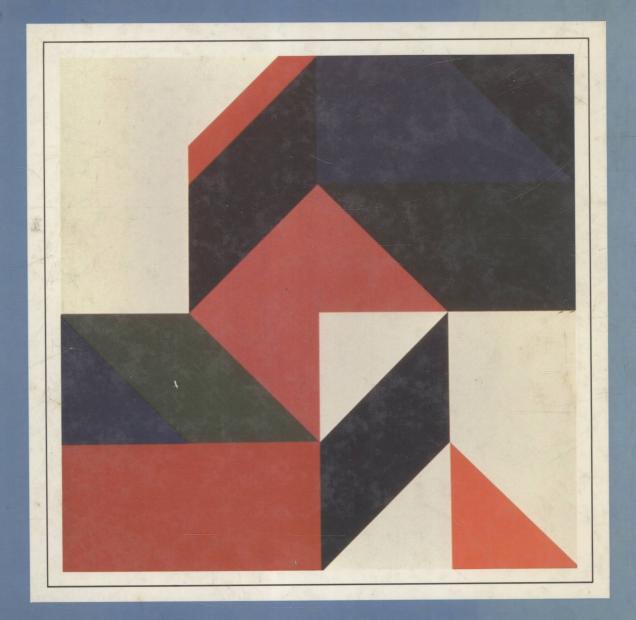
## UNDERSTANDING STATISTICS

Fifth Edition



OTT MENDENHALL

## FIFTH EDITION



# UNDERSTANDING STATISTICS

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A partially programmed study guide with self-correcting exercises has been developed to assist you in mastering difficult statistical concepts you may encounter in this text. It is available from your local bookstore under the name Study Guide for Understanding Statistics, 5th Edition, by Barbara and Robert Beaver.

### **Key to Exercise Symbols**

4	m	ga	-
d	"	b.	
-	-	-	

environmental studies



engineering/technical



business/economics



political science/ government



© agriculture



Population mean

Sample mean

Population variance

Sample variance

Population standard deviation

Sample standard deviation

Population proportion (binomial distribution)

Sample proportion

Confidence interval for  $\mu$ , σ known

Statistical test of  $H_0$ :  $\mu = \mu_0$ ,  $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$ σ known

Confidence interval for  $\mu$ ,  $\bar{x} \pm t \frac{s}{\sqrt{s}}$ σ unknown

Statistical test of  $H_0$ :  $\mu = \mu_0$ , σ unknown

psychology



education



public opinion



medicine

 $s^{2} = \frac{\sum (x - \bar{x})^{2}}{n - 1} = \frac{1}{n - 1} \left[ \sum x^{2} - \frac{(\sum x)^{2}}{n} \right]$ 

biology

🗯 geology

physics \*\*

law

 $\pi$ 

 $\hat{\boldsymbol{\pi}} = \frac{x}{n}$ 

 $\bar{x} \pm z\sigma_{\bar{x}}$ 

Confidence interval for 
$$\mu_1 = \mu_2$$

$$\bar{x}_1 - \bar{x}_2 \pm t s_\rho \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Statistical test of 
$$H_0$$
:  
 $\mu_1 - \mu_2 = 0$ 

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\rho\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Confidence interval for  $\pi$ 

$$\hat{\boldsymbol{\pi}} \pm z \sigma_{\hat{\boldsymbol{\pi}}}$$

Statistical test of 
$$H_0$$
:  $\pi = \pi_0$ 

$$z = \frac{\hat{\boldsymbol{\pi}} - \boldsymbol{\pi}_0}{\boldsymbol{\sigma}_{\hat{\boldsymbol{\pi}}}}$$

Confidence interval for  $\pi_1$  –  $\pi_2$ 

$$\hat{\boldsymbol{\pi}}_1 - \hat{\boldsymbol{\pi}}_2 \pm z \boldsymbol{\sigma}_{\hat{\boldsymbol{\pi}}_1 - \hat{\boldsymbol{\pi}}_2}$$

Statistical test of 
$$H_0$$
:  $\pi_1 - \pi_2 = 0$ 

$$z = \frac{\hat{\boldsymbol{\pi}}_1 - \hat{\boldsymbol{\pi}}_2}{\sigma_{\hat{\boldsymbol{\pi}}_1 - \hat{\boldsymbol{\pi}}_2}}$$

