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To Martina, Andrea, Daniel, Stephan and Claudia

Preface

1. In these notes we present some introductory material on a particular class of dynamical systems, called SEMIFLOWS. This class includes, but is not restricted to, systems that are defined, or modelled, by certain types of differential equations of evolution (DEEs in short). Our purpose is to introduce, in a relatively self-contained manner, the basic results of the theory of dynamical systems that can be naturally extended and applied to study the asymptotic behavior of the solutions of the DEEs we consider. Equations of evolution include ordinary differential equations (ODEs in short), partial differential equations of evolution (PDEEs in short), and other types of equations, such as, for instance, stochastic or difference equations. As such, they provide natural examples of dynamical systems, since one of the independent variables (usually called "time") plays a different role than the other variables (which in some situations may be called "space" variables). Thus, in this context, the heat and wave equations are considered as prototypical examples of PDEEs, while elliptic equations such as Laplace's equation are not considered as evolution equations, because in these equations all the variables have the same role. Here, we make the further distinction that "time" evolves continuously; thus, we do not consider stochastic equations, nor, except for some introductory examples, discrete systems (where "time" varies along a sequence).

2. One of the major goals of the theory of dynamical systems is the study of the evolution of a system, with the purpose of predicting, as accurately as possible, the behavior of the system as time becomes large. A quite general feature of the systems we consider, which is shared with other systems, is a property called DISSIPATIVITY. Loosely speaking, this property can be described by the fact that all solutions of these systems eventually enter, and remain, in a bounded set, called ABSORBING SET. Thus, the evolution of the solutions of the system can be studied in this set; as a result, the long time behavior of the system can be described by means of certain subsets of the absorbing set. Among these, we shall consider three types of sets, called respectively ATTRACTORS, EXPONENTIAL ATTRACTORS, and INERTIAL MANIFOLDS. (Exponential attractors are sometimes also known as INERTIAL SETS.) We will present the fundamental properties of these sets, and then proceed to show the existence of some of these sets for a number of dynamical systems, generated by fairly well known physical models. In particular, we shall consider in full detail two particular PDEEs of evolution: a semilinear version of the heat equation, and a corresponding version of the dissipative wave equation. These examples allow us to illustrate the most important features of the theory of semiflows, and to provide a sort of "template" that can then be applied, in a more or less straightforward fashion, to the analysis of other models, with the help of the many specialized references that exist in the literature.

3. Even a quick survey of much of the existing literature on dynamical systems, both at the introductory and the specialized level, reveals that the notion of "dynamical system" is used with many different meanings, according to the specific point of view of the authors. At the opposite extreme, this notion may well be not defined at all. In these notes, we do not attempt to give a general definition of dynamical system; rather, we confine ourselves to a special class of systems, properly known as CONTINUOUS, SEMI-DYNAMICAL SYSTEMS, or CONTINUOUS SEMIFLOWS. Here, the term "continuous" is used to distinguish these systems from DISCRETE ones, where only a sequence of successive time values are considered, and "semi-" refers to the fact that time evolves, i.e. we only consider nonnegative values of the time variable. For brevity, we shall refer to these systems as SEMIFLOWS (their precise definition is given in section 2.2). In the introductory chapter 1, we consider more general TWO-PARAMETER SEMIFLOWS or DYNAMICAL PROCESSES, which allows us to include some nonautonomous difference or differential equations as generators of dynamical systems. However, when our presentation can proceed in a more discursive way, and rigor is not an issue, we conform to the common use and adopt the general term "dynamical system".

4. In general, we say that an ODE defines a semiflow if the corresponding CAUCHY PROBLEM is globally well posed, in the sense we define in section 1.2.1. We can extend this definition to semiflows defined by PDEEs, by interpreting the PDEE as an abstract ODE in a suitable Banach space $\mathcal{X}$ (see remark 3.2 in chapter 3). This is a generalization of the usual interpretation of a system of ODEs as a single differential equation in the Banach space $\mathcal{X} = \mathbb{R}^n$, or in more general finite dimensional vector spaces, and explains the qualification of the systems generated by PDEEs as "infinite dimensional" ones, since in this case $\mathcal{X}$ is in general no longer a finite dimensional space. Examples of PDEEs that can be put in such abstract form are: the Navier-Stokes equations, the Kuramoto-Sivashinski equations, the "original" Burger's equation, the Chafee-Infante and Cahn-Hilliard reaction-diffusion equations, the Korteweg-de Vries and the Maxwell equations (see chapter 6). Indeed, many basic notions and results in the theory of the asymptotic behavior of infinite dimensional dissipative dynamical systems trace their origin in the study of the Navier-Stokes equations of fluid dynamics, and have been inspired by a detailed analysis of both the qualitative properties of their solutions, and their behavior with respect to numerical computations.

