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FOR SOUND AND VIBRATION ENGINEERS

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Fundamentals of Signal Processing

for Sound and Vibration Engineers

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Fundamentals of Signal Processing

for Sound and Vibration Engineers

Preface

This book has grown out of notes for a course that the second author has given for more years than he cares to remember – which, but for the first author who kept various versions, would never have come to this. Specifically, the Institute of Sound and Vibration Research (ISVR) at the University of Southampton has, for many years, run a Masters programme in Sound and Vibration, and more recently in Applied Digital Signal Processing. A course aimed at introducing students to signal processing has been one of the compulsory modules, and given the wide range of students' first degrees, the coverage needs to make few assumptions about prior knowledge – other than a familiarity with degree entry-level mathematics. In addition to the Masters programmes the ISVR runs undergraduate programmes in Acoustical Engineering, Acoustics with Music, and Audiology, each of which to varying levels includes signal processing modules. These taught elements underpin the wide-ranging research of the ISVR, exemplified by the four interlinked research groups in Dynamics, Fluid Dynamics and Acoustics, Human Sciences, and Signal Processing and Control. The large doctoral cohort in the research groups attend selected Masters modules and an acquaintance with signal processing is a 'required skill' (necessary evil?) in many a research project. Building on the introductory course there are a large number of specialist modules ranging from medical signal processing to sonar, and from adaptive and active control to Bayesian methods.

It was in one of the PhD cohorts that Kihong Shin and Joe Hammond made each other's acquaintance in 1994. Kihong Shin received his PhD from ISVR in 1996 and was then a postdoctoral research fellow with Professor Mike Brennan in the Dynamics Group, then joining the School of Mechanical Engineering, Andong National University, Korea, in 2002, where he is an associate professor. This marked the start of this book, when he began 'editing' Joe Hammond's notes appropriate to a postgraduate course he was lecturing – particularly appreciating the importance of including 'hands-on' exercises – using interactive MATLAB® examples. With encouragement from Professor Mike Brennan, Kihong Shin continued with this and it was not until 2004, when a manuscript landed on Joe Hammond's desk (some bits looking oddly familiar), that the second author even knew of the project – with some surprise and great pleasure.

In July 2006, with the kind support and consideration of Professor Mike Brennan, Kihong Shin managed to take a sabbatical which he spent at the ISVR where his subtle pressures – including attending Joe Hammond's very last course on signal processing at the ISVR – have distracted Joe Hammond away from his duties as Dean of the Faculty of Engineering, Science and Mathematics.

Thus the text was completed. It is indeed an introduction to the subject and therefore the essential material is not new and draws on many classic books. What we have tried to do is to bring material together, hopefully encouraging the reader to question, enquire about and explore the concepts using the MATLAB exercises or derivatives of them.

It only remains to thank all who have contributed to this. First, of course, the authors whose texts we have referred to, then the decades of students at the ISVR, and more recently in the School of Mechanical Engineering, Andong National University, who have shaped the way the course evolved, especially Sangho Pyo who spent a generous amount of time gathering experimental data. Two colleagues in the ISVR deserve particular gratitude: Professor Mike Brennan, whose positive encouragement for the whole project has been essential, together with his very constructive reading of the manuscript; and Professor Paul White, whose encyclopaedic knowledge of signal processing has been our port of call when we needed reassurance.

We would also like to express special thanks to our families, Hae-Ree Lee, Inyong Shin, Hakdoo Yu, Kyu-Shin Lee, Young-Sun Koo and Jill Hammond, for their never-ending support and understanding during the gestation and preparation of the manuscript. Kihong Shin is also grateful to Geun-Tae Yim for his continuing encouragement at the ISVR.

Finally, Joe Hammond thanks Professor Simon Braun of the Technion, Haifa, for his unceasing and inspirational leadership of signal processing in mechanical engineering. Also, and very importantly, we wish to draw attention to a new text written by Simon entitled *Discover Signal Processing: An Interactive Guide for Engineers*, also published by John Wiley & Sons, which offers a complementary and innovative learning experience.

Please note that MATLAB codes (m files) and data files can be downloaded from the Companion Website at www.wiley.com/go/shin_hammond

Kihong Shin
Joseph Kenneth Hammond

About the Authors

Joe Hammond Joseph (Joe) Hammond graduated in Aeronautical Engineering in 1966 at the University of Southampton. He completed his PhD in the Institute of Sound and Vibration Research (ISVR) in 1972 whilst a lecturer in the Mathematics Department at Portsmouth Polytechnic. He returned to Southampton in 1978 as a lecturer in the ISVR, and was later Senior lecturer, Professor, Deputy Director and then Director of the ISVR from 1992–2001. In 2001 he became Dean of the Faculty of Engineering and Applied Science, and in 2003 Dean of the Faculty of Engineering, Science and Mathematics. He retired in July 2007 and is an Emeritus Professor at Southampton.

