EUROMECH 280

EDITED BY
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PROCEEDINGS OF THE INTERNATIONAL SYMPOSIUM ON IDENTIFICATION OF NONLINEAR MECHANICAL SYSTEMS FROM DYNAMIC TESTS – EUROMECH 280
ECULLY / FRANCE / 29 - 31 OCTOBER 1991

Euromech 280

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Ecole Centrale de Lyon & Ecole Nationale des Travaux Publics de l'Etat, France

A.A. BALKEMA / ROTTERDAM / BROOKFIELD / 1992

The texts of the various papers in this volume were set individually by typists under the supervision of each of the authors concerned.

Published by
A.A. Balkema, P.O. Box 1675, 3000 BR Rotterdam, Netherlands
A.A. Balkema Publishers, Old Post Road, Brookfield, VT 05036, USA

ISBN 90 5410 059 1

Printed in the Netherlands

EUROMECH 280

EUROMECH 280, Jezequel & Lamarque (eds) © 1992 Balkema, Rotterdam. ISBN 90 5410 059 1

Scope of the Symposium Euromech 280

Euromech 280 provides an opportunity for discussions of the problems raised by the analysis and identification of nonlinear mechanical systems. Indeed, the origin of nonlinearities in the field of mechanical engineering are numerous:

- Large deflections of beams and plates;
- Plastic and viscoplastic behaviour of materials;
- Dry friction and gaps at the interfaces;
- Inertial nonlinearities and gyroscopic effects;
- Impact phenomena.

The introduction of nonlinear terms in the equilibrium equations induces complicated dynamic behaviour even in the case of low degrees-of-freedom systems:

- Subharmonic and superharmonic responses;
- Internal resonance occurrence;
- Nonlinear instabilities;
- Chaotic behaviour.

When the origin of the nonlinearities is not clearly known, the experimental identification of mechanical systems needs non-parametric modelling:

- Pseudo force methods;
- Equivalent linearization procedures;
- Volterra and Wiener series;
- Chronological series (NARMAX, ...);
- Multi-frequency transfer functions.

When the origin of the nonlinearities are well-known the analysis of the dynamic behaviour of the mechanical systems is linked to the identification of parameters obtained from dynamic tests. This strategy often leads to an inverse problem in order to determine the parameters of interface and material behaviour from dynamic structural responses.

So the main topics in these proceedings are:

A. Non-parametric modelling

- Detection of nonlinearities of mechanical systems from the analysis of dynamic responses.
- Extension of the modal synthesis to the nonlinear case.
- Non-parametric models in the time domain and the frequency domain.
- Modelling of nonlinear systems based on the pseudo force method and linearization procedures.

B. Parametric modelling

- Identification of physical parameters from transient and stationary responses.
- Dynamic behaviour of mechanical systems with friction.
- Stability of self-excited systems.
- Identification of nonlinear model from impact tests.

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3 *Nonlinear behaviour and chaotic motions*

1 Parametric identification

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Influence of nonlinearities on modal tests

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1 INTRODUCTION

There are a number of different strategies which can be adopted by the experimental structural dynamicist when confronted by non-linear behaviour of a test structure. One approach is to address the non-linear effects directly, and to seek a detailed mathematical description of their behaviour. This is effectively the only course when the non-linearity is 'strong'. Alternatively, one can ignore the warning signs of nonlinear behaviour altogether and to carry on with the tests regardless. This is, perhaps, imprudent and, anyway, is unnecessary today. Between these two extremes, there is a third approach that seeks to use 'standard' or near-standard modal testing techniques in such a way as to extract as useful a model of the structure as is possible in the circumstances of its non-linear behaviour. This approach - which is the basis of this paper - is often applicable in cases of weak non-linearities, such as are observed in many practical engineering structures. Thus, this paper will address two specific questions, namely:

- what happens when "standard" modal testing measurement and analysis methods are used on a slightly non-linear structure? and - what can be done with "standard" (or near-standard) modal analysis methods to cope with, even identify, slightly non-linear structures?

