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# Advances in String Theory

## The First Sowers Workshop in Theoretical Physics

Eric Sharpe
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American Mathematical Society · International Press

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Volume 44

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# Preface

Over the last decade string theory has, despite its purely theoretical content, started to make a strong impact on many areas of physics: high energy and hadronic physics, gravitation and cosmology, mathematical physics and even condensed matter physics. The impact has been through many major conceptual and methodological developments in quantum field theory in the past fifteen years. The ideas of duality, holography, extra dimensions, conformal field theory, gauge theory/gravity correspondence, *etc.* have excited the imagination of many theoretical and experimental physicists in these diverse fields of physics. In addition, string theory has exerted a dramatic influence on developments in contemporary mathematics, ranging from mirror symmetry and enumerative geometry in algebraic geometry to Seiberg-Witten theory in four-manifolds.

Nevertheless, despite these advances the fundamentals of string theory are still largely unknown. Thus it seemed appropriate to have a gathering of younger leading practitioners of various aspects of the field around the common theme: "What is string theory?" This gathering took place at Virginia Tech on May 14–18, 2007. This unique event was made possible by a generous donation from a friend and benefactor of the physics department at Virginia Tech, Mr. Mark Sowers, in whose honor the workshop was named, "The first Sowers workshop in theoretical physics."

As can be seen from the proceedings the range of topics was very wide, cutting through many aspects of string theory. We thank the contributors for making this volume possible in a timely manner. We also thank the colleagues at the Department of Physics and the College of Science at Virginia Tech for support and help. Most of all, we sincerely thank our donor and our friend Mr. Mark Sowers for making this meeting, and thus this proceedings possible.

We warmly dedicate this volume to Mark Sowers.

The organizers,
D. Minic, E. Sharpe, T. Takeuchi, and A. Yelnikov

# Acknowledgements

This workshop was made possible through a meeting of two worlds: the world of business and the world of science. Mr. Mark Sowers, a highly successful business man, is fascinated by the frontiers of theoretical physics. The organizers, Djordje Minic, Eric Sharpe, Tatsu Takeuchi, and Alexandr Yelnikov, had a vision: To bring together some of the brightest young minds in string theory to discuss their most recent findings and generate new ideas, in a relaxed and supportive setting. Thanks to Mark Sowers's generous support, the vision became reality, and the first Sowers Theoretical Physics Workshop "What is String Theory?" was held at Virginia Tech in May 2007. The College of Science and the Department of Physics gratefully acknowledge both worlds: We thank Mark Sowers most warmly for his generosity, and the organizers for running a successful and enjoyable workshop. We hope that these proceedings will help continue the discussions.

Lay Nam Chang, Dean, College of Science
Beate Schmittmann, Chair, Department of Physics

# Contents

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# Puff Field Theory

Ori J. Ganor

ABSTRACT. Puff Field Theory is a string-theoretic construction of a nonlocal QFT where fundamental particles puff-up to occupy a three-dimensional volume. This conjectured theory is not Lorentz invariant, but Lorentz invariance is restored in the IR limit. A proposal for the supergravity dual also exists. It has peculiar properties near the boundary, such as infinite redshift in frequency combined with infinite blueshift in wavelength.

## 1. Introduction

On large scales FRW cosmology breaks the microscopic Lorentz symmetry, preserving only spatial rotations, and leaving a preferred time direction. It is therefore interesting to search for quantum field theories with a similar symmetry breaking $SO(3,1) \rightarrow SO(3)$. Denoting by $\Lambda$ the typical scale at which Lorentz violating interactions might become important (if they exist), we will assume that $\Lambda \ll M_{\text{Planck}}$. The question of UV completeness of the QFT is then pertinent. We would like to only consider UV complete theories, and we will also require that in the IR limit they approach $N = 4$ Super Yang-Mills. This generally requires the introduction of nonlocality. Below, we will present a string-theoretic construction that suggests the existence

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I would like to thank Mark Sowers for his generosity and to thank the organizers of the Sowers workshop, Djordje Minic, Eric Sharpe, Tatsu Takeuchi, and Oleksandr Yelnykov for the wonderful hospitality. These notes are based on [**1**]–[**2**], which include work in collaboration with Akikazu Hashimoto (Madison, WI), Sharon Jue (UC Berkeley), Bom Soo Kim (UC Berkeley), and Anthony Ndirango (UC Berkeley). This work was supported in part by the Center of Theoretical Physics at UC Berkeley, and in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098, and in part by the NSF under grant PHY-0098840.

of such a theory. We will also see that such a theory allows superluminal velocities.

