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PREFACE

The 22nd Congress for Applied Mechanics was held at the Science Council of Japan in Tokyo on December 5 and 6, 1972, under the joint sponsorship of the National Committee for Theoretical and Applied Mechanics, Science Council of Japan (NCTAM) and the following nine related societies: The Japan Society for Aeronautical and Space Sciences, the Japan Society of Civil Engineers, the Japan Society of Applied Physics, the Society of Naval Architects of Japan, the Japan Society of Mechanical Engineers, the Architectural Institute of Japan, the Physical Society of Japan, the Mathematical Society of Japan, and the Mining and Metallurgical Institute of Japan.

The Congress was composed of one general and four special sessions. Papers in the field of applied mechanics in general were presented at the general session, while at each of the special sessions a symposium on a selected theme was held and papers on the related subjects were presented. The symposium themes were Stability Problem; Mechanics of Multiphase; Application of Finite Element Method; and Analytical Method in the Field of Applied Mechanics.

After the Congress, fifty-one papers were sent to the Editorial Committee of the NCTAM-22 Proceedings in response to the call for contributions made by the Committee. This volume of the NCTAM-22 Proceedings contains forty-five papers selected by the Committee from those contributed.

Partial financial support was given by the Ministry of Education for the publication of this volume.

March, 1974

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I

STABILITY PROBLEM

Aeroelastic Stability of Structures

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In modern civil engineering structures which are light-weight and slender, the problem of aeroelastic instability under wind action has gained importance. In this paper several causes of such aeroelastic instability, both statical and dynamical, are reviewed.

The steady and unsteady aerodynamic forces acting on bluff sections such as those used in bridges and buildings are discussed first in connection with the possibility of the onset of instability, and typical examples are presented.

The wind-induced behavior of structures referred to in the paper include divergence, lateral buckling, vortex excitation, galloping, stall and coupled flutter. The effects of turbulence in airflow are also discussed. Finally, methods for preventing instability are suggested.

I. INTRODUCTION

Since the old Tacoma Narrows Bridge fell in 1940, the aerodynamic stability of non-aeronautical structures has attracted the attention of researchers. Furthermore, the trends in modern structures, which have become lighter, more slender and more diverse in their configurations due to the use of high-strength materials, advances in fabrication techniques and the development of methods of analysis, have increased the importance of aerodynamic problems in buildings and other structures. The objects of the study are no longer suspension bridges alone, but include cable-stayed girder bridges, steel chimney stacks and hangers of arch bridges, which have sometimes suffered damage or unserviceability due to wind-induced instability.

The behavior of structures subject to wind action may be classified as follows, according to the causes or the nature of the phenomena:

- 1) static behavior (due to steady aerodynamic forces)
 - (a) deformation and stress
 - (b) static instability
 - i. divergence
 - ii. lateral buckling
- 2) dynamic behavior (due to unsteady aerodynamic forces)
 - (a) forced vibrations
 - i. buffeting $\left\{ \begin{array}{l} \text{due to the turbulence of natural wind} \\ \text{due to turbulence in the wakes of windward structures} \end{array} \right.$
 - ii. vortex excitation (aeolian oscillation)
 - (b) self-excited vibrations
 - i. one degree of freedom $\left\{ \begin{array}{l} \text{galloping} \\ \text{stall flutter} \end{array} \right.$
 - ii. coupled flutter (classical flutter)

Among these phenomena, galloping and vortex excitation may not necessarily be distinguishable from one another, and some types of behavior may occur concurrently.

In this paper, static instability, vortex excitation and self-excited vibrations will be discussed in terms of aeroelastic instability.

II. AERODYNAMIC FORCES

The aerodynamic forces causing the wind-induced phenomena classified in the preceding section arise from several sources as discussed below. The difficulties in evaluating these forces acting on buildings and structures lie firstly in that theoretical estimation is almost impossible for bluff-sectional or complicated structures and secondly, that the natural wind contains turbulence.

1. Steady Aerodynamic Forces

Although only the drag component of steady wind force is considered in the design wind loads for most civil engineering structures, the characteristics of lift and moment components can be used to judge the possibility of aerodynamic instability. Figure 1 shows some examples of steady aerodynamic coefficient curves obtained from wind-tunnel tests. The existence of a steep negative slope in the lift (C_L) and moment (C_M) curves indicates the possibility of galloping and stall flutter, respectively.