The paper will not address the special methods (ie non standard 'modal' test procedures) which have been developed recently to study non-linear structures, and especially those with strongly non-linearities, in more detail (Hilbert Transforms; Higher-order FRFs; Volterra series;...).

Between them, these approaches represent two fundamentally different strategies for dealing with non-linearity: (a) "learning to live" in a world with non-linearities when using conventional modal testing methods, without being unduly misled by the effects; or (b) needing to identify and model such effects in detail in order to make a full analysis of the behaviour. At the same time as the first approach is inadequate for the aspirations of the second, so also is it inappropriate to use the much more complicated, and costly, advanced techniques in (ii) for applications where detailed description of the non-linear effects are not required.

It must be acknowledged that once a non-linearity comes into play in the dynamic behaviour of a structure, then that structure ceases to possess normal modes of vibration as we usually define them. This need not deter us from using modal testing and analysis techniques, provided we are sensitive to the scope and lirnitations of our methodology and realistic in our expectations for the results we obtain. There are a number of results we can realistically seek from the application of conventional modal testing techniques to structures with some non-linearity, and these include:

- detection of non-linear behaviour;
- indication of non-linearity type;
- indication of location of non-linearity(ies); all of which might help to eliminate avoidable non-linearities (loose components, joints etc.) prior to conducting a full modal test. In addition,
- identification of the modes of the linear part of the structure;
- identification of the modes of a linearization of the test structure;
- representation of the actual response characteristics of the structure, even if these cannot effectively be described in conventional modal terms.

It is clear that these goals fall into the first of the two strategies above, being relatively unconcerned with the non-linearity itself and it is this area which will be the focus of attention in this paper.

2 SOURCES OF NON-LINEARITY

It is appropriate to begin with a brief summary of the sources of non-linearity encountered in modal testing and, in particular, in tests which do not set out expressly and consciously to study and analyse non-linearity. It is assumed throughout that the types of non-linearity of interest in this context are generally "weak" non-linearities, referring to a degree of

