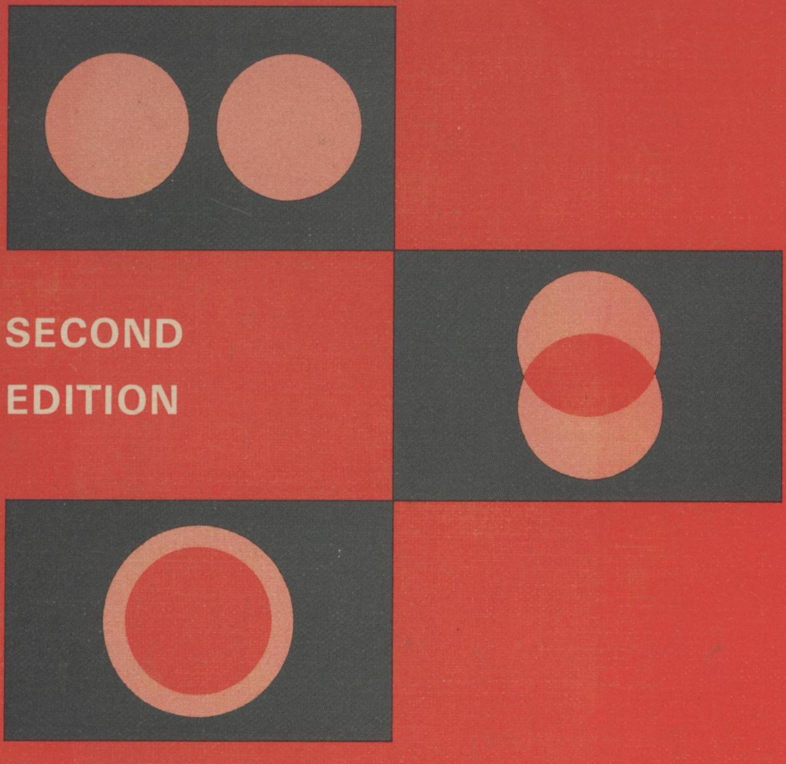
COLLEGE ALGEBRA



SECOND EDITION

Adele Leonhardy

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BY THE SAME AUTHOR

Introductory College Mathematics, Second Edition (1963)

PREFACE

As in the first edition, the purpose of this book is twofold: to promote an understanding of the logical structure of algebra and to develop facility in its essential techniques. The book presupposes completion of a minimum of one and one-half units of high school algebra or equivalent courses offered at the college level.

The two goals are complementary; each reinforces and amplifies the other. Because an understanding of the basic concepts and the logical "pattern" of algebra is one of the chief concerns of this book, emphasis is placed throughout on *why* rather than *how*. This approach leads to greater efficiency in algebraic techniques. In turn, development of the skills of algebra is consistent with, and even necessary to, an understanding of its mathematical concepts and logical structure.

The book is written for the student. In the process of revision, single sentences and sometimes entire sections have been rewritten. In order to clarify the presentation and improve readability, additional explanatory exercises have been added where needed and in a few cases the order of topics has been changed. The exercises that follow the expository material seek to engage the student in actively thinking through what he has read. He may be asked to state a definition in his own words or symbols, to provide proofs for theorems stated in the text or for related theorems, or to compare the various ways in which a given word, such as *equivalent* or *inverse,* is used in mathematics.

In order that the mathematical concepts and principles presented in the context can be brought into full use, the new exercise lists provide much material for the development of algebraic skills. Odd- and even-numbered exercises are quite evenly balanced, and answers to approximately half of the problems are provided at the end of the book. A separate book containing the remainder of the answers is available to instructors from the publisher. For students who need further practice in the fundamental techniques, additional exercises are provided in the appendices at the end of the book. Chapter reading lists suggest material for supplementary work.

The format of the book calls attention to the definitions, postulates, and theorems as they are introduced. The use of a second color points up the logical structure within each chapter and adds to the clarity of the figures. A cumulative summary at the end of each chapter emphasizes the extension of the logical structure that has been accomplished within the chapter. If further study is needed, a section number beside each item in the summary provides a ready reference to the section in which the item is first introduced.