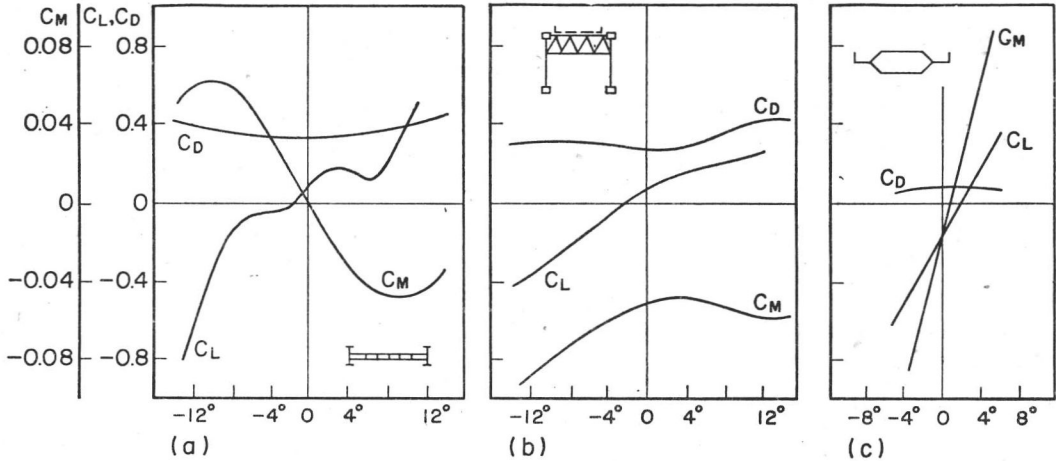


Fig. 1. Steady aerodynamic coefficients.

- (a) old Tacoma Narrows Bridge (Farquharson)
- (b) new Tacoma Narrows Bridge (Farquharson)
- (c) Severn Bridge (Walshe)

2. Unsteady Aerodynamic Forces caused by the Motion of the Body

The lift and moment components of unsteady aerodynamic forces acting on a two-dimensional body vibrating in coupled modes of circular frequency ω in a uniform airflow of velocity V are approximately expressed by¹⁾

$$\left. \begin{aligned} \downarrow L &= \pi \rho b^3 \omega^2 \left[L_\eta \left(\frac{\eta}{b} \right) + L_\phi \phi \right] \\ \curvearrowright M &= \pi \rho b^4 \omega^2 \left[M_\eta \left(\frac{\eta}{b} \right) + M_\phi \phi \right] \end{aligned} \right\} \quad (1)$$

where η and ϕ denote the displacements in vertical and pitching motion, respectively, b is the half-width of the section (see Fig. 2) and ρ is the air density. Because of the phase difference between the motion and the acting force, the aerodynamic coefficients L_η , L_ϕ , M_η and M_ϕ consist of a real and an imaginary part, e.g., $L_\eta = L_{\eta R} + iL_{\eta I}$, and their values are dependent upon the sectional shape, the amplitudes and the reduced frequency

$$k = \frac{\omega b}{V}. \quad (2)$$

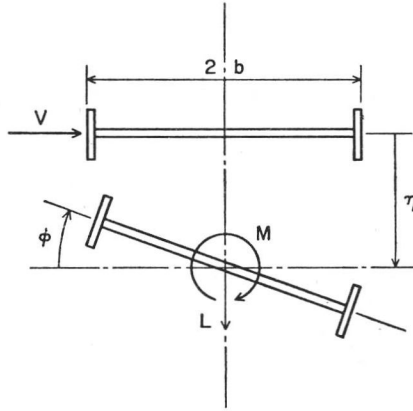


Fig. 2. Motion coordinates for structural section.

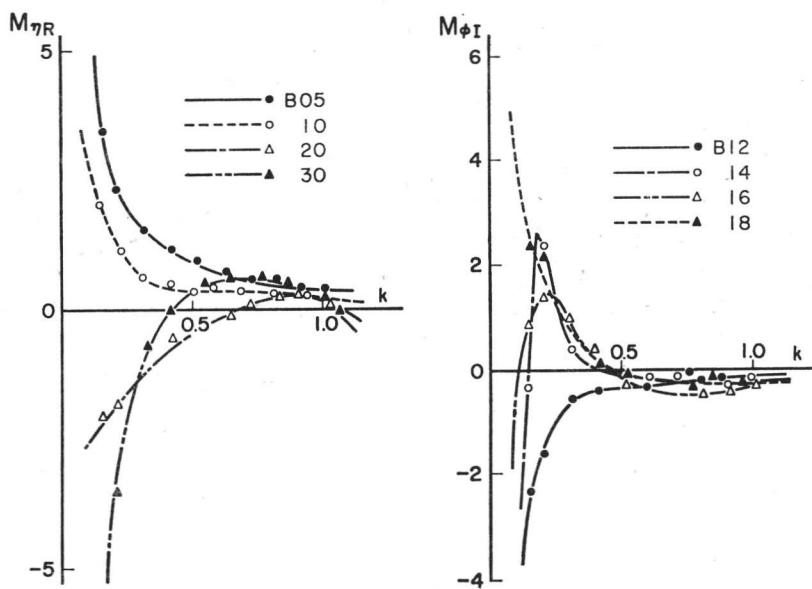
The inertial force of air is usually ignored in discussing the aerodynamic stability of civil engineering structures.