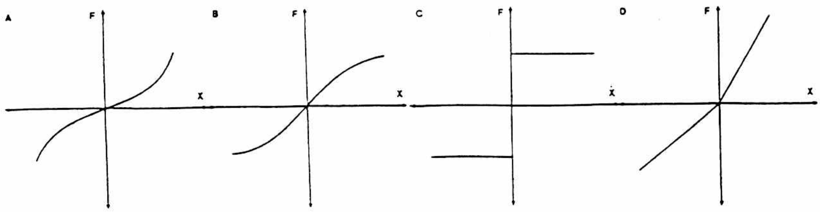


Figure 1. Basis of various non-linear effects encountered in modal tests

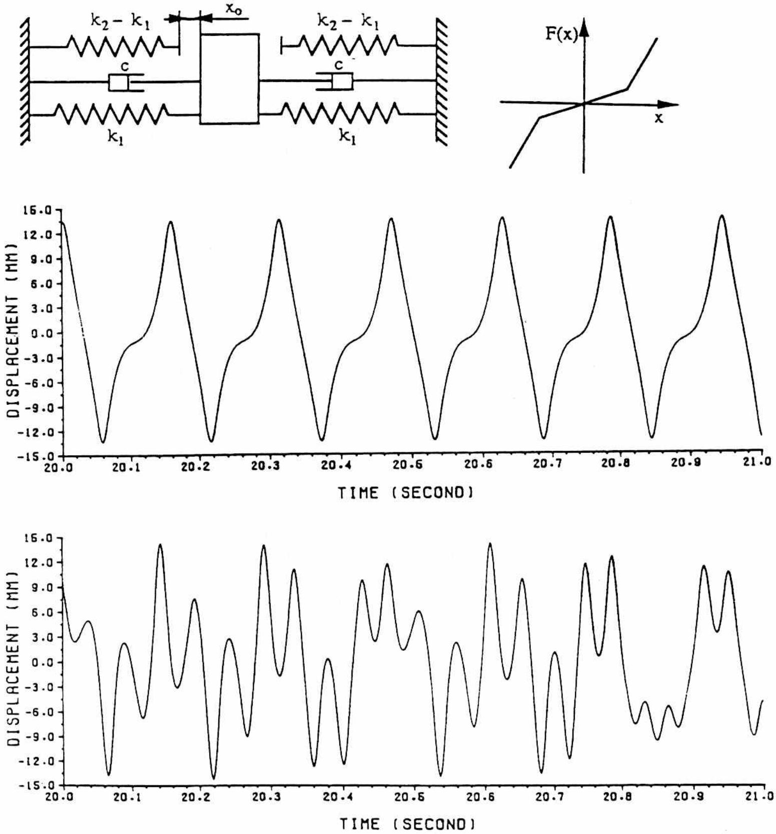


Figure 2. Time histories of non-linear system response to sinusoidal excitations

departure from linearity which is slight to modest.. It is unlikely that a strongly non-linear structure would be subject to a standard modal test.

Some typical features of real structures which give rise to this level of non-linearity include the following, some of which are illustrated by F-x, or F-v characteristics in Fig. 1:

(i) amplitude-dependent stiffness, such as is caused by tension effects in addition to bending (i.e. a hardening stiffness) or a loss of rigidity due to opening of joints;
(ii) dry friction-related damping;
(iii) bilinear stiffness: different stiffnesses in tension and compression loading d irections;
(iv) backlash, and other forms of small-gap clearances in structural joints; (v) non-linear damping, e.g. (velocity)$^{(2)}$ - dependent.

3 MODAL TEST MEASUREMENTS ON NON-LINEAR STRUCTURES

There are two steps in a modal test which are of interest to us here: (i) **measurement** of the response functions which form the basis of the test, and (ii) analysis of these functions to extract the structure's modal parameters. We shall consider the first of these in this section.

Most modal tests use measurements of FRF data, obtained using sinusoidal, periodic, transient, random or burst excitation signals. For linear systems, all of these excitations are theoretically capable of yielding the same FRF properties, but in practice are subject to certain limitations in the means used to measure and analyse the signals with conventional signal processing instrumentation. For non-linear systems, however, the different excitation signals do not yield identical response functions, each reacting differently to the complex characteristics of the system under test. Inevitably, therefore, different modal models will be derived following different test procedures - a not-unknown, but relatively unsatisfactory state of affairs.

Arguably the best choice of excitation signal to investigate this characteristic of nonlinear structures is the sinusoidal (or 'harmonic') excitation. It is known, both from theoretical analysis and practical experience, that a sinusoidal excitation generally produces a periodic response. Although there are situations where the response is not periodic but, instead, is chaotic, the response is heavily influenced in most cases by the frequency component at the excitation frequency, with components at multiples or fractions of this frequency also present - occasionally as significant components. Fig. 2 shows some illustrations of responses to harmonic forces. It is convenient to extract the fundamental component of the response and to relate it to the excitation level to yield the (first-order) frequency response function.

This FRF is just what is measured by a true sinusoidal excitation test, such as provided by a Frequency Response Analyser, and is closely related to the corresponding quantity yielded by an harmonic balance theoretical analysis. Some examples of FRF data obtained by both routes (experimental and theoretical) are illustrated in Fig. 3, using modulus-frequency (Bode) and modulus-phase (Nyquist) plot formats. The nonlinearity is most clearly evident in the different curves produced by different levels of excitation: the distortion in any one curve (by comparison with the shape expected for a linear system) is less unambiguously an indication of non-linear behaviour **unless** it is known that each 'resonance' region harbours only a single mode of vibration.

An important feature of all tests on non-linear systems is the need to record not only the response/excitation ratio but also the individual levels. Indeed, it may well be necessary to control these levels directly. In the case of sinusoidal excitation, if the *excitation* level is kept at a constant amplitude then the response level will vary as the excitation frequency passes through a resonant region and, as a result, any amplitude-dependent properties (stiffness and/or damping) will vary, thereby activating the non-linear effects we have described. Constant-excitation (level) tests therefore will bring out non-linear effects clearly.

