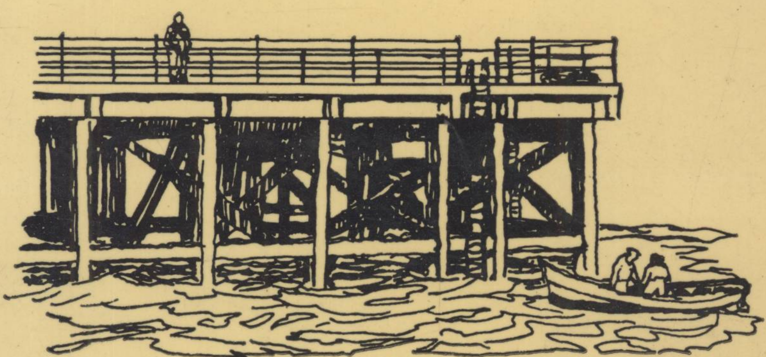
EXAMINATION SUBJECTS FOR ENGINEERS AND BUILDERS



fluid mechanics

R.H. Dugdale & W.S. Bannister

2nd EDITION

EXAMINATION SUBJECTS FOR ENGINEERS AND BUILDERS

FLUID MECHANICS

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GENERAL INTRODUCTION

THIS series is designed as an aid to students in all branches of professional engineering and building. The authors believe that when preparing for an examination the average student's first need is not a complete work of reference but a clear, concise guide to the basic principles of his subject. Throughout the series, therefore, only fundamental theory has been covered in each book and a series of graded worked examples used to show how widely these basic concepts can be applied to actual examination questions.

In view of the changeover to metric units of measurement in Great Britain, all the books are in SI units where applicable as recommended by the Council of Technical Examining Bodies.

In attempting to cover such a wide field as "engineering and building" there will naturally be a certain amount of subject-matter common to all disciplines which will be repeated in two or more books. Conversely, some books will be specifically written for use of students in one particular branch of engineering or building. Each book, therefore, will state in its preface the type of student it is intended for and the examinations for which it is suitable.

Details of the other books in the series will be found in the complete catalogue of Macdonald & Evans Ltd., which can be obtained post free on request.

M. J. Smith
General Editor

AUTHORS' PREFACE

THE more fashionable term "fluid mechanics" has now been adopted as the title of this book. With the adoption of SI units, it was felt worth while to broaden the scope of the book to cover more than just the civil engineering aspects of fluid mechanics.

Students of mechanical and civil engineering should find the book worth reading as its object is to prepare the reader, in about 100 pages, for examinations in fluid mechanics. Both H.N.C. and degree students should find the text and worked examples extremely helpful.

It is not possible in so short a book to include every examination topic. What is shown, however, is how widely the principles of conservation of energy and conservation of momentum can be applied; and the variety of the worked examples indicates the large number of examination questions which relate to these two principles. Six chapters are devoted to problems of energy and momentum, and two chapters to dimensional analysis, a worth-while adjunct of the six. In the first two chapters definitions of fluid properties and certain preliminary results are briefly presented. These provide sufficient information for understanding subsequent chapters—and, incidentally, for answering some common types of examination questions.

November 1970

R. D. H.
W. S. B.

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CHAPTER 1

HYDROSTATICS

INTRODUCTION

System of units

Throughout this book the International System of Units is adopted. The full title "Système International d'Unités" is abbreviated to "SI" units in all languages.

In each problem, regardless of what units the data are given in, length will be converted into metres, mass into kilogrammes and time into seconds for the purpose of calculation. The proper units of force are then *newtons.*

Mass and force are related by Newton's second law of motion:

$$P = ma$$

where P is the force in newtons (N) which, acting on a mass m kilogrammes (kg), produces an acceleration of a metres per second per second (m/s²). The equation can be rewritten as:

$$1\ (\text{N}) = 1\ (\text{kg}) \times 1\ (\text{m/s}^2)$$

To end this section with a word of warning, it is well known that a freely falling body accelerates at about 9·807 m/s². If the body has a mass of 1 kg, then the force acting on the body is 9·807 N which is the *weight* of the body. This can be stated more generally,

$$W = gm$$

where W is the weight of the body in newtons, and g (m/s²) is the *local gravitational acceleration.*

Terms used in hydrostatics

Hydrostatics is the study of liquids at rest. Many of the terms defined in this section, however, apply equally in *hydrodynamics*, the study of liquids in motion.