The unsteady aerodynamic coefficients must also be obtained from wind-tunnel tests, except for, for example, a thin, flat plate placed parallel to the airflow, for which a Theodorsen function is applicable. Figure 3 presents some examples of unsteady aerodynamic coefficients for bluff sections. It should be noted that these forces usually have nonlinear characteristics.

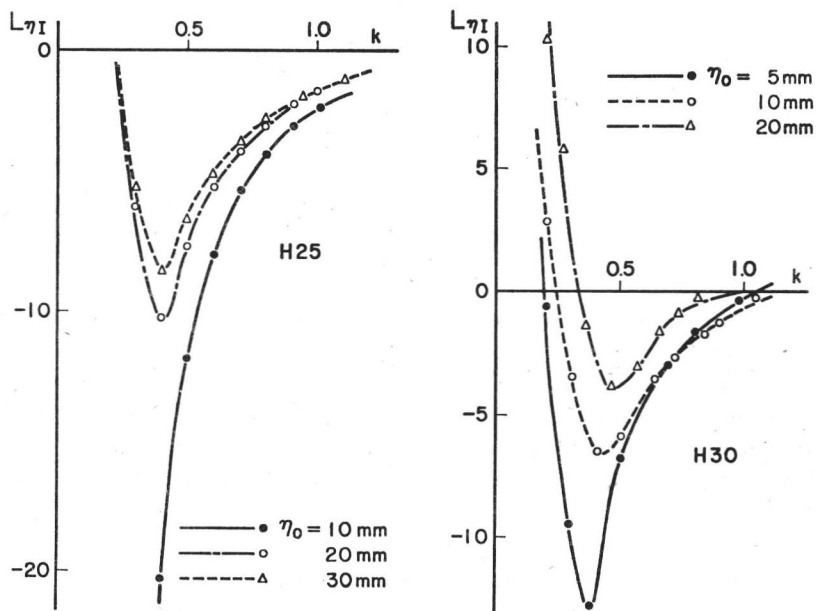
3. Unsteady Aerodynamic Forces due to Vortex Shedding

Vortices shed in the wakes exert mainly alternating lift forces on the structure. The cross-wind force can apparently be written in the form

$$F(t) = \frac{1}{2} \rho V^2 D C_{L'} \cos 2\pi n_s t \quad (3)$$



(a)



(b)

Fig. 3. Examples of unsteady aerodynamic coefficients under horizontal wind (B: box section, H: H-shaped section; the number following indicates the depth-width ratio in %; η_0 : amplitude).

where D is the typical size of the section, C_L' is the nondimensional amplitude of the force and n_s is the frequency of shedding complementary pairs of vortices. However, the quantitative estimation of C_L' is rather difficult, since C_L' is affected by various parameters, such as the amplitudes and the reduced wind velocity. Moreover, once the structure starts to vibrate, the cross-wind forces have a random nature. As seen in Figure 4, the power-spectral density of the fluctuating lift in uniform flow has two peaks,²⁾ corresponding to the structural frequency, n_0 , and the vortex-shedding frequency

$$n_s = \frac{S V}{D} \quad (4)$$

in Eq. (3), where S is the Strouhal number, the value of which is $0.1 \sim 0.2$ depending on the cross-sectional shape and the Reynolds number. The two peaks mentioned above coincide in the locking-in region, where resonance is attained under the condition $n_0 = n_s$.

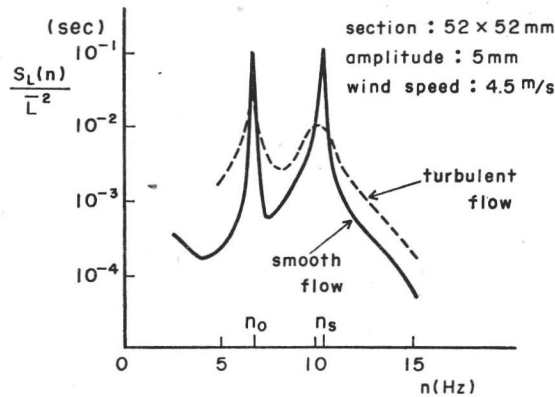


Fig. 4. Power spectra of the aerodynamic lift coefficient of an oscillating square prism in horizontal wind.

4. Effects of Turbulence

The time-dependent and spatial fluctuations of natural wind produce the so-called gust response of structures, but this is not considered in this paper because the phenomenon is not a stability problem.