On the other hand, if the excitation is adjusted such that the *response* level is constant at every frequency of measurement (around a resonance), then the behaviour of the structure will have been linearised since the various amplitude-dependent physical parameters will have been constrained to exhibit constant values in that region. It must be noted here that in the case of a practical (multi-degree of freedom) structure it is only feasible to control the amplitude of vibration at one point. The inevitable possibility of varying amplitudes at other points on the structure means that an exact linearisation is not necessarily attained in this way, although for a narrow frequency band around one resonance it is likely to be a good approximation.

The FRF data shown in Figs. 4 (a) and (b) were obtained for different excitation (and response) levels; those in (a) under constant excitation conditions, while some data measured under constant-response conditions are shown in Fig. 4. (b). It should be noted that, at a practical level, the control of either excitation force or response displacement to a prescribed level is often very difficult to implement because nonlinearity in the test structure can cause the level regulation or control loop to become unstable under some conditions.

Each of the other FRF measurement techniques presents all the information contained in the response signal, including the multiple-frequency components generated by each component of the excitation. It is thus very difficult, if not impossible, to relate correctly the individual response components to the corresponding excitation components, even though the calculations can be made easily. Thus-produced 'FRF' plots are inevitably more difficult to **analyse** and interpret usefully, being distorted (as the single-frequency sinusoidal case, above) but in a very complicated way. In some cases, the resulting FRF plots around each 'resonance' are relatively undistorted and represent the behaviour of an equivalent linearised system, Fig. 5. Of course, the result is excitation-level (and shape) dependent, the linearised FRF curve at a high level of excitation being different to that for a low-level excitation (Fig. 5). Such FRF data have the feature (advantage?) that they are more amenable to modal analysis by conventional linear algorithms without displaying the difficulties encountered on sinusoidal-excitation results (see below), although close scrutiny of the measured data often reveals distortion in the

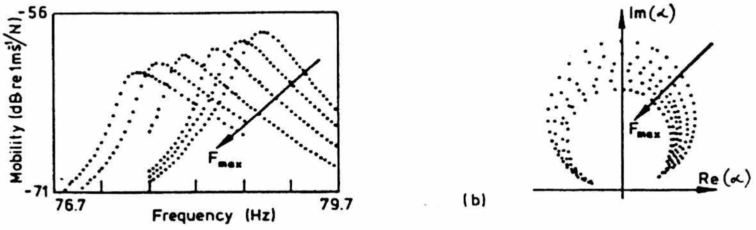


Figure 3. FRF measured on aerospace structure using harmonic excitation at different levels

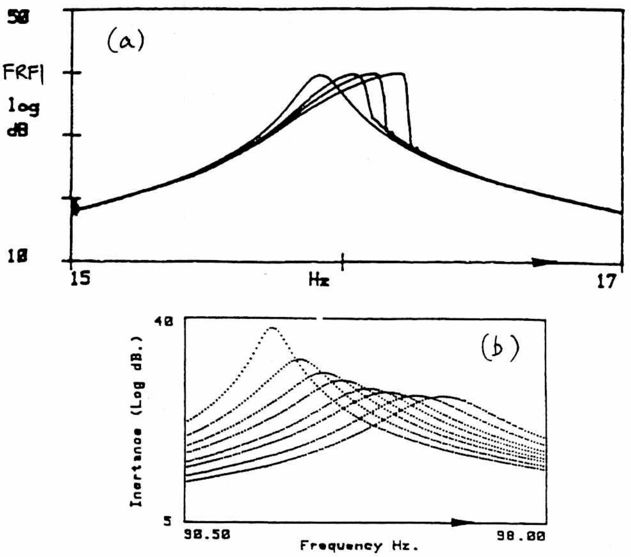
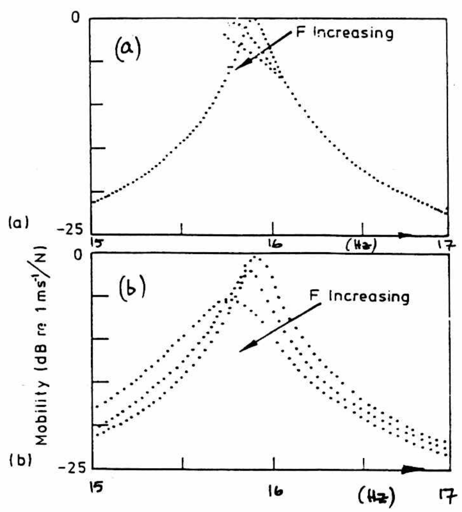


