{iso}	tensile stiffness of an isotropic laminate (Eq. 3.42)
$[A]$, A_{ij}	tensile stiffness of a laminate (Eqs. 3.18, 3.19)
$[a]$, a_{ij}	inverse of the $[A]$ matrix for symmetric laminates (Eq. 3.29)
$[B]$, B_{ij}	stiffness of a laminate (Eqs. 3.18, 3.19)
$[C]$, C_{ij}	3D stiffness matrix in the x_1, x_2, x_3 coordinate system (Eq. 2.22)
$[\overline{C}]$, $\overline{C}_{ij}$	3D stiffness matrix in the x, y, z coordinate system (Eq. 2.19)
c	moisture concentration (Eq. 2.154); core thickness (Fig. 5.2)
$[D]$, D_{ij}	bending stiffness of a laminate (Eqs. 3.18, 3.19)
$[D]^*$, D^*_{ij}	reduced bending stiffness of a laminate (Eq. 4.1)
D^{iso}	bending stiffness of an isotropic laminate (Eq. 3.42)
D, $\overline{D}$, $\widehat{D}$	parameters (Table 6.2, page 222, Eq. 6.157)
$[d]$, d_{ij}	inverse of the $[D]$ matrix for symmetrical laminates (Eq. 3.30)
d, d^{t}, d^{b}	distances for sandwich plates (Fig. 5.2)
E_1, E_2, E_3	Young's moduli in the x_1, x_2, x_3 coordinate system (Table 2.5)
$[E]$	stiffness matrix in the FE calculation (Eq. 9.4)
$\widehat{EA}$	tensile stiffness of a beam (Eq. 6.8)
$\widehat{EI}$	bending stiffness of a beam (Eq. 6.8)
$\widehat{EI}_\omega$	warping stiffness of a beam (Eq. 6.244)
F_i, F_{ij}	strength parameters in the quadratic failure criterion (Eq. 10.2)

f_{ij}	constants in the quadratic failure criterion (Eq. 10.25)
f, f_{ij}	frequency (Eq. 4.190)
f_x, f_y, f_z,	body forces per unit volume (Eq. 2.13)
G_{23}, G_{13}, G_{12}	shear moduli in the x_1, x_2, x_3 coordinate system (Table 2.5)
$\widehat{GI}_t$	torsional stiffness of a beam (Eq. 6.8)
h	plate thickness
h_b, h_t	distances of the bottom and top surfaces of a plate from the reference plane (Eq. 3.9)
i_ω	polar radius of gyration (Eq. 6.340)
$[J]$	inverse of the material stiffness matrix $[E]$ (Eq. 9.16)
K	number of layers in a laminate; number of wall segments; stiffness parameter of a plate (Eq. 4.153)
$\widetilde{k}$	rotational spring constant (Eq. 4.149)
k	equivalent length factor (Eq. 6.340)
L_x, L_y	dimensions of a plate
L	length; number of cells in a multicell beam (Eq. 6.222)
L_i, L_i^f	load and failure load (Eq. 10.42)
l_x, l_x^o	half buckling length (Eq. 4.142), half buckling length corresponding to the lowest buckling load of a long plate (Eq. 4.173)
M_x, M_y, M_{xy}	bending and twist moments per unit length acting on a laminate (Eq. 3.9)
M_x^{ht}, M_y^{ht}, M_{xy}^{ht}	hygrothermal moments per unit length (Eq. 4.247)
$\widehat{M}_y$, $\widehat{M}_z$	bending moments acting on a beam (Fig. 6.2)
$\widehat{M}_\omega$	bimoment acting on a beam (Eq. 6.232)
N_x, N_y, N_{xy}	in-plane forces per unit length acting on a laminate (Eq. 3.9)
N_{x0}, N_{y0}, N_{xy0}	in-plane compressive forces per unit length (Eq. 4.109)
N_x^{ht}, N_y^{ht}, N_{xy}^{ht}	hygrothermal forces per unit length (Eq. 4.246)
$N_{x,\,cr}$	buckling load of a uniaxially loaded plate (Eq. 4.141)
$\widehat{N}$	axial force acting on a beam (Fig. 6.2)
$\widehat{N}_{cr}$, $\widehat{N}_{cr}^B$	buckling load and buckling load due to bending deformation (Eq. 6.337)
$\widehat{N}_{cry}$, $\widehat{N}_{crz}$	buckling load in the x–z and x–y planes, respectively (Eqs. 6.337, 7.110)
$\widehat{N}_{cr\psi}$	buckling load under torsional buckling (Eqs. 6.337, 7.110)
$[P]$, $[\overline{P}]$	stiffness matrix of a beam (Eqs. 6.2, 6.250). Without bar refers to the centroid; with bar to an arbitrarily chosen coordinate system
p	transverse load per unit area; distance between the origin and the tangent of the wall of a beam (Eq. 6.190)
p_x, p_y, p_z	axial and transverse loads (per unit length) acting on a beam (Fig. 6.1); surface forces per unit area (Eq. 2.166)
$[Q]$, Q_{ij}	2D plane-stress stiffness matrix in the x_1, x_2 coordinate system (Eq. 2.134)