Density (ρ) is mass per unit volume and, in keeping with the above remarks, should always be expressed in kg/m³ for the purpose of calculations. For water,

$$\rho = 1000\ \text{kg/m}^3, \quad \text{i.e. } 1{\cdot}0\ \text{Mg/m}^3$$

where 1 megagramme (Mg) = 1000 kg

Specific weight (ρg) is the weight per unit volume. Many writers use the symbol w, but here the double symbol ρg is preferred, as it emphasises that specific weight has a different numerical value from density. It is also important to remember that specific weight is a direct function of the local gravitational acceleration g. For water (strictly, at 4° C) $\rho g =$ 9·807 kN/m³.

The *relative density* of a liquid is the density of the liquid expressed as a ratio of the density of water. Thus if a liquid has a relative density of 0·8 its density is 0·8 Mg/m³ and its specific weight is 0·8 × 9·807, *i.e.* 7·846 kN/m³.

Pressure (p) is force per unit area and is measured in N/m². Fluid pressure exhibits the following properties:

(*a*) At any point within a fluid, the pressure is the same in all directions; and

(*b*) The resultant pressure on the walls of the container of the fluid always acts normal to the surface.

These properties are made use of in, for example, a hydraulic jack in which the effort, F, is applied to a small piston of area a in a cylinder containing oil. This cylinder is connected by a pipe to a large cylinder of area A, the whole system being filled with oil. The rise in pressure in the large cylinder is the same as the rise in pressure in the small one, namely, F/a. Consequently, a piston in the large cylinder would support a load $W = A.F/a$, giving a mechanical advantage of $W/F = A/a$, if the jack were 100% efficient (*see* Fig. 1).

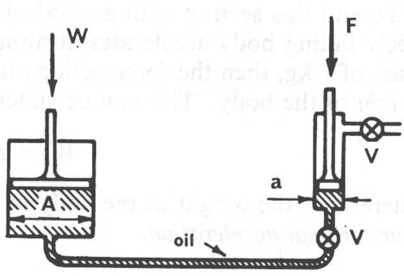

FIG. 1. The principle of the hydraulic jack. The valves which regulate the supply and flow of oil are labelled V.

Absolute pressure. At any point within a perfect vacuum the absolute pressure is zero. The absolute pressure at a point in a fluid is the pressure reckoned from this zero, and will always be positive. In fluid mechanics there are two common ways of expressing pressure: in N/m² and in m *head*. Thus, under normal conditions atmospheric pressure is approximately 101·325 kN/m² absolute, or 10·33 m head of water.

To convert from m head of water into N/m² it is convenient to imagine the column of water as having a cross-sectional area of 1m². Then the pressure at the foot of a 10·33 m column is 10·33 × 9·807 × 10³ N/m² = 101·325 kN/m².

It should be noticed that a 10·33 m depth of water gives the same liquid pressure, whatever the shape of the vessel containing the water, by virtue of the property that fluid pressure at a point is the same in all directions.

Gauge pressure. Usually, in fluid mechanics, pressures are reckoned from atmospheric pressure. In other words, atmospheric pressure is taken as zero, giving a scale of pressures which differs from the absolute scale. Pressures on this scale are described as *gauge pressures*, and are related to absolute pressures by the following equation:

Gauge pressure = Absolute pressure − Atmospheric pressure

MEASUREMENT OF PRESSURE

Of the large number of different kinds of pressure-measuring devices, the three types of manometer shown in Fig. 2 meet the needs of this book. The diligent student should seek a more complete knowledge of pressure gauges by reading up differential gauges, which are used for measuring small differences of pressure, and the Bourdon gauge, which is perhaps the commonest device in engineering for measuring gauge pressures.

In Fig. 2 (*a*) liquid of density ρ is flowing along the pipe. If the piezometer is tapped into the pipe the liquid will rise to height h above the centre-line. The pressure at the centre-line is then said to be h m of the liquid or, alternatively, $\rho g h$ N/m² gauge.