Figure 4. First-order FRF measured using sinusoidal excitations under (a) constant-excitation levels; (b) constant response levels



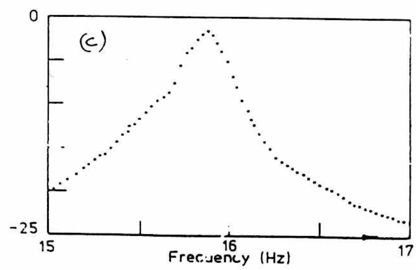


Figure 5. FRF measurements on analogue computer system using
(a) sinusoidal excitation,
(b) random excitaion,
(c) transient excitation

immediate vicinity of the resonance frequency.

It is thus clear from the above discussion that different measurement techniques yield different FRF curves for a non-linear structure and also, that a given excitation signal produces different curves when applied at different levels. None of these variations are found on linear structures. The sinusoidal excitation signal probably represents the best choice when some information concerning the non-linearity is required, while random excitation may provide the least-troublesome approach when conventional modal analysis procedures are to be applied to the measured data. In any event, except where a detailed model of the non-linear behaviour is sought, a proper awareness of non-linear effects is appropriate as is the use of measurement techniques which will result in the minimum interference in the modal test processes by the non-linearities.

4 MODAL ANALYSIS OF NON-LINEAR SYSTEM FRF DATA

The second of the two stages of modal testing of interest here is the (modal) analysis of measured FRF data, undertaken in order to determine the structure's modal parameters. We shall divide this section into two parts: first, examining the outcome of a 'blind' use of some standard modal analysis methods and second, exploring the possibilities for alternative usage of these analysis procedures in order to extract some useful (if incomplete) information about the non-linearity.

4. 1 Standard Use of Modal Analysis Procedures

First, we can record the results of applying a standard circle-fit SDOF analysis to typical data from a non-linear system. The method used [1] provides a detailed check on the quality of the analysis results by scrutiny of a 'damping plot' and is able to indicate clearly the presence of distortions in the FRF data plot - whether these originate from non-linear or other effects. One major feature of this analysis is the tendency to overestimate considerably the extent of modal complexity (implying particular damping distributions). FRF data measured using sinusoidal excitation are illustrated by the results shown in Fig 6 and include examples of clearly-evident non-linearities, Figs. 6 (a) and (b), as well as those in which the non-linear characteristics have been suppressed by making the FRF measurement under constant-response conditions, Fig. 6 (c) and (d).

Next we consider an alternative SDOF approach which uses the reciprocal-FRF format as the basis of analysis [2]. This approach has the feature that **data points** around the resonance peaks become the least important of those used in the curve-fit procedure (by virtue of the reciprocal function used) and, consequently, the modal analysis concentrates on the data points furthest from the resonance frequency for which the actual response levels are generally low and the non-linear effects least prominent. As a result, modal analysis undertaken by this approach tends to yield the low-level (often the linear part of the) behaviour of the structure and to be relatively insensitive to strongly amplitude-dependent non-linear effects.

A third illustration of the application of a conventional modal analysis to a nonlinear system is illustrated in Fig. 7 and relates to the application of the Rational Fraction Polynomial MDOF method of analysis to data measured on a lightly-nonlinear system. The original measured FRF curve and the successful curve-fit are shown in Figs. 7 (a) and (b). The result was obtained by the identification of several 'modes', although only one degree of freedom is present in the (admittedly nonlinear) system. Here, the response of the modal analysis procedure to the non-linear system data is to compensate for the distorted FRF characteristic by adding computational modes to the identified model. In more complex situations, where it is not known that only a single mode is present, such an outcome would be more difficult to detect.