However, airflow turbulence seems to affect the aerodynamic forces mentioned above, though comparatively little is known about its effects. Sometimes random signals due to turbulence are separated from and superimposed on the unsteady aerodynamic forces, Eq. (1), in smooth flow.³⁾ An experimental result by the authors²⁾ shows that turbulence lowers the power-spectral density but broadens the peak of the spectral curve at the vortex frequency n_s (Fig. 4), and that the turbulence seems not to change the unsteady aerodynamic coefficients substantially (Fig. 5). However, more thorough investigations will be necessary to reach general conclusions.

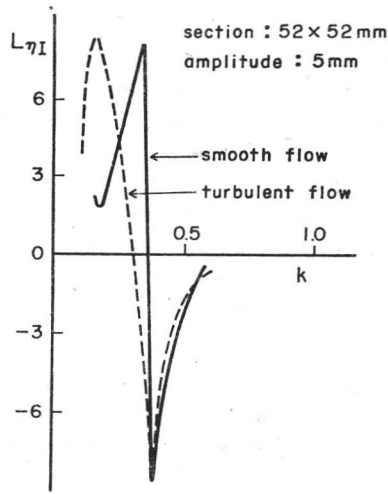


Fig. 5. Unsteady aerodynamic lift force on a square (52×52 mm) prism in horizontal wind (intensity of turbulence: 9%, scale of turbulence: 120 mm, amplitude of motion: 10 mm).

III. STATIC INSTABILITY

1. Divergence

Referring to Figure 6, the aerodynamic moment curve is assumed to be a straight line at small incidence α , that is,

$$C_M = C_{M0} + s_t \alpha$$

where s_t is the slope of the moment coefficient curve. The condition of static equilibrium is then

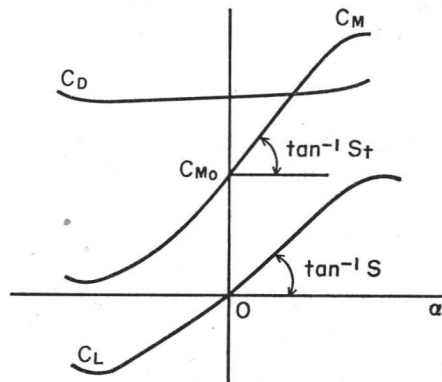


Fig. 6. Idealized steady aerodynamic coefficients.

$$M = \frac{1}{2} \rho V^2 (C_{M0} + s_t \alpha) b^2 = K_\theta \cdot \alpha$$

in which K_θ is the torsional resistance of the structure. Therefore,

$$\alpha = \left(\frac{1}{2} \rho V^2 b^2 \frac{C_{M0}}{K_\theta} \right) \left/ \left(1 - \frac{\rho V^2 s_t b^2}{2K_\theta} \right) \right.$$

and the rotational displacement α diverges if

$$V_D = \sqrt{\frac{2K_\theta}{\rho s_t b^2}} \quad (5)$$

is attained. This is the critical wind velocity for static divergence of the torsional type. A discussion from the viewpoint of vibratory phenomena will be given in V.

2. Lateral Buckling

If the drag component is predominant, and the torsional and flexural rigidity in the cross-wind direction are relatively small, statically coupled instability similar to the lateral buckling of a beam may occur due to the wind forces. This phenomenon was pointed out for the first time in 1942 by Hirai,⁴⁾ who studied stability criteria for coupled oscillations of torsion and vertical bending of suspension bridges.

When one considers a beam with a span length l subject to a horizontal wind, and the steady aerodynamic forces (see Fig. 6) are assumed as

$$D = C_D p b, \quad L = -s p b \phi, \quad M = -s_t p b^2 \phi \quad (6)$$

where

$$p = \frac{1}{2} \rho V^2,$$

the equations of statical equilibrium at the coupled displacements (vertical η and torsional ϕ) can be written as

$$\left. \begin{aligned} EI \frac{d^4 \eta}{dx^4} &= s p b \phi + C_D p b \phi - \frac{d^2}{dx^2} (M_0 \phi) \\ EC_w \frac{d^4 \phi}{dx^4} - GK \frac{d^2 \phi}{dx^2} &= s_t p b^2 \phi - M_0 \frac{d^2 \eta}{dx^2} \end{aligned} \right\} \quad (7)$$

in which EI , EC_w and GK are the flexural, warping and torsional rigidity of the beam, respectively, and the bending moment due to drag force, $M_0 = (1/2) \times (l - x) C_D p b$, is considered as playing a predominant role. Assuming the displacements as

$$\eta = A \sin \lambda x, \quad \phi = B \sin \lambda x, \quad \lambda = \frac{n\pi}{l}$$

and applying Galerkin's method to Eq. (7), the buckling condition becomes