Kihong Shin Kihong Shin graduated in Precision Mechanical Engineering from Hanyang University, Korea in 1989. After spending several years as an electric motor design and NVH engineer in Samsung Electro-Mechanics Co., he started an MSc at Cranfield University in 1992, on the design of rotating machines with reference to noise and vibration. Following this, he joined the ISVR and completed his PhD on nonlinear vibration and signal processing in 1996. In 2000, he moved back to Korea as a contract Professor of Hanyang University. In Mar. 2002, he joined Andong National University as an Assistant Professor, and is currently an Associate Professor.

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1

Introduction to Signal Processing

Signal processing is the name given to the procedures used on measured data to reveal the information contained in the measurements. These procedures essentially rely on various transformations that are mathematically based and which are implemented using digital techniques. The wide availability of software to carry out digital signal processing (DSP) with such ease now pervades all areas of science, engineering, medicine, and beyond. This ease can sometimes result in the analyst using the wrong tools – or interpreting results incorrectly because of a lack of appreciation or understanding of the assumptions or limitations of the method employed.

This text is directed at providing a user's guide to linear system identification. In order to reach that end we need to cover the groundwork of Fourier methods, random processes, system response and optimization. Recognizing that there are many excellent texts on this,¹ why should there be yet another? The aim is to present the material from a user's viewpoint. Basic concepts are followed by examples and structured MATLAB® exercises allow the user to 'experiment'. This will not be a story with the punch-line at the end – we actually start in this chapter with the intended end point.

The aim of doing this is to provide reasons and motivation to cover some of the underlying theory. It will also offer a more rapid guide through methodology for practitioners (and others) who may wish to 'skip' some of the more 'tedious' aspects. In essence we are recognizing that it is not always necessary to be fully familiar with every aspect of the theory to be an effective practitioner. But what is important is to be aware of the limitations and scope of one's analysis.

¹ See for example Bendat and Piersol (2000), Brigham (1988), Hsu (1970), Jenkins and Watts (1968), Oppenheim and Schaffer (1975), Otnes and Enochson (1978), Papoulis (1977), Randall (1987), etc.

The Aim of the Book

We are assuming that the reader wishes to understand and use a widely used approach to ‘system identification’. By this we mean we wish to be able to characterize a physical process in a quantified way. The object of this quantification is that it reveals information about the process and accounts for its behaviour, and also it allows us to predict its behaviour in future environments.

The ‘physical processes’ could be anything, e.g. vehicles (land, sea, air), electronic devices, sensors and actuators, biomedical processes, etc., and perhaps less ‘physically based’ socio-economic processes, and so on. The complexity of such processes is unlimited – and being able to characterize them in a quantified way relies on the use of physical ‘laws’ or other ‘models’ usually phrased within the language of mathematics. Most science and engineering degree programmes are full of courses that are aimed at describing processes that relate to the appropriate discipline. We certainly do not want to go there in this book – life is too short! But we still want to characterize these systems – with the minimum of effort and with the maximum effect.

This is where ‘system theory’ comes to our aid, where we employ descriptions or models – abstractions from the ‘real thing’ – that nevertheless are able to capture what may be fundamentally common, to large classes of the phenomena described above. In essence what we do is simply to watch what ‘a system’ does. This is of course totally useless if the system is ‘asleep’ and so we rely on some form of activation to get it going – in which case it is logical to watch (and measure) the particular activation and measure some characteristic of the behaviour (or response) of the system.

In ‘normal’ operation there may be many activators and a host of responses. In most situations the activators are not separate discernible processes, but are distributed. An example of such a system might be the acoustic characteristics of a concert hall when responding to an orchestra and singers. The sources of activation in this case are the musical instruments and singers, the system is the auditorium, including the members of the audience, and the responses may be taken as the sounds heard by each member of the audience.

The complexity of such a system immediately leads one to try and conceptualize something simpler. Distributed activation might be made more manageable by ‘lumping’ things together, e.g. a piano is regarded as several separate activators rather than continuous strings/sounding boards all causing acoustic waves to emanate from each point on their surfaces. We might start to simplify things as in Figure 1.1.