In developing the materials that will realize the two goals of the book, the recommendations of the various curriculum study groups have been followed. The assistance of a number of reviewers was also sought. In final analysis, however, the book represents my own ideas and experience as to what students need in mathematics and what will provide a more modern approach. Much of the new material of the book has been tried out at Stephens College and revised on the basis of our experience with it.

The treatment of the logical structure of algebra presented here is compatible with the level of development of the student. Rigor is relative, and complete logical rigor is neither possible nor desirable at this point. Hence the presentation is introductory and not exhaustive. The student gains a respect for precise definitions and an appreciation of the need for stating the underlying assumptions of a mathematical system. He sees how the resulting deductive structure is built on this foundation. Thus he increases his understanding of the mathematical method and of the nature of mathematical systems and models.

In keeping with the present trend, the use of symbols has been increased somewhat in the second edition. However, the notation is relatively simple, and the extension is made with a view toward clarifying ideas. For example, throughout the book emphasis is placed on the correspondence between "and" and the intersection of two sets ($\wedge$ and $\cap$) and between "or" and the union of the two sets ($\vee$ and $\cup$).

The concept of sets and operations with sets is introduced in the first chapter, and these ideas are carried along in later chapters wherever they clarify and extend the student's knowledge of algebra. The theory of sets adds materially to understanding and skill in working with numbers in the real and complex number systems, in the solution of equations, inequalities, and systems of equations, and to the clarification of the concept of functions and relations, of permutations and combinations, and of the theory of probability. The theory of sets is not relegated to a single chapter and then laid aside. It lends a modern flavor to the entire book, but it is used as a means to an end rather than as an end in itself.

The separate chapter on Groups and Fields has not been included in this edition. A section on Field Postulates for the Real Numbers appears in Chapter 4.

A short chapter on Circular and Trigonometric Functions has been added.

Its purposes is to round out the presentation of transcendental functions and to present the circular functions as functions whose domain is the set of real numbers. The sections on operations with complex numbers in polar form have been incorporated in this chapter.

The book is flexible enough to permit variation among groups of different abilities, interests, and mathematical backgrounds. It is intended as material for a course meeting from three to five times a week for one semester or for two three-unit quarter courses. It may serve, also, as preparation for more advanced courses in mathematics.

For those students who have been away from mathematics for some time, a basic course might include the following:

Chapter 1. *The Nature of Mathematics.* In this chapter the student is introduced to deductive reasoning, some of the laws of logic, and sets and operations on sets.
Chapter 2. *The Set of Integers*
Chapter 3. *The Set of Rational Numbers*
Chapter 4. *The Set of Real Numbers*
Chapter 5. *The Set of Complex Numbers* (Omit Section 4)
Chapter 7. *Functions and Relations and Their Graphs* (Omit Sections 12 and 13)
Chapter 9. *Systems of Equations*

Other chapters may be included if time permits. Attention is called to the fact that Chapter 6 on mathematical induction may be omitted without loss of continuity in the development.

For students who have recently completed two years of high-school algebra, Chapters 1 through 5 may serve as a brief but thorough review. Special emphasis in these chapters should be placed on the logical structure of algebra and exercises directed toward this goal.

The major part of the semester may then be devoted to Chapters 7 through 9 and to other suitable chapters selected from the remainder of the book. Following is a list of the remaining chapters together with brief comments about them. Any one of these chapters may be omitted without destroying the continuity of presentation.

Chapter 6. *Mathematical Induction.* This is a short chapter presenting mathematical induction as a form of deductive reasoning.

Chapter 10. *Matrices and Determinants.* If time does not permit the use of the entire chapter, the footnotes suggest possible choices from the chapter. Matrices have been given a prominent place because of their many applications in modern mathematics.

Chapter 11. *The Theory of Equations.* This chapter is included primarily for those who will specialize in mathematics but may be omitted or used for future reference if needed.

Chapter 12. *Exponential and Logarithmic Functions.* Here the teacher may exercise choice as to how much to include in theory and number of applications.