## 2. Lorentz violation

Our theory will come with a dimensionful parameter

$$\zeta \sim \Lambda^{-\Delta},$$

where $\Lambda$ has dimensions of mass and sets the scale of Lorentz violations, and we will consider only the case $\Delta > 0$, which corresponds to a UV-relevant parameter.

In the real world, Lorentz-violating interactions can affect the dispersion relation of the photon. There are, however, excellent experimental bounds on this dispersion relation. One technique, for example, uses the observations of gamma ray bursts (GRBs). There have been observations of GRBs at distances of billions of light-years away, which last on the order of milliseconds, during which photons in the energy range of typically 100 KeV–100 MeV are detected [**3**]. This implies that photons with different energies must be traveling at almost exactly the same speed. The bounds from these observations on the variation in the speed of the photon are [**3**],

$$\frac{\Delta c}{c} < \left|\frac{E}{10^{16}\mathrm{GeV}}\right|, \qquad E < 200\,\mathrm{KeV}.$$

These bounds have also been extended to higher energies [**4**],

$$\frac{\Delta c}{c} < \left|\frac{E}{10^{17}\,\mathrm{GeV}}\right|, \qquad 1\mathrm{MeV} < E < 17\,\mathrm{MeV}.$$

So, if we wish our theories to have potentially realistic applications in phenomenology, we had better preserve the Lorentz invariant photon dispersion relation, at least to a very good accuracy.

There is a nice generic way to construct theories with Lorentz violation using an extra dimension and a brane [**5**]. Letting $x_4$ denote the extra (fifth) dimension, one postulates a 3-brane at $x_4 = 0$ with $g_{00}$ a function of $x_4$ such that at $x_4 = 0$, $g_{00} = -1$ (by a choice of coordinates). If $-g_{00} > 1$ somewhere in the bulk ($x_4 \neq 0$) then particles may appear to travel at superluminal velocities, from the point of view of an observer on the brane. The new bound on velocities is $|v| < \sqrt{-g_{00}}$. Our construction is related to this idea, but we will recast it in a dual setting where we will have a better control of the background.

## 3. Nonlocality

There are several examples of self-consistent nonlocal QFTs that can be constructed within the framework of string theory. $U(N)$ $\mathcal{N} = 4$ Super-Yang-Mills theory on a noncommutative $\mathbf{R}^4$ (NCSYM) [**6**]–[**8**] is perhaps the most famous one. In this theory fundamental particles can become extended objects – electric dipoles whose length $L$ in one particular direction is proportional to their momentum $p_\perp$ in another [9], so that $L \sim \theta p_\perp$, where $\theta$ is the fundamental noncommutativity parameter. Noncommutative geometry can be constructed directly in field theory, by replacing products of $U(N)$ matrices with the "star-product" [6]–[8],

$$\Phi_1 \star \Phi_2 = \left(\exp \frac{i}{2}\theta^{ij}\partial_i^{(1)}\partial_j^{(2)}\right)[\Phi_1\Phi_2]$$

where $\Phi_1$ and $\Phi_2$ are $N \times N$ matrix fields and $\theta^{ij}$ is an antisymmetric spacetime tensor.

Another related example of a nonlocal theory that can be constructed from string theory is *dipole-theory* [10]–[11]. There, fundamental particles are dipoles of length proportional not to momentum, but to R-symmetry charge. It can be constructed from $U(N)$ Yang-Mills theory by replacing covariant derivatives of adjoint fields $\Phi$ with

$$D_\mu\Phi(x) \longrightarrow \partial_\mu\Phi(x) - iA_\mu\left(x - \frac{1}{2}QL\right)\Phi(x) + i\Phi(x)A_\mu\left(x + \frac{1}{2}QL\right)$$

where $Q$ is (a certain component of) the R-charge of $\Phi$ and $L$ is a constant 4-vector, which is the fundamental parameter of the theory.