This diagram is a model of a greatly simplified system with several actuators – and the several responses as the sounds heard by individual members of the audience. The arrows indicate a ‘cause and effect’ relationship – and this also has implications. For example, the figure implies that the ‘activators’ are unaffected by the ‘responses’. This implies that there is no ‘feedback’ – and this may not be so.

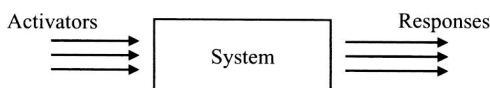


Figure 1.1 Conceptual diagram of a simplified system

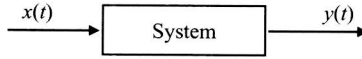


Figure 1.2 A single activator and a single response system

Having got this far let us simplify things even further to a single activator and a single response as shown in Figure 1.2. This may be rather ‘distant’ from reality but is a widely used model for many processes.

It is now convenient to think of the activator $x(t)$ and the response $y(t)$ as time histories. For example, $x(t)$ may denote a voltage, the system may be a loudspeaker and $y(t)$ the pressure at some point in a room. However, this time history model is just one possible scenario. The activator x may denote the intensity of an image, the system is an optical device and y may be a transformed image. Our emphasis will be on the time history model generally within a sound and vibration context.

The box marked ‘System’ is a convenient catch-all term for phenomena of great variety and complexity. From the outset, we shall impose major constraints on what the box represents – specifically systems that are **linear**² and **time invariant**.³ Such systems are very usefully described by a particular feature, namely their response to an **ideal impulse**,⁴ and their corresponding behaviour is then the **impulse response**.⁵ We shall denote this by the symbol $h(t)$.

Because the system is linear this rather ‘abstract’ notion turns out to be very useful in predicting the response of the system to any arbitrary input. This is expressed by the **convolution**⁶ of input $x(t)$ and system $h(t)$ sometimes abbreviated as

$$y(t) = h(t) * x(t) \quad (1.1)$$

where ‘*’ denotes the convolution operation. Expressed in this form the system box is filled with the characterization $h(t)$ and the (mathematical) mapping or transformation from the input $x(t)$ to the response $y(t)$ is the convolution integral.

System identification now becomes the problem of measuring $x(t)$ and $y(t)$ and deducing the impulse response function $h(t)$. Since we have three quantitative terms in the relationship (1.1), but (assume that) we know two of them, then, in principle at least, we should be able to find the third. The question is: how?

Unravelling Equation (1.1) as it stands is possible but not easy. Life becomes considerably easier if we apply a transformation that maps the convolution expression to a multiplication. One such transformation is the **Fourier transform**.⁷ Taking the **Fourier transform of the convolution**⁸ in Equation (1.1) produces

$$Y(f) = H(f)X(f) \quad (1.2)$$

* Words in bold will be discussed or explained at greater length later.

² See Chapter 4, Section 4.7.

³ See Chapter 4, Section 4.7.

⁴ See Chapter 3, Section 3.2, and Chapter 4, Section 4.7.

⁵ See Chapter 4, Section 4.7.

⁶ See Chapter 4, Section 4.7.

⁷ See Chapter 4, Sections 4.1 and 4.4.

⁸ See Chapter 4, Sections 4.4 and 4.7.

where f denotes frequency, and $X(f)$, $H(f)$ and $Y(f)$ are the transforms of $x(t)$, $h(t)$ and $y(t)$. This achieves the unravelling of the input–output relationship as a straightforward multiplication – in a ‘domain’ called the **frequency domain**.⁹ In this form the system is characterized by the quantity $H(f)$ which is called the system **frequency response function (FRF)**.¹⁰

The problem of ‘system identification’ now becomes the calculation of $H(f)$, which seems easy: that is, divide $Y(f)$ by $X(f)$, i.e. divide the Fourier transform of the output by the Fourier transform of the input. As long as $X(f)$ is never zero this seems to be the end of the story – but, of course, it is not. Reality interferes in the form of ‘uncertainty’. The measurements $x(t)$ and $y(t)$ are often not measured perfectly – disturbances or ‘noise’ contaminates them – in which case the result of dividing two transforms of contaminated signals will be of limited and dubious value.

Also, the actual excitation signal $x(t)$ may itself belong to a class of **random**¹¹ signals – in which case the straightforward transformation (1.2) also needs more attention. It is this ‘dual randomness’ of the actuating (and hence response) signal and additional contamination that is addressed in this book.

