JOHN P. BURGESS

Mathematics, Models and Modality

SELECTED PHILOSOPHICAL ESSAYS

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Selected Philosophical Essays

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MATHEMATICS, MODELS, AND MODALITY

John Burgess is the author of a rich and creative body of work which seeks to defend classical logic and mathematics through counter-criticism of their nominalist, intuitionist, relevantist, and other critics. This selection of his essays, which spans twenty-five years, addresses key topics including nominalism, neo-logicism, intuitionism, modal logic, analyticity, and translation. An introduction sets the essays in context and offers a retrospective appraisal of their aims. The volume will be of interest to a wide range of readers across philosophy of mathematics, logic, and philosophy of language.

JOHN P. BURGESS is Professor in the Department of Philosophy, Princeton University. He is co-author of *A Subject With No Object* with Gideon Rosen (1997) and *Computability and Logic*, 5th edn with George S. Boolos and Richard C. Jeffrey (2007), and author of *Fixing Frege* (2005).

Dedicated to the memory of my sister
Barbara Kathryn Burgess

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Preface

The present volume contains a selection of my published philosophical papers, plus two items that have not previously appeared in print. Excluded are technical articles, co-authored works, juvenilia, items superseded by my published books, purely expository material, and reviews. (An annotated partial bibliography at the end of the volume briefly indicates the contents of such of my omitted technical papers as it seemed to me might interest some readers.) The collection has been divided into two parts, with papers on philosophy of mathematics in the first, and on other topics in the second; references in the individual papers have been combined in a single list at the end of the volume. Bibliographic data for the original publication of each item reproduced here are given source notes on pp. xi–xiii, to which the notes of personal acknowledgment, dedications, and epigraphs that accompanied some items in their original form have been transferred; abstracts that accompanied some items have been omitted.

It has become customary in volumes of this kind for the author to provide an introduction, relating the various items included to each other, as an editor would in an anthology of contributions by different writers. I have fallen in with this custom. The remarks on the individual papers in the introduction are offered primarily in the hope that they may help direct readers with varying interests to the various papers in the collection that should interest them most. But such introductions also serve another purpose: they provide an opportunity for an author to note any changes of view since the original publication of the various items, thus reducing any temptation to tamper with the text of the papers themselves on reprinting. I have made only partial use of the opportunity to note changes in view, but nonetheless I have felt no temptation to make substantial changes in the papers, since my own occasional historical research has convinced me of the badness of the practice of revising papers on reprinting.

I have tried to acknowledge in each individual piece those to whom I have been most indebted in connection with that item, though I am sure there are some I have unintentionally neglected, whose pardon I must beg. Here I would like to acknowledge those who have been helpful specifically with the preparation of the present collection: Hilary Gaskin, who first suggested such a volume, and Joanna Breeze, along with Gillian Dadd and the rest of the staff who saw the work through publication.

Source notes

"Numbers and ideas" was first delivered orally as part of a public debate at the University of Richmond (Virginia), 1999. Ruben Hersh argued *for* the thesis "Resolved: that mathematical entities and objects exist within the world of shared human thoughts and concepts." I argued against. It was first published in a journal for undergraduates edited at the University of Richmond (England), the *Richmond Journal of Philosophy*, volume 1 (2003), pp. 12–17. (There is no institutional connection between the universities of the two Richmonds, and my involvement with both is sheer coincidence.)

"Why I am not a nominalist" was first delivered orally under the title "The nominalist's dilemma," to the Logic Club, Catholic University of Nijmegen, 1981. It was first published in the *Notre Dame Journal of Formal Logic*, volume 24 (1983), pp. 93–105.

"Mathematics and *Bleak House*" was first delivered orally at a symposium "Realism and anti-realism" at the Association for Symbolic Logic meeting, University of California at San Diego, 1999. The other symposiast was my former student Penelope Maddy, and the Dickensian title of my paper is intended to recall the Dickensian title of her earlier review, "Mathematics and *Oliver Twist*" (Maddy 1990). First published in *Philosophia Mathematica*, volume 12 (2004), pp. 18–36.

"Quine, analyticity, and philosophy of mathematics" was first delivered orally at the conference "Does Mathematics Require a Foundation?," Arché Institute, University of St. Andrews, 2002. Identified in its text as a sequel to the preceding item, this paper circulated in pre-publication draft under the title "Mathematics and *Bleak House*, II." First published in the *Philosophical Quarterly*, volume 54 (2004), pp. 38–55.

"Being explained away" is a shortened version (omitting digressions on technical matters) of a paper delivered orally to the Department of Philosophy, University of Southern California, 2004. (I wish not only to thank that department for the invitation to speak, but especially to thank

Stephen Finlay, Jeff King, Zlatan Damnjanovic, and above all Scott Soames for their comments and questions, as well as for their hospitality during my visit.) It was first published in the *Harvard Review of Philosophy*, volume 13 (2005), pp. 41–56.