In both these examples nonlocality is *linear* – fundamental particles expand into nonlocal objects which are segments with a linear dimension. *Puff Field Theory* (PFT), the theory that we will construct in these notes, is different in that fundamental particles expand into objects with volume, and the volume is proportional to (a component of) the R-charge of the particle, as we shall see.

At low energies, all the examples above of nonlocal QFTs, including the conjectured PFT, are deformations of $\mathcal{N} = 4$ SYM with gauge group $U(N)$. Their Lagrangians are schematically of the form

$$L = L_{\mathcal{N}=4} + \zeta\mathcal{O}^{(\Delta+4)} + \cdots \tag{3.1}$$

where $\zeta$ is a parameter and $\mathcal{O}^{(\Delta+4)}$ is an operator of dimension $\Delta + 4$. The various dimensions as well as the schematic structure of the operators $\mathcal{O}$ are listed below (see the references above for more details):

| Theory | Deformation $\zeta\mathcal{O}$ | $\Delta$ | |
|---|---|---|---|
| NCYM | $\frac{1}{2}g_{\text{YM}}^2\theta^{\mu\nu}\ \text{tr}\{F_{\mu\sigma}F_{\nu\tau}F^{\sigma\tau} + F_{\sigma\tau}F^{\mu\nu}F_{\mu\nu} + \cdots\}$ | 2 | $\zeta \to \theta$ |
| Dipole | $\frac{1}{2}g_{\text{YM}}^2 L_{IJ}^{\mu}\ \text{tr}\{F_{\mu\nu}(\Phi^I D^\nu \Phi^J + \cdots)\}$ | 1 | $\zeta \to L$ |
| PFT | $\zeta\mathcal{O}^{(7)}$ | 3 | |

Here $g_{\text{YM}}$ is the coupling constant, $F_{\mu\nu}$ are the field strength components ($\mu, \nu = 0, \ldots, 3$), $\Phi^I$ are the scalars ($I, J = 1, \ldots, 6$), fermions as well as other less illuminating interaction terms have been suppressed, and we will discuss $\mathcal{O}^{(7)}$ in more detail below.

## 4. Constructing PFT through string theory

Our construction can be regarded as a variant of the Douglas-Hull construction [7] of NCSYM, so we will begin by reviewing the latter. One starts with type-IIA string theory compactified on $T^2$, which for simplicity we take to be of the form $S^1 \times S^1$. We denote the compactification radii by $R'_1, R'_2$. Now add $N$ coincident D0-branes, and take the limit $R'_1, R'_2 \to 0$. T-duality can be applied if there are no other fluxes, and then one gets $N$ D2-branes compactified on a large $T^2$ with compactification radii $\alpha'/R'_i$ ($i = 1, 2$). The low-energy limit of this construction then has a decoupled sector of 2+1D $U(N)$ $\mathcal{N} = 4$ SYM. But Douglas and Hull added an obstruction to this small/large area T-duality, in the form of a constant NSNS $B$-flux. Then, T-duality does not yield a simpler description, but instead Douglas and Hull showed that the low-energy description of the system in the limit $R'_1, R'_2 \to 0$ can be described by the nonlocal NCSYM.

We now turn to the construction of PFT. We start with type-IIA $T^3$ in the form of $S^1 \times S^1 \times S^1$ with compactification radii $R'_i$ ($i = 1, 2, 3$) and string coupling constant $g'_{\text{st}}$. We add $N$ Kaluza-Klein particles in the 1$^{\text{st}}$ direction and seek the low-energy description of this configuration in the limit

(4.1)
$$\alpha'^{-1/2} R'_1 \longrightarrow 0, \quad \alpha'^{-1/2} R'_2 \to \text{finite}, \quad \alpha'^{-1/2} R'_3 \to \text{finite}, \quad g'_{\text{st}} \to \text{finite}.$$