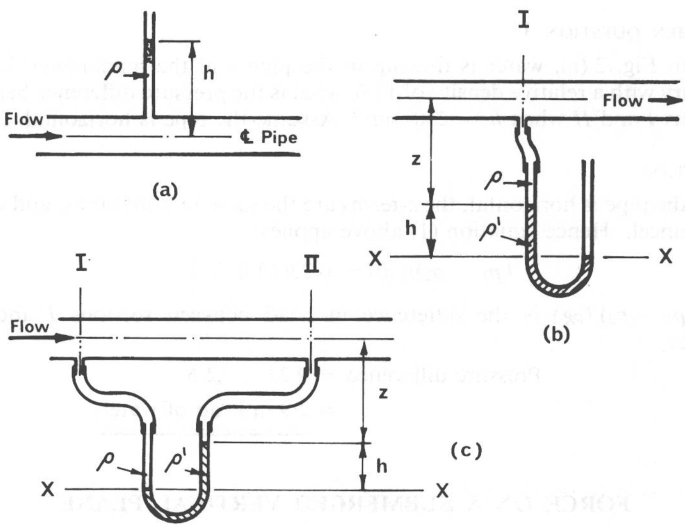

FIG. 2. Three kinds of manometer; (*a*) piezometer or stand-pipe; (*b*) and (*c*) two types of U-tube

Fig. 2 (*b*) shows a U-tube manometer which can be used to measure negative gauge pressures (as shown) or positive gauge pressures of greater magnitude than can conveniently be measured by a piezometer. The density of the liquid flowing along the pipe is ρ and the density of the liquid filling the bend of the U-tube is ρ'. To find the pressure at the centre-line of the pipe at section *I*, we note that the pressure in the two limbs of the manometer at section *XX* must be equal, for otherwise the liquid in the bend would move, being acted on by different forces. But the pressure on the right-hand limb is atmospheric, while the pressure on the left-hand limb at *XX* is p_1, the pressure at the centre-line of the pipe, plus $\rho gz + \rho' gh$. Hence,

$$p_1 + \rho gz + \rho' gh = \text{atmospheric pressure}$$
$$\therefore\ p_1 = -\rho gz - \rho' gh, \text{ gauge pressure}$$

Fig. 2(*c*) shows an adaptation of the U-tube for comparing the pressures at sections *I* and *II* of the pipe. By equating the pressures at *XX* as above, the difference between the pressure at section *I*, p_1, and at section *II*, p_2 is

$$p_1 - p_2 = \rho' gh - \rho gh$$

provided that the pipe is horizontal. This can, alternatively, be written

$$(p_1 - p_2)/(\rho g) = h(\rho'/\rho - 1) \quad . \quad . \quad . \quad (1)$$

If the liquid in the pipe is water, ρ'/ρ = relative density of the manometer fluid.

SPECIMEN QUESTION 1

If, in Fig. 2 (*c*), water is flowing in the pipe and the manometer fluid is mercury with a relative density of 13·6, what is the pressure difference between sections *I* and *II* when h = 230 mm? Assume the pipe is horizontal.

SOLUTION

As the pipe is horizontal, the *z*-terms are the same in both limbs, and therefore cancel. Hence equation (1) above applies:

$$(p_1 - p_2)/(\rho g) = 0{\cdot}23(13{\cdot}6 - 1)$$

But $(p_1 - p_2)/(\rho g)$ is the difference in head between sections *I* and *II*. Hence:

$$\text{Pressure difference} = 0{\cdot}23 \times 12{\cdot}6$$
$$= \underline{\underline{2{\cdot}9 \text{ m head of water.}}}$$

FORCE ON A SUBMERGED VERTICAL PLANE

In many hydraulics problems the thrust due to hydrostatic pressure on a submerged plane is required. The thrust on a vertical plane can *always* be found by the formula developed below; in the special case of a rectangular plane, however, diagrams showing the distribution of hydrostatic pressure on the plane can be drawn and the resulting thrust calculated from the area of these pressure diagrams.