5. Not surprisingly, much of the general terminology in the theory of dynamical systems, as well as the general spirit of its qualitative results, borrows directly from the qualitative theory of ODEs in $\mathbb{R}^n$. For example, we shall need to recall some basic results on stability, equilibrium points, periodic orbits, ω-limit sets, etc. On the other hand, in an effort to keep these notes within a reasonable length, we shall be forced to not discuss many other important topics. In particular, we regretfully do not include any result on bifurcation theory. Among the many excellent and fairly complete references on the qualitative theory of ODEs, including ODEs as dynamical systems, we refer for example to Hirsch and Smale, [HS93], Jordan and Smith, [JS87], Perko, [Per91], Amann, [Ama90], and Verhulst, [Ver90]. A few other references, specifically on dynamical systems, are listed in the bibliography. Since so many articles and books are continually being published, it is almost impossible to compile an exhaustive list of references; on the other hand, an internet search can provide all necessary updated references on any particular topic.

6. These notes have their origin in a series of graduate seminars we held at the Universities of Dresden, Wisconsin-Milwaukee and Tsukuba. Most of the material we cover is relatively well known, although some of the results we present, in particular on the existence of an exponential attractor and of an inertial manifold for semilinear dissipative wave equations, even if not entirely new, do not seem to enjoy the recognition we feel they deserve. In part, our intention in writing these notes is to be of some help to "beginners" in the area of infinite dimensional dynamical systems; that is, anyone who, having a solid background in the classical theory of ODEs and some knowledge of functional analysis in Sobolev spaces, wishes to proceed to the study of examples of semiflows arising from DEEs, but may need some "smoothing into" the subject, before turning to more general introductory texts, such as those of Temam, [Tem88], the cycle of lectures by Oleinik, [Ole96], or, most recently, Sell-You, [SY02], and Robinson, [Rob01]. We also hope that these notes may serve as a ready reference to researchers in more applied fields, who feel the need for a clear presentation of the background material and results that are necessary for the study of the specific systems they are interested in. To this end, we have tried to "build up" the material in as careful and gradual progression as possible, with the goal of presenting the main topics (in particular, the construction of the exponential attractor and the inertial manifold), with a larger degree of detail than generally found in most sources in the literature. If successful, our effort should put the reader in a better position to refer to more specific texts on global attractors, exponential attractors, and inertial manifolds, such as, respectively, the books by Babin and Vishik, [BV92], Eden, Foias, Nicolaenko and Temam, [EFNT94], and Constantin, Foias, Nicolaenko and Temam, [CFNT89].

7. These notes are organized as follows. As an introduction to the main ideas of the abstract theory of semiflows, in chapter 1 we present some well known and well studied examples of finite dimensional dynamical systems, generated by such celebrated ODEs as Duffing's equations and Lorenz' equations. In chapter 2 we introduce the general definitions of SEMIFLOWS and their GLOBAL ATTRACTORS, and we present two sufficient conditions that guarantee the existence of the attractor under different assumption on the asymptotic properties of the semiflow. We also describe an alternate construction of the attractor, based on the idea of α-contracting maps. In chapter 3 we apply these results to show that the semiflows generated by two types of semilinear dissipative evolution PDEEs (one parabolic and the other hyperbolic) admit a global attractor in a suitable space of weak solutions. In chapter 4 we briefly develop the theory of EXPONENTIAL ATTRACTORS, and apply this theory to the models of PDEEs considered in chapter 3. In chapter 5 we present Hadamard's GRAPH TRANSFORMATION METHOD for the construction of an INERTIAL MANIFOLD, and apply this method to a one-dimensional version of the PDEEs considered in chapter 3. In chapter 6, we consider a number of other dynamical systems, generated by PDEEs that model various mathematical physics problems, and briefly show how the methods developed in the previous chapters can be applied. In chapter 7 we present a result, due to Mora and Solà-Morales, on the nonexistence of inertial manifolds for the semiflow generated by a one-dimensional version of the hyperbolic model of PDEE considered in chapter 3. Finally, in the Appendix we collect, for the reader's convenience, a list of various definitions and results from the classical theory of ODEs and PDEs, functional and nonlinear analysis, semigroup theory and Lebesgue-Sobolev spaces, that we use in these notes, and provide at least one reference for each of the definitions and theorems we state.

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Contents

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Chapter 1

Dynamical Processes

In this chapter we introduce the definition of DYNAMICAL PROCESS, and the main ideas of the theory of dynamical systems that we want to investigate. We illustrate these ideas by examining some simple examples of dynamical processes generated by finite systems of ODEs and by iterated maps.