By 'low-energy' here we generally mean energies low compared to $\alpha'^{-1/2}$, but in our case we actually need a somewhat stronger condition – energies small compared to $(\alpha'^{-1/2} R'_1)^{1/2} \alpha'^{-1/2}$. Using U-duality, it is not hard to see that the sought-after description is simply 3+1D $U(N)$ $\mathcal{N} = 4$ SYM. In fact, T-duality in the 1$^{\text{st}}$ direction, followed by S-duality, followed by T-dualities in the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ directions, converts the system to $N$ D3-branes in type-IIB compactified on another $T^3$ with compactification radii

$$R_1 = \alpha'^{\frac{3}{4}} g'^{-\frac{1}{2}}_{\text{st}} R'^{-\frac{1}{2}}_1, \qquad R_k = \alpha'^{\frac{5}{4}} g'^{\frac{1}{2}}_{\text{st}} R'^{-\frac{1}{2}}_1 R'^{-1}_k \qquad (k = 2, 3),$$

and string coupling constant $g_{\text{st}} = \alpha' R'^{-1}_3 R'^{-1}_2$. The limit (4.1) was chosen so that the ratios $R_2/R_1$ and $R_3/R_1$ and the coupling constant $g_{\text{st}}$ remain finite, while $\alpha'^{-1/2} R_k \longrightarrow \infty$ ($k = 1, 2, 3$).

Following Douglas and Hull, we now add an obstruction to U-duality. The obstruction that seems to lead to interesting results in our case is not a flux but a geometrical twist. Let $x_1, x_2, x_3$ be the compact coordinates on $T^3$ with periodicities $2\pi R_1, 2\pi R_2, 2\pi R_3$ and let $\vec{y}$ denote the vector of coordinates in the six transverse directions. We then replace the original periodicity conditions of $T^3$ on the type-IIA side with

$$(x_1, x_2, x_3, \vec{y}) \sim (x_1 + 2\pi R_1, x_2, x_3, \Omega\vec{y})$$

where $\Omega$ is some element of the rotation group SO(6). Strictly speaking, we should let $\Omega$ be an element of Spin(6), but since we will eventually only need an infinitesimal $\Omega$, the distinction will be immaterial. We have to decide, though, on how to scale $\Omega$ in the limit (4.1). We want $\Omega$ to be infinitesimally close to the identity element such that

$$\Omega = \exp\left(\frac{2\pi}{R_1 R_2 R_3}\zeta\right) \longrightarrow I, \qquad \zeta \to \text{finite}.$$

Here,

$$\zeta \equiv \begin{pmatrix} & \beta_1 & & & & \\ -\beta_1 & & & & & \\ & & & \beta_2 & & \\ & & -\beta_2 & & & \\ & & & & & \beta_3 \\ & & & & -\beta_3 & \end{pmatrix} \in so(6). \tag{4.2}$$

We note in passing that if $\zeta$ is in an appropriate $su(3)$ ($su(2)$) subgroup of $so(6)$ then $\mathcal{N}=1$ ($\mathcal{N}=2$) SUSY is preserved, respectively. For the $\mathcal{N}=2$ case, we set $\beta \equiv \beta_1 = \beta_2$ and $\beta_3 = 0$.

With the introduction of $\zeta$, U-duality is less useful in the limit (4.1). However, we will argue below that the low-energy limit still describes a decoupled QFT on $\mathbf{R}^{3,1}$, but a nonlocal one.

Heuristically, the construction is designed so that fundamental particles that carry R-charge acquire a fundamental volume proportional to that R-charge and to $\zeta$. To see this, consider the original type-IIA geometry with $N$ Kaluza-Klein particles. For simplicity, let's focus on two out of the six transverse directions, and combine them into a complex variable $z \equiv y_1 + iy_2$. The twist $\Omega$ reduces to an element of $SO(2)$ which we write as $\omega \equiv \exp(\pi i\beta/R_1R_2R_3)$. Now consider a field $\phi$ in this geometry. Suppressing the coordinates $x_2, x_3$, we get the boundary conditions $\phi(x_1 + 2\pi R_1', \omega z) = \phi(x_1, z)$. Expanding in a Fourier series we get

$$\phi(x_1, z) = \sum_{p,\ell} C_{n,\ell}(|z|) z^{-\ell} e^{ipx_1/R_1'}, \qquad p \in \mathbf{Z} + \frac{2\beta\ell}{R_1R_2R_3}.$$

Thus, formally the Kaluza-Klein momentum is not an integer anymore, and its fractional part is proportional to both the angular momentum $\ell$ and the parameter $\beta$, which is a component of $\zeta$. If we formally perform the U-duality transformation that we used in the $\zeta = 0$ case and if we interpret our result above literally, we get a non-integer number of D3-branes that occupy a volume of $(2\pi)^3(R_1R_2R_3N + 2\beta\ell)$. In the next section we will make this statement more concrete.