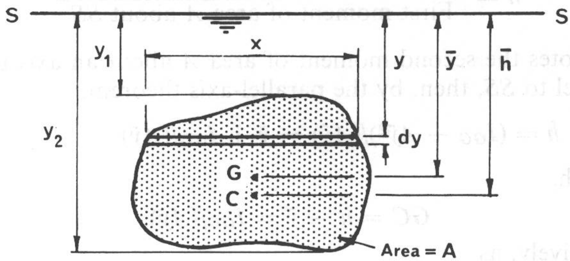

FIG. 3. An irregular-shaped submerged vertical plane

The hydrostatic force on one side of the plane shown in Fig. 3, due to liquid of density ρ, can be found by calculus:

$$\text{Pressure at depth } y = \rho g y,$$

$$\therefore \text{ Force on the element of width } dy = \text{Pressure} \times \text{Area} = \rho g y \,.\, x dy$$

Hence, over the whole area, A,

$$\text{Total force} = \rho g \int_{y_1}^{y_2} yx \,.\, dy$$

$$= \rho g \,.\, A\bar{y}$$

because the first moment of area of A about the surface SS is defined as $\int_{y_1}^{y_2} yx \,.\, dy$, and is also equal to $A\bar{y}$, where $\bar{y}$ is the depth of the centroid, G, of the area. Also, $\rho g\bar{y}$ is the pressure at G. Hence,

$$\text{Total force} = \text{Area} \times \text{Pressure at } G$$

The *centre of pressure*, C, is the point at which the total force may be considered to act, and, in general, does not coincide with the centroid, G. The depth, $\bar{h}$, of C is found by taking moments about SS. Thus

$$\text{Moment of total force about } SS = \rho g A\bar{y} \times \bar{h} \text{ and}$$

$$\text{Sum of the moments due to the forces on elements of width } dy$$

$$= \rho g \int_{y_1}^{y_2} yxy \,.\, dy$$

These two expressions represent the same thing. Hence,

$$A\bar{y} \times \bar{h} = \int_{y_1}^{y_2} y^2 x \,.\, dy$$

But $A\bar{y}$ is the first moment of area about SS and $\int_{y_1}^{y_2} y^2x \, . \, dy$ is the second moment of the area A about SS, whence

$$\bar{h} = \frac{\text{Second moment of area } A \text{ about } SS}{\text{First moment of area } A \text{ about } SS}$$

If I_{GG} denotes the second moment of area A about an axis through G and parallel to SS, then, by the parallel-axis theorem,

$$\bar{h} = (I_{GG} + A\bar{y}^2)/(A\bar{y}) = \bar{y} + I_{GG}/(A\bar{y})$$

from which,

$$GC = \bar{h} - \bar{y} = I_{GG}/(A\bar{y})$$

Alternatively, as

$$I_{GG} = Ak_G^2$$

where k_G = the radius of the gyration of the plane about G, then

$$GC = k_G^2/\bar{y}$$

i.e. C is always below G by an amount $k_G^2/\bar{y}$, which is the same as $I_{GG}/(A\bar{y})$.

In problems, $\bar{y}$ is usually known exactly. Consequently the most accurate, and often the most convenient, way of calculating $\bar{h}$ is to calculate GC by the formula and add it to $\bar{y}$.

SPECIMEN QUESTION 2

Deduce expressions for the force and depth of centre of pressure on a vertical plane surface subject to water on one side.

A culvert of 2·5 m diameter circular cross-section is closed by a vertical disc of the same diameter which is pivoted about a horizontal diameter. Determine the net turning moment about the axis if the head of water upstream of the disc is 12 m and that downstream is 6 m above the axis.

SOLUTION

In Fig. 4, G is the centre of the disc and, therefore, its centroid; P_1 is the thrust due to the water upstream and acts at A. Downstream the thrust is P_2, and the centre of pressure is B.

For a circle turning about a diameter, $I_{GG} = \pi d^4/64$.
Since $A = \pi d^2/4$,

$$I_{GG}/(A\bar{y}) = (\pi d^4/64)/(\pi \bar{y} d^2/4) = d^2/(16\bar{y}) = 0{\cdot}391/\bar{y}$$

Upstream, $\bar{y} = 12$ m and, therefore, $GA = 0{\cdot}391/12 = 0{\cdot}0326$ m, and downstream, $\bar{y} = 6$ m and, therefore, $GB = 0{\cdot}0652$ m.