1.1 Introduction

1. Roughly speaking, the theory of dynamical systems studies mathematical models of physical "systems" which evolve in time from a "state" which is known at an initial moment; more specifically, how the evolution of a system depends, or is influenced by, its initial state. The changing density of a population from a known number of individuals (e.g., sharks in a regional sea; bacteria in an infected organism; prey-predator models); the changing of weather patterns in a particular region; the spreading of a rumor; the vapor trail in the wake of an airplane; the propagation of a fire: all these would be examples of dynamical systems.

To study the evolution of a system, we assume that its state at each time t can be described generally by means of a function $t \mapsto u(t)$, where the independent "time" variable t is measured in a parameter set $\mathcal{T} \subset \mathbb{R}$, and the corresponding dependent variable is in a set $\mathcal{X}$, called STATE SPACE. We also assume that the state $u(t)$ of the system at any given time t depends not only on the value of t, but also on its initial configuration, i.e. on the value u_0 of the system at a previous time t_0, with u_0 and t_0 given or known. A natural goal of the theory is then to study the dependence of the state $u \in \mathcal{X}$ on the time $t \in \mathcal{T}$ and the INITIAL VALUES $u_0 \in \mathcal{X}$, $t_0 \in \mathcal{T}$. In particular, we can think of a dynamical system as a way of transforming an initial state u_0 into a family of subsequent states $u(t)$, parametrized by $t \in \mathcal{T}$. We shall indeed assume that there is a specified functional dependence of $u \in \mathcal{X}$ from $u_0 \in \mathcal{X}$ and t, $t_0 \in \mathcal{T}$, described by a map

$$(t, t_0, u_0) \mapsto u(t, t_0, u_0) . \tag{1.1}$$

By specifying certain properties of this map, we come to a definition of a special kind of dynamical systems.

DEFINITION 1.1 *Let $\mathcal{X}$ be an arbitrary set, and $\mathcal{T}$ be one of the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}_{\geq 0}$ or $\mathbb{R}$, where $\mathbb{R}_{\geq 0} := [0, +\infty[$. Set*

$$\mathcal{T}_*^2 := \{(t, \tau) \in \mathcal{T} \times \mathcal{T} : t \geq \tau\}.$$

A TWO-PARAMETER SEMIFLOW, *or* DYNAMICAL PROCESS *in $\mathcal{X}$ is a family $S = (S(t,\tau))_{(t,\tau)\in\mathcal{T}_*^2}$ of maps $S(t,\tau)\colon \mathcal{X} \to \mathcal{X}$, which satisfies the following conditions:*

$$\forall t \in \mathcal{T}: \quad S(t,t) = I_{\mathcal{X}} \tag{1.2}$$

(the identity in $\mathcal{X}$), and

$$\forall t_1, t_2, t_3 \in \mathcal{T}: \quad S(t_1,t_2)S(t_2,t_3) = S(t_1,t_3). \tag{1.3}$$

The following are familiar examples of dynamical processes.

Example 1.2
Let $\mathcal{X} = \mathbb{R}$ and $\mathcal{T} = \mathbb{R}$. Let f be a continuous function on $\mathbb{R}$, and $S = (S(t,\tau))_{(t,\tau)\in\mathcal{T}_*^2}$ be the family of maps $S(t,\tau)\colon \mathbb{R} \to \mathbb{R}$ defined by

$$S(t,\tau)x := \left(\exp\left(\int_\tau^t f(s)\,\mathrm{d}s\right)\right)x, \qquad x \in \mathbb{R}. \tag{1.4}$$

Then, S is a dynamical process in $\mathbb{R}$. Indeed, verification of (1.2) and (1.3) is immediate. □

Example 1.3
Let $\mathcal{X} = \mathbb{R}^n$, and A be an $n \times n$ matrix. Then, the family $T = (\mathrm{e}^{tA})_{t\in\mathbb{R}}$ of the exponentials of the matrices tA is a linear semigroup in $\mathcal{X}$ (see section A.3). Consequently, the family S defined by

$$S(t,\tau) := \mathrm{e}^{(t-\tau)A}, \qquad (t,\tau) \in \mathbb{R}^2,$$

is a dynamical process. □

Note that, in these examples, each map $S(t,\tau)$ is linear; as we shall see, this needs not be the case in general.

According to definition 1.1, a dynamical process S on a set $\mathcal{X}$ consists of a family of transformations of $\mathcal{X}$ into itself, each defined by the map (1.1), that is,

$$\mathcal{X} \ni u_0 \mapsto u(t,\tau,u_0) =: S(t,\tau)u_0 \in \mathcal{X}. \tag{1.5}$$

We are then mainly interested in the dependence of the map $t \mapsto S(t,t_0)u_0$ on the "initial values" t_0 and u_0 or, sometimes, on u_0 only, for fixed t_0. Of course, this requires $\mathcal{X}$ to have some kind of topological structure, and we shall remove the provisional nature of definition 1.1, supplementing it by a number of continuity conditions on

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