## 5. Electric and magnetic fluxes

So, we treat PFT as a formal QFT. If we replace the $\infty$ in (4.1) with "very large" we get a compactified (on $T^3$) version of PFT. As above, we denote the compactification radii by $R_1, R_2, R_3$. We also denote $V \equiv R_1 R_2 R_3$. In the $\mathcal{N} = 2$ supersymmetric PFT we can calculate the energies of BPS states that include electric and magnetic fluxes, as well as momentum. Suppose we have $\ell$ units of R-charge, $k_i$ units of momentum, $e_i$ units of electric flux, and $m_i$ units of magnetic flux in the $i^{th}$ direction, for $i = 1, 2, 3$. (All of these are, of course, integers.) With the notation

$$\mathbf{P} \equiv \sum_{i=1}^{3} \frac{k_i}{R_i} \hat{\mathbf{n}}_i, \qquad \mathbf{E} \equiv \sum_{i=1}^{3} \frac{e_i R_i}{2\pi V} \hat{\mathbf{n}}_i, \quad \mathbf{B} \equiv \sum_{i=1}^{3} \frac{m_i R_i}{2\pi V} \hat{\mathbf{n}}_i,$$

we get the BPS energy

$$E = \frac{2\ell\beta}{g_{\text{st}} \alpha'^2} + \frac{2\pi^2 V^2}{|NV + 2\ell\beta|} \left( \frac{g_{\text{YM}}^2}{2\pi} \mathbf{E}^2 + \frac{2\pi}{g_{\text{YM}}^2} \mathbf{B}^2 \right) + \left| \mathbf{P} - \frac{4\pi^2 V^2}{|NV + 2\ell\beta|} \mathbf{E} \times \mathbf{B} \right| .$$

Analogous formulas for NCSYM have been derived, for instance, in [**12**]–[**14**]. The first term is a manifestation of the Volume/R-charge relation discussed above, and the appearance of $\ell$ in the denominator of the remaining terms clearly shows that the theory is nonlocal, since $\ell$ is an integral of the R-charge density over the entire volume. We also see from the last term that the dispersion relation of massless Kaluza-Klein particles remains relativistic.

## 6. Supergravity dual

The supergravity dual was found in [**2**] using techniques similar to those applied in the NCSYM case [**15**]–[**16**] and the dipole-theory case [**11**]. The result is type-IIB supergravity on a space with metric

$$\begin{aligned} ds^2 &= \frac{R^2}{r^2} K^{-\frac{1}{2}} \left[ dx^2 + dy^2 + dz^2 - \left( dt - \frac{4\pi N}{r^2} \vec{n}^T \zeta d\vec{n} \right)^2 \right] \\ &\quad + \frac{R^2}{r^2} K^{\frac{1}{2}} dr^2 + R^2 K^{\frac{1}{2}} d\Omega_5^2, \\ C_4' &= \frac{\pi N}{r^4} K^{-1} dt \wedge dx \wedge dy \wedge dz - \frac{\pi N}{g_{\text{st}} \alpha'^2 r^6} K^{-1} \vec{n}^T \zeta d\vec{n} \wedge dx \wedge dy \wedge dz, \end{aligned} \tag{6.1}$$

where $C_4'$ is the RR flux, and

$$K \equiv 1 + \frac{16\pi^2 N^2}{r^6} \vec{n}^T \zeta^T \zeta \vec{n}, \quad \vec{n} \in S^5, \quad d\Omega_5^2 = \sum_{I=1}^{6} dn_I^2, \quad R^4 \equiv 4\pi g_{\text{st}} N \alpha'^2.$$

We can now explore in more detail the IR and UV limits of PFT. In the IR limit $r \longrightarrow \infty$ we expand the background as

$$ds^2 = \frac{R^2}{r^2}[dr^2 + dx^2 + dy^2 + dz^2 - dt^2] + R^2 d\Omega_5^2 + \frac{8\pi N R^2}{r^4}\vec{n}^T \zeta d\vec{n}\, dt + \cdots$$
$$C_4' = \frac{\pi N}{r^4} dt \wedge dx \wedge dy \wedge dz - \frac{\pi N}{g_{\mathrm{st}}\alpha'^2 r^6}\vec{n}^T \zeta d\vec{n} \wedge dx \wedge dy \wedge dz + \cdots$$