$[\overline{Q}]$, $\overline{Q}_{ij}$	2D plane-stress stiffness matrix in the x, y coordinate system (Eq. 2.126)
$\widehat{Q}_{cr}$	buckling load resulting in lateral buckling (Eq. 6.359)
q	shear flow (Eq. 6.189).
R	stiffness parameter (Eq. 3.46)
$\widetilde{R}$	stress ratio (Eq. 10.42)
R_x, R_y, R_{xy}	radii of curvatures of a shell (Eq. 8.1)
$[R]$, R_{ij}	compliance matrix under plane-strain condition in the x_1, x_2 coordinate system (Eq. 2.79)
$[\overline{R}]$, $\overline{R}_{ij}$	compliance matrix under plane-strain condition in the x, y coordinate system (Eq. 2.65)
$[S]$, S_{ij}	3D compliance matrix in the x_1, x_2, x_3 coordinate system (Eq. 2.23)
$[\overline{S}]$, $\overline{S}_{ij}$	3D compliance matrix in the x, y, z coordinate system (Eq. 2.21)
$\widehat{S}_{ij}$	shear stiffness of a beam, $i, j = z, y, \omega$ (Eqs. 7.13, 7.36)
$\widetilde{S}_{ij}$	shear stiffness of a plate, $i, j = 1, 2$ (Eq. 5.15)
$\widehat{s}_{ij}$	shear compliance of a beam, $i, j = z, y, \omega$ (Eq. 7.38)
s_1^+, s_2^+, s_3^+	tensile strengths (Eq. 10.13)
s_1^-, s_2^-, s_3^-	compression strengths (Eq. 10.13)
s_{23}, s_{13}, s_{12}	shear strengths (Eq. 10.15)
$\widehat{T}$	torque acting on a beam (Fig. 6.2)
$\widehat{T}_\omega$	restrained warping-induced torque (Eq. 6.235)
$\widehat{T}_{sv}$	Saint-Venant torque (Eq. 6.239)
$[T_\sigma]$	2D stress transformation matrix (Eq. 2.182)
$[\hat{T}_\sigma]$	3D stress transformation matrix (Eq. 2.179)
$[T_\epsilon]$	2D strain transformation matrix (Eq. 2.188)
$[\hat{T}_\epsilon]$	3D strain transformation matrix (Eq. 2.185)
t	torque load acting on a beam (Fig. 6.1)
t^t, t^b	thicknesses of the top and bottom facesheets (Eq. 5.26)
U	strain energy (Eq. 2.200)
U	displacement in the x direction; varies with the x and y coordinates only (Eq. 2.50)
u	displacement in the x direction
u^o	displacement of the reference surface in the x direction
u_1, u_2, u_3	displacements in the x_1, x_2, and x_3 direction
V	displacement in the y direction; varies with the x and y coordinates only (Eq. 2.51)
V_f, V_m, V_v	volume of fibers, matrix, and void
V_x, V_y	out-of-plane shear forces per unit length (Eq. 3.10)
$\widehat{V}_y$, $\widehat{V}_z$	transverse shear forces acting on a beam (Fig. 6.2)
v	displacement in the y direction
v^o	displacement of the reference surface in the y direction
v_f, v_m, v_v	volume fraction of fibers, matrix, and void

