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# Principles of Discrete Event Simulation

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# Preface

Many developments have occurred in the field of simulation since 1973 when my earlier book *Concepts and Methods in Discrete Event Digital Simulation* was published. At that time the use of a regenerative representation to characterize a phenomenon of interest in a simulation was barely in its infancy. In fact, that book contains only a limited example of its use. Since then the attractiveness of the regenerative representation, when it applies, has become apparent to an increasing number of analysts who wish to exploit the features of their simulation models in order to simplify the ensuing statistical analyses of output. The present book places considerable emphasis on this type of model exploitation by describing many features of the regenerative representation in detail. Presumably, the attractiveness of these features will encourage simulation modelers to consider incorporating the representation into their models and reaping the benefits it offers for statistical analysis.

Not all simulations are amenable to the regenerative representation. Therefore the book also describes methods of analysis that are more time series oriented and apply to sample records of autocorrelated observations in general. Moreover, it contains a progress report on the current state of random number generation and a comprehensive account of sampling from diverse distributions on a computer.

In my earlier book I emphasized the central roles that modeling, programming, and statistical considerations play in a discrete event simulation. Although the present book retains the same emphasis, the order of presentation differs. Instead of discussing programming after modeling and statistical considerations after programming, I now present these issues in parallel. For example, Chapter 2 describes the event scheduling approach to modeling, the use of SIMSCRIPT II to translate the model into executable code, and methods of statistical inference that apply when the model admits a particular regenerative representation. The change

from a series to parallel presentation aims to shorten the time it takes to enable a student to begin building, programming, and analyzing a simulation. Experience at the University of North Carolina at Chapel Hill confirms that students derive considerably more satisfaction from the parallel presentation than from the earlier series presentation. They also appear to learn more.

The book contains an additional parallelism. Chapter 4 describes the process interaction approach to simulation modeling and applies it to the sample problem studied in Chapter 2 by way of the event scheduling approach. Then Chapter 4 shows how to convert this process interaction model into executable code in SIMSCRIPT II.5, GPSS, SIMPL/1, and SIMULA. This parallelism for all four simulation programming languages allows the reader to absorb the similarities and differences among these languages quickly and expeditiously.

My gratitude goes to Barry Margolin of the National Institute of Environmental Health Sciences who provided me with constructive comments on Chapter 6, to Vicki Horton of IBM for her assistance with the SIMPL/1 example in Chapter 4, to Richard Nance and Anil Chatterji of Virginia Polytechnic Institute and Chris Nevison of Colgate University for their assistance with the SIMULA example in Chapter 4, and to William Kwapil, Mark Miller, and David Raber of the University of North Carolina at Chapel Hill for their programming assistance. Alan Pritsker of Purdue University deserves my gratitude for allowing me to use several homework exercises that he prepared for his own books on Q-GERT. I also thank George Kastner of the University of North Carolina for bringing to my attention numerous typographical errors found during his conscientious reading of the original manuscript. Special thanks go to Louis Moore of the University of North Carolina whose thoughtful suggestions and assistance have contributed to many sections of the book. My thanks also go to the Office of Naval Research which has supported my research on statistical methods in discrete event simulation. Many of these developments appear in this book.

George S. Fishman

*Chapel Hill, North Carolina  
August 1978*



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