The last term on each line corresponds to a deformation of $\mathcal{N} = 4$ by an irrelevant operator. This is the operator $\mathcal{O}^{(7)}$ from (3.1) which can be identified using the general AdS/CFT rules [**17**]–[**18**]. It has dimension 7, it transforms as a vector under Lorentz transformations, and it is in the adjoint representation of the R-symmetry group $SU(4)$. Further analysis using supersymmetry [**2**] shows that in the case of a $U(1)$ gauge symmetry

$$(6.2)\quad \zeta\mathcal{O} \longrightarrow \zeta^{\mu}_{AB} T_{\mu\nu} J^{\nu AB} + \zeta^{\mu}_{AB}\epsilon^{ABCDEF}\epsilon_{\mu\nu\sigma\tau}\Phi^C \partial_\nu \Phi^D \partial_\sigma \Phi^E \partial_\tau \Phi^F + \cdots,$$

where fermions have been suppressed, $\Phi^A$ $(A, \ldots, F = 1, \ldots, 6)$ are the scalars of the $\mathcal{N} = 4$ vector multiplet, $J^{\nu AB} = -J^{\nu BA}$ is the R-charge, and $T^{\mu\nu}$ is the energy-momentum tensor. In the case of a generic $U(N)$ gauge group, the expression is similar, but requires a symmetrized trace and additional commutator terms, and has not been calculated.

## 7. UV limit

The supergravity dual allows us to make a few plausible statements with regard to the UV behavior of the (large $N$ limit of) PFT on $\mathbf{R}^{3,1}$. Whereas in the IR the metric (6.1) approaches $AdS_5 \times S^5$, in the UV limit $r \longrightarrow 0$ the metric differs markedly from $AdS_5 \times S^5$, and in fact becomes singular. For simplicity, it is convenient to restrict the form of the parameter $\zeta$ and set $\beta_1 = \beta_2 = \beta_3 \equiv \beta$ in (4.2). This form does not preserve any supersymmetry, but it is convenient because it preserves a $U(3)$ subgroup of the R-symmetry group $SU(4)$. This subgroup has a simple geometrical interpretation in terms of the $S^5$ component of the supergravity dual.

$S^5$ can be realized as an $S^1$ fibration over $\mathbf{CP}^2$ (the *Hopf* fibration). The R-symmetry $SU(4)$ is a double cover of the isometry group $SO(6)$ of $S^5$, and $U(3) \subset SU(4)$ is the subgroup of isometries that preserve the Hopf fibration structure. It is the isometry group of the base $\mathbf{CP}^2$.

In $AdS_5 \times S^5$ the radius of $S^5$ is a constant $R$. When $\zeta$ is turned on, the coordinates $\vec{n}$ in (6.1) still describe a manifold that is diffeomorphic to $S^5$, but the metric is different. If, in addition, $\zeta$ preserves $U(3) \subset SU(4)$ as above, the metric on the deformed $S^5$ is determined by specifying both the size of the fiber $S^1$ and the size of the base $\mathbf{CP}^2$. From (6.1) we find

$$R_{\mathrm{fiber}} = R\left(1 + \frac{16\pi^2 N^2 \beta^2}{r^6}\right)^{-1/4}, \qquad R_{\mathrm{base}} = R\left(1 + \frac{16\pi^2 N^2 \beta^2}{r^6}\right)^{1/4}.$$

In the UV limit we find $R_{\text{fiber}} \sim r^{3/2}$ and $R_{\text{base}} \sim r^{-3/2}$. (Note that, in contrast, in the IR limit both $R_{\text{fiber}}$ and $R_{\text{base}}$ approach the constant $R$.) Thus, in the UV limit the base $\mathbf{CP}^2$ expands to infinite size, and the fiber $S^1$ shrinks to zero. The type-IIB supergravity description is therefore inadequate. However, we can contemplate a local duality transformation along the fiber that would convert type-IIB on a small $S^1$ to M-theory on $T^2$. A similar transformation in the context of $AdS_5 \times S^5$ was discussed in [**20**]. The final result is an M-theory background with