W	displacement in the z direction; varies with the x and y coordinates only (Eq. 2.52)
$[W]$, $[\overline{W}]$	compliance matrix of a beam (Eq. 6.17). No bar refers to the centroid; bar to an arbitrarily chosen coordinate system
w	deflection in the z direction
$\widetilde{w}$	maximum deflection in the z direction (Eq. 4.29)
w^{o}	deflection of the reference surface in the z direction
w^{B}, w^{S}	deflections due to bending and shear deformations (Eq. 7.85)
y_{c}, z_{c}	coordinates of the centroid of a beam (Eqs. 6.54, 6.73)
y_{sc}, z_{sc}	coordinates of the shear center of a beam (Eq. 6.311)
z_k, z_{k-1}	coordinates of the top and bottom surfaces of the kth ply in a laminate (Eq. 3.20)

Greek letters

α	parameter describing shear deformation (Eq. 7.253)
α_i	parameter describing shear deformation, $i = w, \psi, N, \omega$ (Eq. 7.244)
$[\alpha]$, α_{ij}	compliance matrix of a laminate (Eq. 3.23)
α, β	parameters describing buckled shape of a shell (Eq. 8.78)
$\widehat{\alpha}_{ij}$	compliances for closed-section beams (Eq. 6.156)
$\widetilde{\alpha}_i$, $\widetilde{\alpha}_{ij}$	thermal expansion coefficients (Eqs. 2.153, 2.158)
β, λ	parameters in the displacements of a cylinder (Eq. 8.30)
$[\beta]$, β_{ij}	compliance matrix of a laminate (Eq. 3.23)
$\overline{\beta}_{ij}$	compliance of symmetrical cross-section beams (Table 6.2)
$\widehat{\beta}_{ij}$	compliance of closed-section beams (Eq. 6.156)
$\widetilde{\beta}_i$, $\widetilde{\beta}_{ij}$	moisture expansion coefficients in the x, y, z directions (Eqs. 2.154, 2.159)
β_1	property of the cross section (Eq. 6.360)
γ_y, γ_z	shear strain in a beam in the x–y and x–z planes (Eq. 7.2)
γ_{yz}, γ_{xz}, γ_{xy}	engineering shear strain in the x, y, z coordinate system (Eq. 2.9)
γ_{23}, γ_{13}, γ_{12}	engineering shear strain in the x_1, x_2, x_3 coordinate system
Δh	change in thickness (Eq. 4.282)
ΔT	temperature change (Eq. 2.153)
$[\delta]$, δ_{ij}	compliance matrix of a laminate (Eq. 3.23)
$\widehat{\delta}_{ij}$	compliance of closed-section beams (Eq. 6.157)
$\overline{\epsilon}_x, \ldots$	average strains in a sublaminate (Eq. 9.14)
ϵ_x, ϵ_y, ϵ_z	engineering normal strains in the x, y, z coordinate system
ϵ_1, ϵ_2, ϵ_3	engineering normal strains in the x_1, x_2, x_3 coordinate system
ϵ_x^{o}, ϵ_y^{o}, γ_{xy}^{o}	strains of the reference surface
$\epsilon_x^{\text{o,ht}}$, $\epsilon_y^{\text{o,ht}}$, $\gamma_{xy}^{\text{o,ht}}$	hygrothermal strains in a laminate (Eq. 4.250)
ζ	parameter of restraint (Eq. 4.152)
Θ	polar moment of mass (Eq. 6.411)