Chapter 13. *Circular and Trigonometric Functions.* This chapter emphasizes circular functions as functions of real numbers and applies trigonometric functions to operations with complex numbers in polar form.

Chapter 14. *Permutations, Combinations, and the Binomial Theorem*

Chapter 15. *Theory of Probability.* This chapter presupposes a knowledge of combinations from Chapter 14.

If possible, grouping students in sections according to their mathematical proficiency is desirable. If this is not feasible, provision for differences may be made by giving assignments for different levels or by assigning certain of the later chapters to the more able students.

I wish to express my appreciation to the students, teachers, and reviewers whose suggestions have been invaluable in the preparation of this new edition.

Adele Leonhardy

Columbia, Missouri
December 1967

COLLEGE ALGEBRA

CONTENTS

1
THE NATURE OF MATHEMATICS

1. COLLEGE ALGEBRA

Does the term *college algebra* refer merely to any algebra that is studied in college, or does it refer to a particular section of the large field of algebra? Actually, some courses entitled college algebra are offered in high school, and some courses in algebra offered in college are not called college algebra. Obviously, then, the term college algebra does not merely mean algebra that is studied in college.

College algebra does, in a rather loose sense, refer to a certain body of algebra, but the exact starting point and the content of the course are determined for each class largely by the abilities and needs of its members and their background in mathematics. Because many students in college algebra plan to continue in mathematics, engineering, or other related fields, the student should leave the course proficient in the skills and techniques of algebra. To insure a proper start, a review of the basic skills of high school algebra is necessary, bringing ourselves up to date, as it were. This review is then followed by a study of new concepts and the development of additional skills and techniques, thus enlarging our understanding of algebra and extending our ability to use it. In this sense, college algebra is a tool subject to later courses in mathematics and to other fields to which it may be applied, such as science, engineering, and business.

A second and most important goal of college algebra, as we see it, is to develop an understanding of the nature of algebra—and of mathematics in general—and a knowledge of *why* we do things as we do. In other words, we are not interested alone in developing skills, but we consider a knowledge of underlying principles essential to efficient performance of the techniques of algebra and to the further study of mathematics. Consequently, at the beginning of this course we shall review some of the algebra that you have studied previously, but our approach will be from a new and more advanced standpoint. In order to do this, let us first look at the question: What is mathematics?

2. MATHEMATICS AS AN INVENTION

At the outset, we must make clear that mathematics is something man has *invented* and not something he has *discovered*. Number systems grew out of primitive man's need for counting, and, because he counted on his fingers, the number ten is the base of the Hindu-Arabic number system, which we use. Numbers other than ten have been used as the bases of number systems. The early Sumerians and Babylonians used the base sixty. A residual of this base is found in our division of the hour and of the degree into sixty minutes and division of the minute into sixty seconds. The Mayans of Central America, who made considerable progress in mathematics and astronomy, used a number system with a base of twenty. Other ancient tribes, and some primitive tribes of today, have used five as their number base.

But now that the Hindu-Arabic system is the established number system, you will say, "Isn't it an absolute truth that ten plus four is fourteen?" But, is it? On a clock, ten plus four is two. Obviously, the arithmetic on a clock is a different system of arithmetic.

Similarly, there are different algebras and geometries, most of them invented in the last one hundred fifty years. Others may be invented in the future, the number being dependent solely on the ingenuity and creativeness of man. No practical uses for many of these new systems of mathematics were known at the time of their development, but applications for many of them were found later. Thus some mathematical tools were invented in advance of the need for them. In other cases mathematicians have been asked to develop a system of mathematics to fit the needs of a particular field of business, industry, or natural or social science.

Classical algebra, the algebra that you studied in high school, is the algebra of numbers. For example, in ordinary algebra x times y means that two numbers x and y are multiplied. In the modern algebras the letters, or elements, may mean anything we wish and the operations whatever we specify. In one of the algebras x plus x is x and x times x is equal to x. This is a very useful algebra, but, as you have already guessed, in this algebra x does not represent a number, and "addition" and "multiplication" have meanings different from those used in ordinary arithmetic and algebra.