The forces on the disc are:

$$P_1 = A_1 \times 12 \times 10^3 \times 9{\cdot}807 \text{ and } P_2 = A_1 \times 6 \times 10^3 \times 9{\cdot}807$$

where A_1 is the area of the plate and need not be evaluated.

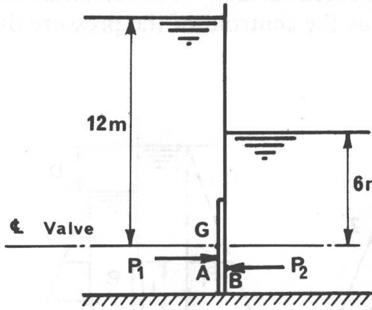

FIG. 4

Taking moments about G, the net turning moment on the disc is:

$$A_1 \times 12 \times 10^3 \times 9{\cdot}807 \times 0{\cdot}333 - A_1 \times 6 \times 10^3 \times 9{\cdot}807 \times 0{\cdot}667 = \underline{\underline{0}}$$

Specimen question 2 is a particular illustration of a general result: that the turning moment on a disc valve, under the following conditions, will be zero:

(*a*) The valve must have the same wetted area both sides.
(*b*) It must be pivoted about a horizontal diameter.
(*c*) Both sides must be submerged.

Apart from these considerations, the depth of water either side of the valve is immaterial, for on the upstream side of the valve, for example, the force on the disc is proportional to y, whereas the distance GA is proportional to $1/y$, so that the product of these two quantities is independent of y—*i.e.* it is constant. It is left to the reader, as an exercise well worth while, to write out the formal proof.

The thrust due to hydrostatic pressure on *rectangular* surfaces can sometimes be found most simply by drawing pressure diagrams and basing the calculation on them (*see* Specimen question 3).

Referring to Fig. 5, (*a*) shows a plane, vertical surface retaining water to a depth d. Triangle XYZ is the pressure diagram, and is so drawn that $Y'Z'$ represents the pressure at depth z to scale. In other words, $Y'Z'$ represents the quantity ρgz to scale. z can vary from 0 to d; to each value of z there corresponds a line, parallel to $Y'Z'$, whose length is proportional to z. Consequently the pressure diagram *is* a triangle. Moreover, YZ, the base of the triangle, scales ρgd. The total force on the wall, P, the sum of all the pressures such as that represented by $Y'Z'$ and is therefore the area of the triangle, *i.e.*

$$P = \tfrac{1}{2}\rho g d^2$$

Furthermore, P acts through the centroid of triangle XYZ, a fact easily established by calculus. But, by definition, P also acts at the centre of pressure of the submerged surface. It follows that the centre of pressure is at the same level as the centroid of the pressure diagram, in this case at a depth of $\frac{2}{3}d$.

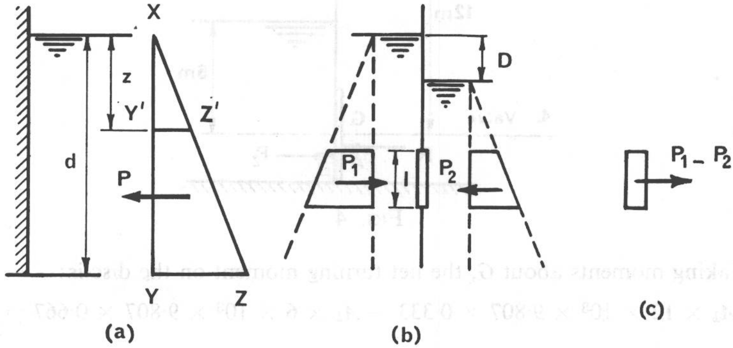

FIG. 5. Pressure diagrams. (*a*) *XYZ* is the pressure diagram for fluid pressure on the plane vertical wall, (*b*) shows the diagrams for pressure on a submerged rectangular valve, (*c*) is the resulting pressure diagram for (*b*)

Fig. 5 (*b*) shows the hydrostatic pressure distribution on a rectangular valve which is submerged on both sides. The pressure diagrams are both trapezoidal; their difference, shown in Fig. 5 (*c*), is a rectangle of width $\rho g D$ because the difference between the upstream and downstream pressures at every point on the valve is $\rho g D$. Hence the resulting thrust on the valve is

$$P_1 - P_2 = \rho g D l, \text{ per unit width of valve}$$

and acts on the horizontal centre-line of the valve.