"*E pluribus unum*" evolved from a paper "From Frege to Friedman" delivered orally at the Logic Colloquium of the University of Pennsylvania and the Department of Logic and Philosophy of Science at the University of California at Irvine. It was first published in *Philosophia Mathematica*, volume 12 (2004), pp. 193–221. (I am grateful to Harvey Friedman for introducing me to his recent work on reflection principles, to Kai Wehmeier and Sol Feferman for drawing my attention to the earlier work of Bernays on that topic, and to Penelope Maddy for pressing the question of the proper model theory for plural logic, which led me back to the writings of George Boolos on this issue. From Feferman I also received valuable comments leading to what I hope is an improved exposition.)

"Logicism: a new look" was first delivered orally at the conference marking the inauguration of the UCLA Logic Center, and later (under a different title) as part of the annual lecture series of the Center for Philosophy of Science, University of Pittsburgh, both in 2003. It has not previously been published.

"Tarski's tort" was first delivered orally at Timothy Bays' seminar on truth, Notre Dame University, Saint Patrick's Day, 2005. It was previously unpublished. The paper should be understood as dedicated to my teacher Arnold E. Ross, mentioned in its opening paragraphs.

"Which modal logic is the right one?" was first delivered orally at the George Boolos Memorial Conference, University of Notre Dame, 1998. It was first published in the *Notre Dame Journal of Formal Logic*, volume 40 (1999), pp. 81–93, as part of a special issue devoted to the proceedings of that conference. Like all the conference papers, mine was dedicated to the memory of George Boolos.

"Can truth out?" was first delivered orally under the title "Fitch's paradox of knowability" as a keynote talk at the annual Princeton–Rutgers Graduate Student Conference in Philosophy, 2003. It was first published in Joseph Salerno, ed., *New Essays on Knowability*, Oxford: Oxford University Press (2007). The paper originally bore the epigraph "Truth will come to light; murder cannot be hid long; a man's son may, but at the length truth will out" (*Merchant of Venice* II: 2). Thanks are due to Michael Fara, Helge Rückert, and Timothy Williamson for perceptive comments on earlier drafts of this note.

"*Quinus ab omni naevo vindicatus*" was first delivered orally to the Department of Philosophy, MIT, 1997. It was first published in Ali Kazmi, ed., *Meaning and Reference*, *Canadian Journal of Philosophy Supplement*, volume 23 (1998), pp. 25–65. (The present paper is a completely rewritten version of an unpublished paper, "The varied sorrows of modality, part II." I am indebted to several colleagues for information used in writing that paper, and for advice given on it once written, and I would like to thank them all – Gil Harman, Dick Jeffrey, David Lewis – even if the portions of the paper with which some of them were most helpful have disappeared from the final version. But I would especially like to thank Scott Soames, who was most helpful with the portions that have *not* disappeared.)

"Translating names" was first published in *Analysis*, volume 65 (2005), pp. 96–204. I am grateful to Pierre Bouchard and Paul Égré for linguistic information and advice.

"Relevance: a fallacy?" was first published in the *Notre Dame Journal of Formal Logic*, volume 22 (1981), pp. 76–84. Its sequels were Burgess (1983c) and Burgess (1984b).

"Dummett's case for intuitionism" was first published in *History and Philosophy of Logic*, volume 5 (1984), pp. 177–194. The paper originally bore the epigraph from Chairman Mao "Combat Revisionism!" I am indebted to several colleagues and students for comments, and especially to Gil Harman, who made an earlier draft of this paper the topic for discussion at one session of his summer seminar. Comments by editors and referees led to what it is hoped are clearer formulations of many points.

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Introduction

ABOUT "REALISM"

A word on terminology may be useful at the outset, since it is pertinent to many of the papers in this collection, beginning with the very first. The label "realism" is used in two very different ways in two very different debates in contemporary philosophy of mathematics. For nominalists, "realism" means acceptance that there exist entities, for instance natural or rational or real numbers, that lack spatiotemporal location and do not causally interact with us. For neo-intuitionists, "realism" means acceptance that statements such as the twin primes conjecture may be true independently of any human ability to verify them. For the former the question of "realism" is ontological, for the latter it is semantico-epistemological. Since the concerns of nominalists and of neo-intuitionists are orthogonal, the double usage of "realism" affords ample opportunity for confusion.