The Effect of Uncertainty

We have referred to randomness or uncertainty with respect to both the actuation and response signal and additional noise on the measurements. So let us redraw Figure 1.2 as in Figure 1.3.

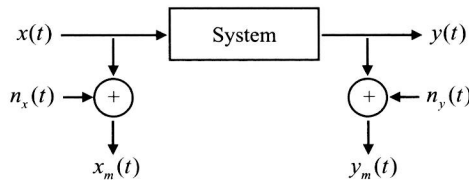


Figure 1.3 A single activator/response model with additive noise on measurements

In Figure 1.3, x and y denote the actuation and response signals as before – which may themselves be random. We also recognize that x and y are usually not directly measurable and we model this by including disturbances written as n_x and n_y which add to x and y – so that the actual measured signals are x_m and y_m . Now we get to the crux of the system identification: that is, on the basis of (noisy) measurements x_m and y_m , what is the system?

We conceptualize this problem pictorially. Imagine plotting y_m against x_m (ignore for now what x_m and y_m might be) as in Figure 1.4.

Each point in this figure is a ‘representation’ of the measured response y_m corresponding to the measured actuation x_m .

System identification, in this context, becomes one of establishing a relationship between y_m and x_m such that it somehow relates to the relationship between y and x . The noises are a

⁹ See Chapter 2, Section 2.1.

¹⁰ See Chapter 4, Section 4.7.

¹¹ See Chapter 7, Section 7.2.

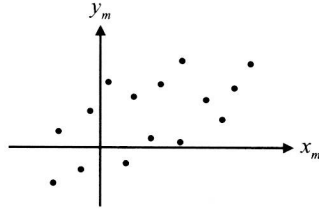


Figure 1.4 A plot of the measured signals y_m versus x_m

nuisance, but we are stuck with them. This is where ‘optimization’ comes in. We try and find a relationship between x_m and y_m that seeks a ‘systematic’ link between the data points which suppresses the effects of the unwanted disturbances.

The simplest conceptual idea is to ‘fit’ a linear relationship between x_m and y_m . Why linear? Because we are restricting our choice to the simplest relationship (we could of course be more ambitious). The procedure we use to obtain this fit is seen in Figure 1.5 where the slope of the straight line is adjusted until the match to the data seems best.

This procedure must be made systematic – so we need a measure of how well we fit the points. This leads to the need for a specific measure of fit and we can choose from an unlimited number. Let us keep it simple and settle for some obvious ones. In Figure 1.5, the closeness of the line to the data is indicated by three measures e_y , e_x and e_T . These are regarded as errors which are measures of the ‘failure’ to fit the data. The quantity e_y is an error in the y direction (i.e. in the output direction). The quantity e_x is an error in the x direction (i.e. in the input direction). The quantity e_T is orthogonal to the line and combines errors in both x and y directions.

We might now look at ways of adjusting the line to minimize e_y , e_x , e_T or some convenient ‘function’ of these quantities. This is now phrased as an optimization problem. A most convenient function turns out to be an average of the squared values of these quantities (‘convenience’ here is used to reflect not only physical meaning but also mathematical ‘niceness’). Minimizing these three different measures of closeness of fit results in three correspondingly different slopes for the straight line; let us refer to the slopes as m_y , m_x , m_T . So which one should we use as the best? The choice will be strongly influenced by our prior knowledge of the nature of the measured data – specifically whether we have some idea of the dominant causes of error in the departure from linearity. In other words, some knowledge of the relative magnitudes of the noise on the input and output.

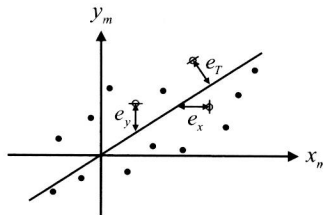


Figure 1.5 A linear fit to measured data

We could look to the figure for a guide:

- m_y seems best when errors occur on y , i.e. errors on output e_y ;
- m_x seems best when errors occur on x , i.e. errors on input e_x ;
- m_T seems to make an attempt to recognize that errors are on both, i.e. e_T .

We might now ask how these rather simple concepts relate to ‘identifying’ the system in Figure 1.3. It turns out that they are directly relevant and lead to three different estimators for the system frequency response function $H(f)$. They have come to be referred to in the literature by the notation $H_1(f)$, $H_2(f)$ and $H_T(f)$,¹² and are the analogues of the slopes m_y , m_x , m_T , respectively.