Test of 
$$H_0$$
:  $\sigma^2 = \sigma_0^2$ 

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Test of 
$$H_0$$
:  $\sigma_1^2 = \sigma_2^2$ 

$$F = \frac{s_1^2}{s_2^2}$$

Least squares line

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

Slope of least squares line

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$$

Intercept of least squares line

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \, \bar{x}$$

Test of  $H_0$ :  $\beta_1 = 0$ 

$$t = \frac{\hat{\beta}_1}{\sqrt{s_{\epsilon}^2/S_{rr}}}$$

Population correlation coefficient

Sample correlation coefficient

$$\hat{\rho} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

Paired-difference test

$$t = \frac{\overline{d}}{s_d/\sqrt{n}}$$

Chi-square test of independence

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Spearman's rank correlation coefficient

$$\hat{\rho}_s = 1 - \frac{6(\Sigma d^2)}{n(n^2 - 1)}$$

## UNDERSTANDING STATISTICS

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### **PREFACE**

This text is designed for a one-quarter or one-semester introductory course in statistics. It can be used in those colleges and universities that teach a general course in statistics appropriate for students majoring in, or intending to major in, many different areas. The approach, examples, and exercises provide a basic knowledge of statistical concepts that will be useful in business and in the biological, social, and physical sciences.

The focus of this edition is similar to that for the fourth edition. One of the primary objectives of an introductory statistics course is to develop in the student an understanding and appreciation of the role of statistics in society. The fifth edition of Understanding Statistics emphasizes that statistics, as a subject, is the study of making sense of data. Thus, after completing an introductory course in statistics, students should have a basic understanding of how to make sense of data. In this text, we approach the study of statistics by considering four steps in making sense of data: (1) gathering methods, (2) methods for summarizing data, (3) methods for analyzing data, and (4) ways to report the results of analyses. Most texts focus on the summarization and analysis steps; we emphasize that all problems involve the collection, summarization, analysis, and reporting steps of making sense of data. We do not, however, spend a great deal of time on teaching statistical calculations. Rather, with calculators and computers available to do the calculations required in analyzing data, we try to focus on the inferences that are made after the calculations have been done.

The use of this text does not require that students have access to a computer and standard statistical software. However, we do discuss the use of calculators and computers in statistics (Chapter 2) and, in an optional section at the end of each chapter, we give reusable programs and output to show how two software systems (SAS and Minitab) can be used to perform some of the calculations of that chapter. Exercises with computer output also provide opportunities for students to understand and interpret the results of a statistical analysis presented as computer output.

To summarize, the fifth edition provides an introduction to statistics with strong emphasis on making sense of data. From this text, students gain an understanding and appreciation of the role of statistics in society and of the steps and methods used in making sense of data. Since computers are becoming so much a part of our lives, an optional section is included in each chapter to show where computer software can be of assistance in performing the calculations of statistics. Important features of this edition include the following:

- Organizational and textual changes to emphasize the four steps in making sense of data: collecting data, summarizing data, analyzing data, and reporting data.
- A new chapter on methods for gathering data (Chapter 3). This chapter discusses some of the survey methods and designs of scientific studies used in collecting data.
- A new chapter on the reporting of results of statistical analyses (Chapter 18).
- A brief discussion of observational studies (Section 5.7).
- An early discussion of methods for summarizing data from more than one variable (Chapter 5).
- New sections on discussing assumptions (Sections 8.7, 9.5, and 15.4).
- Discussion of inferences about  $\beta_0$  (Section 13.2).
- A new section on predicting y for a given value of x (Section 13.4).
- Updated and expanded computer output for examples and exercises.
- Expanded and improved exercise sets and examples.

A special note of appreciation is extended to Susan Reiland, who provided an extensive prerevision review of this edition. We are also indebted to Will Sullivan for redoing the computer output and to Jim Stegeman for preparation of the solutions manual. Special thanks are also due to Barbara and Robert Beaver for preparing the study guide.

Many students and professors who used previous editions of this text have contacted us with suggestions, and these are greatly appreciated. The authors especially want to acknowledge the reviewers for this edition:

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