The arch-nominalists Charles Chihara and Hartry Field, for instance, are anti-intuitionists and "realists" in the neo-intuitionists' sense. They do not believe there are any unverifiable truths about numbers, since they do not believe there are any numbers for unverifiable truths to be about. But they do believe that the facts about the possible production of linguistic expressions, or about proportionalities among physical quantities, which in their reconstructions replace facts about numbers, can obtain independently of any ability of ours to verify that they do so. Michael Dummett, the founder of neo-intuitionism, was an early and forceful anti-nominalist, and though he calls his position "anti-realism," he and his followers are "realists" in the nominalists' sense, accepting some though not all classical existence theorems, namely those that have constructive proofs, and agreeing that it is a category mistake to apply spatiotemporal or causal predicates to mathematical subjects.

On top of all this, even among those of us who are "realists" in both senses there are important differences. *Metaphysical* realists suppose, like

Galileo and Kepler and Descartes and other seventeenth-century worthies, that it is possible to get behind all human representations to a God's-eye view of ultimate reality as it is in itself. When *they* affirm that mathematical objects transcending space and time and causality exist, and mathematical truths transcending human verification obtain, they are affirming that such objects exist and such truths obtain *as part of ultimate metaphysical reality* (whatever that means). *Naturalist* realists, by contrast, affirm only (what even some self-described anti-realists concede) that the existence of such objects and obtaining of such truths is an implication or presupposition of science and scientifically informed common sense, while denying that philosophy has any access to exterior, ulterior, and superior sources of knowledge from which to "correct" science and scientifically informed common sense. The naturalized philosopher, in contrast to the alienated philosopher, is one who takes a stand as a citizen of the scientific community, and not a foreigner to it, and hence is prepared to reaffirm while doing philosophy whatever was affirmed while doing science, and to acknowledge its evident implications and presuppositions; but only the metaphysical philosopher takes the status of what is affirmed while doing philosophy to be a revelation of an ultimate metaphysical reality, rather than a human representation that is the way it is in part because a reality outside us is the way it is, and in part because we are the way we are.

My preferred label for my own position would now be "naturalism," but in the papers in this collection, beginning with the first, "realism" often appears. Were I rewriting, I might erase the R-word wherever it occurs; but as I said in the preface above, I do not believe in rewriting when reprinting, so while in date of composition the papers reproduced here span more than twenty years, still I have left even the oldest, apart from the correction of typographical errors, just as I wrote them. *Quod scripsi, scripsi.*

This collection begins with five items each pertinent in one way or another to nominalism and the problem of the existence of abstract entities. The term "realism" is used in an ontological sense in the first of these, "Numbers and ideas" (2003). This paper is a curtain-raiser, a lighter piece responding to certain professional mathematicians turned amateur philosophers who propose a cheap and easy solutions to the problem. According to their proposed compromise, numbers exist, but only "in the world of ideas." Since acceptance of this position would render most of the professional literature on the topic irrelevant, and since the amateurs often offer unflattering accounts of what they imagine to be the reasons why professionals do not accept their simple proposal, I thought it worthwhile to accept an invitation to try to state, for a general audience, our real reasons,

which go back to Frege. The distinction insisted upon in this paper, between the kind of thing it makes sense to say about a number and the kind of thing it makes sense to say about a mental representation of a number (and the distinction, which exactly parallels that between the two senses of "history," between mathematics, the science, and mathematics, its subject matter) is presupposed throughout the papers to follow.

Some may wonder where my emphatic rejection of "idealism or conceptualism" in this paper leaves intuitionism. The short answer is that I leave intuitionism entirely out of account: I am concerned in this paper with descriptions of the mathematics we have, not prescriptions to replace it with something else. Intuitionism is orthogonal to nominalism, as I have said, and issues about it are set aside in the first part of this collection. I will add that, though I do not address the matter in the works reprinted here, my opinion is that Frege's anti-psychologistic and anti-mentalistic points raise some serious difficulties for Brouwer's original version of intuitionism, but no difficulties at all for Dummett's revised version. Neither opinion should be controversial. Dummett's producing a version immune to Fregean criticism can hardly surprise, given that the founder of neo-intuitionism is also the dean of contemporary Frege studies. That Brouwer's version, by contrast, faces serious problems was conceded even by so loyal a disciple as Heyting, and all the more so by contemporary neo-intuitionists.

AGAINST HERMENEUTIC AND REVOLUTIONARY NOMINALISM

"Why I am not a nominalist" (1983) represents my first attempt to articulate a certain complaint about nominalists, namely, their unclarity about the distinction between *is* and *ought*. It was this paper that first introduced a distinction between *hermeneutic* and *revolutionary* nominalism. The formulations a decade and a half later in *A Subject With No Object* (Burgess and Rosen, 1997) are, largely owing to my co-author Gideon Rosen, who among other things elaborated and refined the hermeneutic/revolutionary distinction, more careful on many points than those in this early paper. This piece, however, seemed to me to have the advantage of providing a more concise, if less precise, expression of key thoughts underlying that later book than can be found in any one place in the book itself. Inevitably I have over the years not merely elaborated but also revised (often under Rosen's influence) some of the views expressed in this early article.