Θ_k	ply orientation
ϑ	rate of twist (Eq. 6.1)
ϑ^B, ϑ^S	rate of twist due to bending and shear deformation (Eq. 7.5)
$\kappa_x, \kappa_y, \kappa_{xy}$	curvatures of the reference surface (Eq. 3.8)
$\kappa_x^{ht}, \kappa_y^{ht}, \kappa_{xy}^{ht}$	hygrothermal curvatures of a laminate (Eq. 4.250)
$\lambda, \lambda_{cr}, \lambda_{ij}$	load parameter (Eq. 4.109); buckling load parameter (Eq. 4.121); eigenvalue (Eq. 4.225)
$\mu_{Bi}, \mu_{Gi}, \mu_{Si}$	parameters in the calculation of natural frequencies (Eqs. 6.398, 6.400, 7.203)
ν_{ij}	Poisson's ratio
ξ, η, ζ	coordinates attached to the wall of a beam (Fig. 6.13)
ξ, ξ'	parameters in the expressions of the buckling loads of plates with rotationally restrained edges (Eq. 4.151)
π_p	potential energy (Eq. 2.204)
ρ_x, ρ_y, ρ_z	radius of curvature in the y–z, x–z, and x–y planes (Eq. 2.45)
ρ_1, ρ_2, ρ_3	radius of curvature in the x_2–x_3, x_1–x_3, and x_1–x_2 planes (Eq. 2.53)
$\rho_{comp}, \rho_f, \rho_m$	densities of composite, fiber, and matrix
ρ	mass per unit area or per unit length
$\sigma_1, \sigma_2, \sigma_3$	normal stresses in the x_1, x_2, x_3 coordinate system
$\sigma_x, \sigma_y, \sigma_z$	normal stresses in the x, y, z coordinate system
$\overline{\sigma}$	average stress
$\tau_{23}, \tau_{13}, \tau_{12}$	shear stresses in the x_1, x_2, x_3 coordinate system
$\tau_{yz}, \tau_{xz}, \tau_{yx}$	shear stresses in the x, y, z coordinate system
χ_{xz}, χ_{yz}	rotation of the normal of a plate in the x–z and x–y planes (Eqs. 3.2 and 5.1)
χ_y, χ_z	rotation of the cross section of a beam in the x–y and x–z planes (Eq. 7.2)
ψ	angle of rotation of the cross section about the beam axis (twist) (Fig. 6.3)
Ψ	bending stiffness of an unsymmetrical long plate (Eq. 4.52)
Ω	potential energy of the external loads (Eq. 2.203)
ω	circular frequency (Eq. 4.190)
ω^B, ω^S	circular frequency of a beam due to bending and shear deformation (Eq. 7.198)
ω_y, ω_z	circular frequency of a freely vibrating beam in the x–z and x–y planes, respectively (Eq. 6.398)
ω_ψ	circular frequency of a freely vibrating beam under torsional vibration (Eq. 6.400)
$\varrho, \widetilde{\varrho}, \bar{\varrho}, \widehat{\varrho}$	distances between the new and the old reference surfaces (Eqs. 3.47, 6.105, 6.107, A.3)

Contents

CHAPTER ONE

Introduction

In this book we focus on fiber-reinforced composites composed of fibers embedded in a matrix. The fibers may be short or long, continuous or discontinuous, and may be in one or in multiple directions (Fig. 1.1). Such materials offer advantages over conventional isotropic structural materials such as steel, aluminum, and other types of metal. These advantages include high strength, low weight, and good fatigue and corrosion resistance. In addition, by changing the arrangements of the fibers, the properties of the material can be tailored to meet the requirements of a specific design.

The excellent properties of composites are achieved by the favorable characteristics of the two major constituents, namely the fiber and the matrix. In low-performance composites, the reinforcements, usually in the form of short or chopped fibers (or particles), provide some stiffening but very little strengthening; the load is mainly carried by the matrix. In high-performance composites, continuous fibers provide the desirable stiffness and strength, whereas the matrix provides protection and support for the fibers, and, importantly, helps redistribute the load from broken to adjacent intact fibers.