4. 2 Modified Modal Analysis Methods

Next, we consider the possibility of adapting any of the existing modal analysis methods, such as those mentioned above, to yield more explicit information on the linear and non-linear components of the structure under study. Although the circlefit analysis described above is effective at detecting the presence of non-linearity in a test structure, it is not amenable to providing more precise information, such as the type, extent or location of the non-linear element(s). The so-called 'Inverse Method' does have some possibilities, however. For a single degree of freedom system, the pair of plots Re $(H^{-1}(\omega)$ vs ω^2 and $Im(H^{-1}(\omega)$ vs ω should both yield straight lines in the region of resonance, so long as the system parameters are constant. Failure to satisfy this condition, such as occurs during variable-displacement tests on non-linear systems, will cause one or both of the aforementioned plots to be curved and the form of such curves can be analysed to yield information on the non-linearity in the form of $k_e(x)$ and $C_e(x)$ where k_e, C_e are the effective stiffness and damping coefficients, and x represents the amplitude of vibration displacement. The basis of the analysis of this form is illustrated in Fig. 8 where, using only the conventional FRF data measurement, an indication of the effective non-linearity can be extracted from the equivalent stiffness of the system at each individual measured frequency. Of course, this result does not indicate, nor permit the determination of, the actual type of nonlinearity present - only its effect on the steady-state response of the structure.

One limitation of this method is an implicit assumption, in addition to those applicable to all

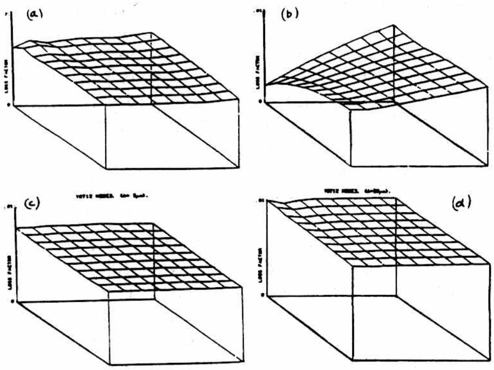


Figure 6. SDOF Circle-fit modal analysis of aerospace structure FRF data: (a) constant force - low level; (b) constant force - high level; (c) constant response - low level; (d) constant response - high level

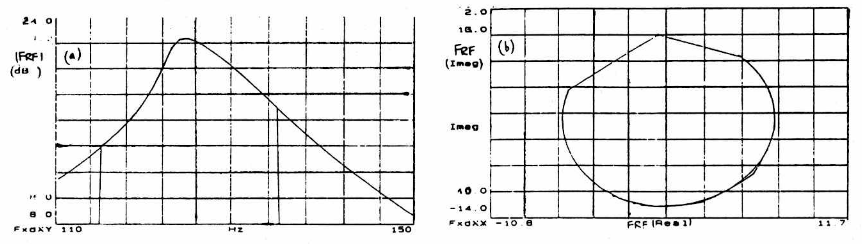


Figure 7. MDOF Rational Fraction Polynomial modal analysis of non-linear circuit (a) Bode plot; (b) Nyquist plot

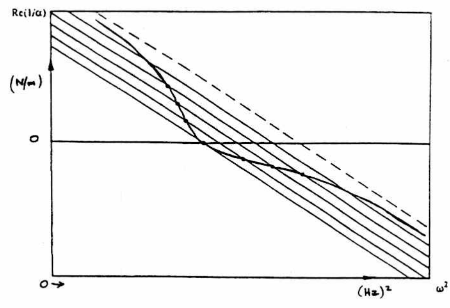


Figure 8. Basis of Inverse-Receptance analysis: (straight lines represent linear systems with different stiffnesses)

SDOF analyses, that the modal constants (mode shapes) of the structure are all real. This is not always realistic and so restricts the usefulness of the inverse method. An alternative approach has been proposed [3] in which pairs of FRF points are taken from the same FRF plot, each pair comprising one point below resonance together with another above **resonance with both sharing the same response level** (so that they both relate to the same effective stiffness and damping properties). Analysis of these FRF values permits estimation of the modal properties which apply to that response level. Repeated application using different pairs of points across the resonance region can yield indications of the variation of system properties with vibration amplitude, as before. Some results obtained in this way are shown in Fig.9, both for an electronic circuit, Fig.9 (a), and for a practical aerospace structure, Fig.9 (b).