$$
\begin{aligned}
\ell_P{}^{-2} ds_M^2 &= (4\pi N)^{-1/3} g_{\text{st}}^{-1} \rho^2 \left[\Delta^{-2/3}(dx_1^2 + dx_2^2 + dx_3^2) - \Delta^{1/3} dx_0^2\right] \\
&\quad + (4\pi N)^{2/3} \Delta^{1/3} \rho^{-2} d\rho^2 + (4\pi N)^{2/3} \Delta^{1/3} ds_B^2 \\
&\quad + (4\pi N)^{-1/3} \Delta^{1/3} (g_{\text{st}}^{-1} d\xi^2 + g_{\text{st}} d\eta^2), \\
\ell_P{}^{-3} G_4 &= 2\pi N \omega \wedge \omega + 2\omega \wedge d\eta \wedge d\xi \\
&\quad + \frac{2}{3} (4\pi N)^{-2} g_{\text{st}}^{-3} \beta d\left(\frac{\rho^6}{\Delta}\right) \wedge dx_1 \wedge dx_2 \wedge dx_3,
\end{aligned} \tag{7.1}
$$

where

$$
\Delta \equiv 1 + (4\pi N)^{-1} g_{\text{st}}^{-3} \beta^2 \rho^6,
$$

$\ell_P$ is the Planck length, $x_0, x_1, x_2, x_3$ are coordinates on $\mathbf{R}^{3,1}$, $\eta$ and $\xi$ are periodic coordinates with period $2\pi$ (parameterizing $T^2$), $ds_B^2$ is the metric on $\mathbf{CP}^2$, $\omega$ is a harmonic 2-form whose cohomology class generates $H^2(\mathbf{CP}^2, \mathbf{Z})$, and $G_4$ is the 4-form flux. We also replaced the coordinate $r$ with $\rho \propto 1/r$, so that $\rho \to \infty$ is the UV limit. The background (7.1) becomes weakly coupled as $\rho \to \infty$. It is not completely clear that local duality is reliable in our case, especially since the background is not supersymmetric, but we take the fact that the final result is weakly coupled as encouraging.

Let us study the UV limit in more detail. As $\rho \to \infty$ we can approximate the background (7.1) as

$$
\begin{aligned}
\ell_P{}^{-2} ds_M^2 &\approx -(4\pi N)^{-2/3} g_{\text{st}}^{-2} \beta^{2/3} \rho^4 dx_0^2 \\
&\quad + (4\pi N)^{1/3} g_{\text{st}} \beta^{-4/3} \rho^{-2} (dx_1^2 + dx_2^2 + dx_3^2) \\
&\quad + (4\pi N)^{1/3} g_{\text{st}}^{-1} \beta^{2/3} (d\rho^2 + \rho^2 ds_B^2) \\
&\quad + (4\pi N)^{-2/3} \beta^{2/3} \rho^2 (g_{\text{st}}^{-2} d\xi^2 + d\eta^2), \\
\ell_P{}^{-3} G_4 &\approx 2\pi N \omega \wedge \omega + 2\omega \wedge d\eta \wedge d\xi - 4 g_{\text{st}}^3 \beta^{-3} \rho^{-7} d\rho \wedge dx_1 \wedge dx_2 \wedge dx_3.
\end{aligned} \tag{7.2}
$$

This background exhibits several interesting properties as $\rho \to \infty$:

- It is geodesically complete;
- The curvature tends to zero;
- There is an infinite redshift in frequency $g_{00} \to \infty$;

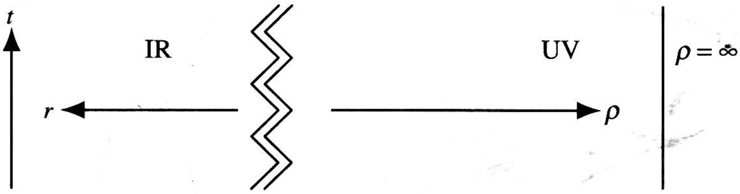


FIGURE 1. The Penrose diagram of the conjectured supergravity dual of PFT. The IR region is described by (6.1), while the UV region is described by (7.1). In the intermediate region neither description is weakly coupled.

- There is an infinite blueshift in wavelength $g_{11}, g_{22}, g_{33} \to 0$.