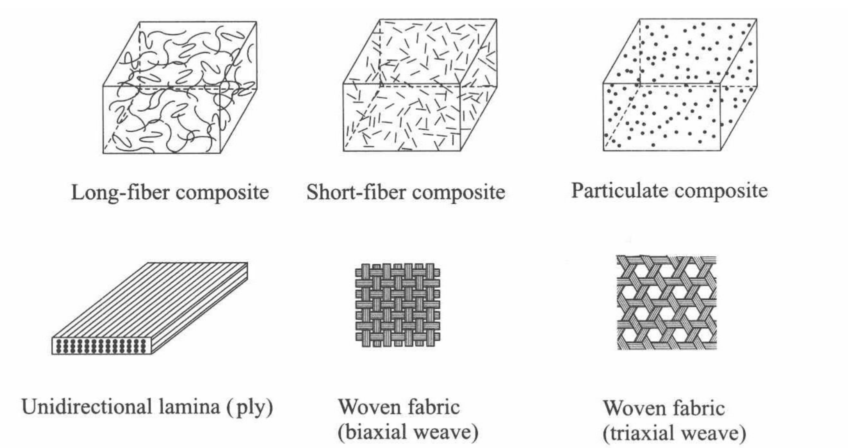

Figure 1.1: Composite material systems.

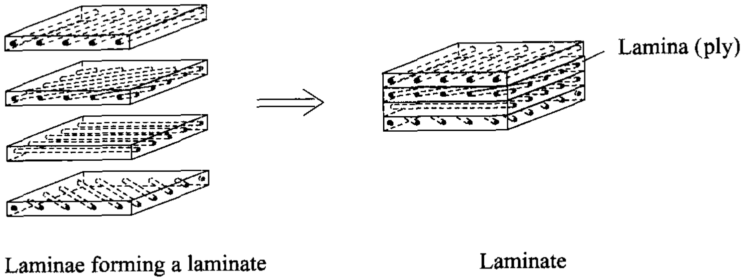

Figure 1.2: Laminated composite.

The arrangement of the fibers in a structure is governed by the structural requirements and by the process used to fabricate the part. Frequently, though not always, composite structures are made of thin layers called laminae or plies. Within each lamina, the fibers may be aligned in the same direction (unidirectional ply, Fig. 1.1) or in different directions. The latter configuration is produced, for example, by weaving the fibers in two or more directions (woven fabric). The lamina may also contain short fibers either oriented in the same direction or distributed randomly. Several laminae are then combined into a laminate to form the desired structure (Fig. 1.2).

The mechanical and thermal behaviors of a structure depend on the properties of the fibers and the matrix and on the amount and orientations of the fibers. In this book, we consider the design steps from micromechanics (which takes into account the fiber and matrix properties) through macromechanics (which treats the properties of the composite) to structural analysis. These steps are illustrated in Figure 1.3 for a structure made of laminated composite.

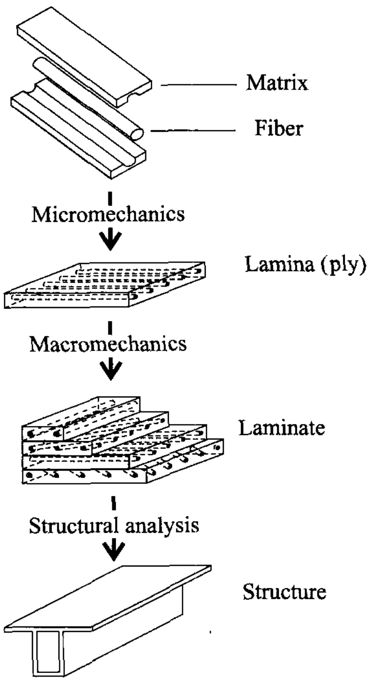

Figure 1.3: The levels of analysis for a structure made of laminated composite.