We have now mapped out what the book is essentially about in Chapters 1 to 10. The book ends with a chapter that looks into the implications of multi-input/output systems.

1.1 DESCRIPTIONS OF PHYSICAL DATA (SIGNALS)

Observed data representing a physical phenomenon will be referred to as a time history or a *signal*. Examples of signals are: temperature fluctuations in a room indicated as a function of time, voltage variations from a vibration transducer, pressure changes at a point in an acoustic field, etc. The physical phenomenon under investigation is often translated by a transducer into an electrical equivalent (voltage or current) and if displayed on an oscilloscope it might appear as shown in Figure 1.6. This is an example of a *continuous* (or *analogue*) signal.

In many cases, data are *discrete* owing to some inherent or imposed sampling procedure. In this case the data might be characterized by a sequence of numbers equally spaced in time. The sampled data of the signal in Figure 1.6 are indicated by the crosses on the graph shown in Figure 1.7.

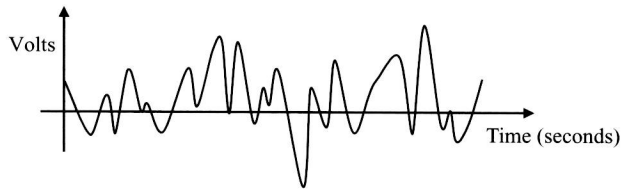


Figure 1.6 A typical continuous signal from a transducer output

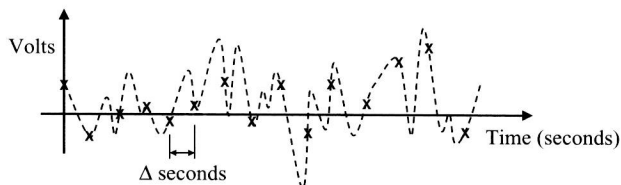


Figure 1.7 A discrete signal sampled at every Δ seconds (marked with \times)

¹² See Chapter 9, Section 9.3.

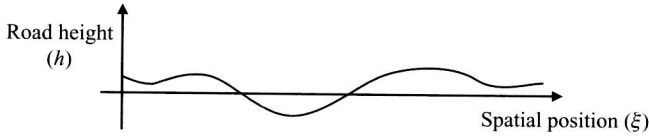


Figure 1.8 An example of a signal where time is not the natural independent variable

For continuous data we use the notation $x(t)$, $y(t)$, etc., and for discrete data various notations are used, e.g. $x(n\Delta)$, $x(n)$, x_n ($n = 0, 1, 2, \dots$).

In certain physical situations, ‘time’ may not be the natural independent variable; for example, a plot of road roughness as a function of spatial position, i.e. $h(\xi)$ as shown in Figure 1.8. However, for uniformity we shall use time as the independent variable in all our discussions.

1.2 CLASSIFICATION OF DATA

Time histories can be broadly categorized as shown in Figure 1.9 (chaotic signals are added to the classifications given by Bendat and Piersol, 2000). A fundamental difference is whether a signal is *deterministic* or *random*, and the analysis methods are considerably different depending on the ‘type’ of the signal. Generally, signals are mixed, so the classifications of Figure 1.9 may not be easily applicable, and thus the choice of analysis methods may not be apparent. In many cases some prior knowledge of the system (or the signal) is very helpful for selecting an appropriate method. However, it must be remembered that this prior knowledge (or assumption) may also be a source of misleading the results. Thus it is important to remember the First Principle of Data Reduction (Ables, 1974)

The result of any transformation imposed on the experimental data shall incorporate and be consistent with all relevant data and be maximally non-committal with regard to unavailable data.

It would seem that this statement summarizes what is self-evident. But how often do we contravene it – for example, by ‘assuming’ that a time history is zero outside the extent of a captured record?

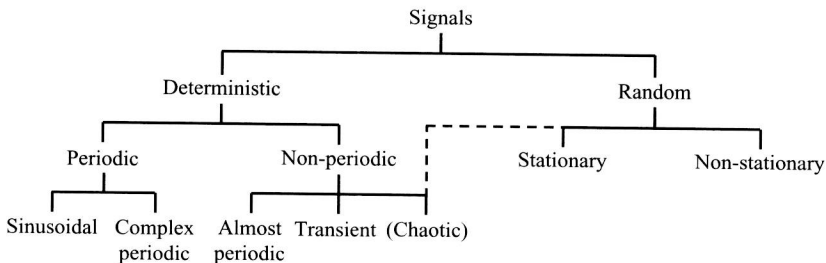


Figure 1.9 Classification of signals