The Penrose diagram of this background is depicted in Figure 1, and is similar to that of AdS space.

## 8. Degrees of freedom

We argued in the previous section that the UV regime of PFT is holographically dual to a weakly-coupled M-theory background. This raises the question of whether the spectrum of PFT is really discrete. Since the $\rho$ direction of (7.1) is infinite in extent, it looks as if there is an "infinite amount" of weakly coupled M-theory in the holographic dual, which would suggest a continuous spectrum. (Similar issues arise in Little String Theory [**21**].) A more careful analysis, however, suggests that the spectrum of PFT is indeed discrete. First, let's be more precise about what we mean by "discrete." The question is whether when compactified on $T^3$ with generic boundary conditions (that should include R-symmetry twists in order to remove zero modes of scalar fields) the spectrum of the theory is discrete. And the answer might very well be "yes," due to the following remarkable property of (7.1).

Consider particle trajectories with fixed energy $E$ and spatial momentum $\vec{p}$ (defined with respect to the Killing vectors $\partial/\partial x_0$ and $\partial/\partial x_1, \ldots \partial/\partial x_3$, respectively). Then the following inequality can easily be derived from the condition that the trajectory be timelike or lightlike:

$$\rho \leq \rho_{\max} \equiv (4\pi N)^{1/6} g_{\rm st}^{1/2} \beta^{-1/3} \left(\frac{E}{|\vec{p}|}\right)^{1/3}. \tag{8.1}$$

Returning to the question of the spectrum, note that if PFT is compactified on a fixed $T^3$ and all possible zero modes of bosonic fields are removed, then $|\vec{p}|$ is bounded from below (by a value of the order of the inverse of the longest side of the $T^3$). If we now also put an upper bound on $E$, we see from (8.1) that $\rho$ is bounded from above, and only a finite portion of the background (7.1) is accessible. This suggests a discrete spectrum.

## 9. Superluminal velocities

Without Lorentz invariance there is no a priori bound on velocities. For example, it was demonstrated in NCYM [**22**–**24**] that under various circumstances certain particles can have a nonrelativistic dispersion relation, and can therefore travel faster than the speed of light.

Superluminal velocities are also possible in PFT. To see this, consider the IR expansion (3.1) with the operator given in (6.2). We will concentrate on the first term of (6.2) – the $TJ$ coupling. In a background with a nonvanishing R-charge density $\langle\zeta J^0\rangle \equiv \langle\zeta_{AB} J^{AB\,0}\rangle$ we have

$$L = L_{(N=4)} + \langle\zeta J_0\rangle T^{00} + \cdots$$

Thus, in R-charged matter the effect of PFT, to lowest order in $\zeta$, can be mimicked by a shift of the time-time component of the background metric,

$$g_{00} \to g_{00} - \zeta\langle J^0\rangle + \cdots,$$

and the upper bound on velocities is therefore

$$v_{\max} \approx 1 + \frac{1}{2}\langle\zeta J^0\rangle.$$

Thus, superluminal velocities are possible inside R-charged matter, at least if $\langle\zeta J^0\rangle$ is positive.

Using the extreme UV supergravity dual (7.2), we can estimate the energy required to travel at a velocity $v \gg 1$. Consider a supergravity particle with mass $m > 0$ and energy $E$ in the metric (7.2). We have

$$-\ell_P{}^2 m^2 \geq -(4\pi N)^{2/3} g_{\rm st}^2 \beta^{-2/3}\frac{E^2}{\rho^4},$$

(derived by requiring a timelike worldline). The upper bound on velocity is given by

$$|v| < v_{\max} = \sqrt{-\frac{g_{00}}{g_{11}}} = (4\pi N)^{-1/2} g_{\rm st}^{-3/2}\beta\rho^3.$$

Combining the last two inequalities we get

$$E \geq \ell_P m |v|^{2/3}\beta^{-1/3}.$$

Thus, in this context, higher velocity (surpassing the speed of light) requires higher energy. To be sure, this discussion does not address the massless case $m = 0$.

## 10. D3-branes in strong RR flux

Formally, we can set $N = 0$ in the construction of section §4, perform the U-duality transformation on the background with $\Omega$, and place $N$ D3-branes in the resulting (strongly coupled) type-IIB background, instead of $N$ Kaluza-Klein particles in the original type-IIA background.