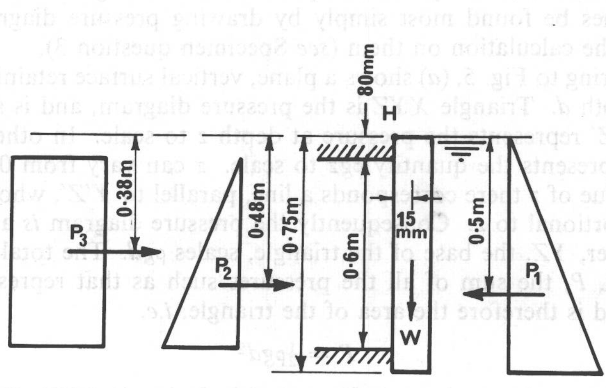

FIG. 6. Configuration, pressure diagrams and forces acting on the valve (*see* Specimen question 3)

SPECIMEN QUESTION 3

The fully submerged sea outfall of a 0·6-m-square sewer is provided with a vertical top-hung cast-steel flap valve, 0·75 m square, with the hinged edge 80 mm above the top of the sewer.

The vertical line through the centre of gravity of the flap is 15 mm outwards from the centre-line of the hinge. The weight of the flap in air is 2·5 kN; the relative density of the cast-steel is 7·75; and the density of sewage may be considered as equal to that of sea water, *i.e.* 1·028 Mg/m³.

Calculate the differential head required just to open the flap when the surface of the sea is at the level of the hinge.

SOLUTION

Effective weight of flap, allowing for buoyancy, is:

$$W = 2{\cdot}5 \times 10^3\left(1 - \frac{1{\cdot}028}{7{\cdot}75 \times 1{\cdot}000}\right)\text{N}$$
$$= 2168\ \text{N}$$

Let x m = head of sewage above top of sewer. Then, referring to Fig. 6,

$$P_1 = (0{\cdot}75)^2 \times \tfrac{1}{2} \times 0{\cdot}75 \times 1028 \times 9{\cdot}81 = 2128\ \text{N}$$
$$P_2 = (0{\cdot}6)^2 \times \tfrac{1}{2} \times 0{\cdot}6 \times 1028 \times 9{\cdot}81 = 1088\ \text{N}$$
$$P_3 = (0{\cdot}6)^2 \times 1028 \times 9{\cdot}81 \times x = 3627x\ \text{N}$$

Since P_1, P_2, P_3 act at the centroids of their respective pressure diagrams,

$$\text{depth of } P_1 \text{ below the hinge} = \tfrac{2}{3} \times 0{\cdot}75 = 0{\cdot}50\ \text{m}$$
$$\text{depth of } P_2 \text{ below the hinge} = \tfrac{2}{3} \times 0{\cdot}6 + 0{\cdot}08 = 0{\cdot}48\ \text{m}$$
$$\text{and depth of } P_3 \text{ below the hinge} = \tfrac{1}{2} \times 0{\cdot}6 + 0{\cdot}08 = 0{\cdot}38\ \text{m}$$

Hence, taking moments about H,

$$(0{\cdot}500 \times 2128) + (0{\cdot}015 \times 2168) = (1088 \times 0{\cdot}48) + (3627x \times 0{\cdot}38)$$

from which,

$$x = 0{\cdot}416\ \text{m}$$
$$\therefore \text{differential head} = 0{\cdot}416 - 0{\cdot}080 = 0{\cdot}416 - 0{\cdot}080$$
$$= \underline{\underline{0{\cdot}336\ \text{m}}}$$

SUBMERGED CURVED SURFACES

Referring to Fig. 7 (*a*), to find the forces due to a depth *d* of water, on the curved surface *LM*, we consider the equilibrium of the forces acting on the water in the region *LMN*. The forces acting on it are its weight, *W*, the force due to the water to the left of *NM*, *i.e.* *P*, and the reaction from the curved surface which is equal and opposite to *R*. To be in equilibrium the three forces must be concurrent.