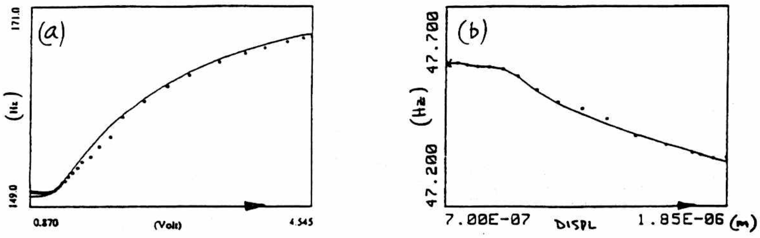


Figure 9. Effective Stiffness estimates from non-linear systems using both Inverse-Stiffness (——) and Two-point analysis (····) methods (a) non-linear electronic circuit; (b) aerospace structure

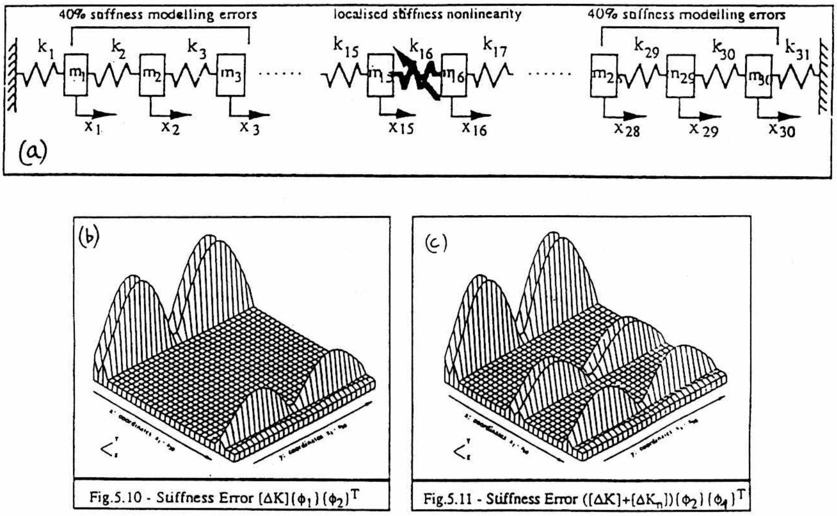


Figure 10. Location of non-linear elements in MDOF system using modal test data (a) Model; (b) Location of modelling errors + non-linear elements; (c) Location of modelling errors + non-linear elements

In summary, it is seen that proper use of conventionally-measured FRF data, and associated modal analysis methods, can both detect and help avert the influences of non-linearities. It is also seen that a start can be made on identification of the nonlinear effects, without the use of advanced or non-standard analysis processes, although the scope of such identification is limited to determining the **effective** stiffness and damping properties. If these are sufficient for the eventual application of the model which is sought from the test, then this is an appropriate approach to take.

5. LOCATION OF NON-LINEARITIES

We shall conclude this survey of modal testing and non-linear structures with a mention of the applicability of some of the recently-developed techniques for model updating. The essential purpose of model updating is to locate, and then to correct, differences between an analytical model of a structure and the 'true' model, based on measurements of the structure's vibration properties. The interest here lies in the possibility of adapting the usual Analysis/Test comparison to include a Test/Test comparison where data from the two tests relate to different amplitudes. Thus, a test conducted at a low response level could be compared with one undertaken at a high response level in order to identify which stiffness and/or damping elements in the structure cause the recorded differences between the test results by virtue of their amplitude-dependent characteristics.

Most model updating methods are based on a direct comparison of the predicted and measured modal properties [4], but at least one exists based on FRF data (instead of modal data) [5]. Both types can be applied in the case of non-linear behaviour although the latter, FRF-based, approach is the more suitable (indeed, the method in [5] was developed specifically for non-linear applications).