First, the brief sketches of projects of Charles Chihara and Hartry Field in the appendix to the paper (which I include on the recommendation of an

anonymous referee, having initially proposed dropping it in the reprinting) are in my present opinion more accurate as descriptions of aspirations than of achievements, and even then as descriptions only to a first approximation; moreover the later approach of Geoffrey Hellman is not discussed at all. My ultimate view of the technical side of the issue is given in full detail in the middle portions of *A Subject*, superseding several earlier technical papers.

Further, though I still see no serious linguistic evidence in favor of any hermeneutic nominalist conjectures, I no longer see the absence of such evidence as the main objection to them. For reasons that in essence go back to William Alston, such conjectures lack relevance *even if correct*. Even if we grant that "There are prime numbers greater than a million" does just mean, say, "There could have existed prime numerals greater than a million," the conclusion that should be drawn is that "Numbers exist" means "Numerals could have existed," and is therefore *true*, as anti-nominalists have always maintained, and not *false*, as nominalists have claimed. There is no threat at all to a naturalist version of anti-nominalism in such translations, though there might be to a metaphysical version. This line I first developed in a very belatedly published paper (Burgess 2002a) of which a condensed version was incorporated into *A Subject*.

Finally, I now recognize that there is a good deal more to be said for the position I labeled "instrumentalism" than I or almost anyone active in the field was prepared to grant back in the early 1980s when I wrote "Why I am not," or even in the middle 1990s, when I wrote my contributions to *A Subject*. The position in question is that of those philosophers who speak with the vulgar in everyday and scientific contexts, only to deny on entering the philosophy room that they meant what they said seriously. This view is now commonly labeled "fictionalism," and it deserves more discussion than it gets in either "Why I am not" or *A Subject*. It should be noted that while I originally opposed fictionalism (or instrumentalism) to both the revolutionary and hermeneutic positions, Rosen has correctly pointed out that fictionalism itself comes in a revolutionary version (this is the attitude philosophers *ought to* adopt) and a hermeneutic version (this is the attitude commonsense and scientific thinkers *already do* adopt). What I originally called the "hermeneutic" position should be called the "content-hermeneutic" position, and the hermeneutic version of fictionalism the "attitude-hermeneutic" position, in Rosen's refined terminology.

On two points my view has not changed at all over the past years. First, while nominalists would wish to blur what for Rosen and myself is a key distinction, and avoid taking a stand on whether they are giving a

description of the mathematics we already have (hermeneutic) or a prescription for a new mathematics to replace it (revolutionary), gesturing towards a notion of "rational reconstruction" that would somehow manage to be neither the one nor the other, I did not think this notion had been adequately articulated when I first took up the issue of nominalism, and I have not found it adequately articulated in nominalist literature of the succeeding decades.

Second, as to the popular epistemological arguments to the effect that even if numbers or other objects "causally isolated" from us do exist, we cannot *know* that they do, I have not altered the opinions that I expressed in my papers Burgess (1989) and the belatedly published Burgess (1998b), and that Rosen expressed in his dissertation, and that the two of us jointly expressed in *A Subject*. The epistemological argument, according to which belief in abstract objects, even if conceded to be implicit in scientific and commonsense thought, and even if perhaps true – for the aim of going epistemological is precisely to avoid direct confrontation over the question of the *truth* of anti-nominalist existence claims – cannot constitute *knowledge*, surely is not intended as a Gettierological observation about the gap between justified true belief and what may properly be called *knowledge*. It follows that it must be an issue about *justification*; and here to the naturalized anti-nominalist the nominalist appears simply to be substituting some extra-, supra-, praeter-scientific *philosophical* standard of justification for the ordinary standards of justification employed by science and common sense: the naturalist anti-nominalist's answer to nominalist skepticism about mathematics is skepticism about philosophy's supposed access to such non-, un-, and anti-scientific standards of justification.

AGAINST FICTIONALIST NOMINALISM

Returning to the issue of fictionalism, in our subsequent work Rosen and I have generally dealt with it separately and in our own ways. A chapter bearing the names of Rosen and myself, "Nominalism reconsidered," does appear in Stuart Shapiro's *Handbook of Philosophy of Mathematics and Logic* (2005), and it is a sequel to our book adding coverage of fictionalist nominalism, with special reference to the version vigorously advocated over the past several years by Steve Yablo; but this chapter is substantially Rosen's work, my contributions being mainly editorial.

My own efforts to address a fictionalist position are to be found rather in "Mathematics and *Bleak House*," which revisits, in a sympathetic spirit, Rudolf Carnap's ideas on the status of ontological questions and nominalist

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