NOTES ON
NUMERICAL FLUID MECHANICS

Volume 7

Maurizio Pandolfi/
Renzo Piva (Eds.)

Proceedings of the Fifth GAMM-Conference on Numerical Methods in Fluid Mechanics

Vieweg

Maurizio Pandolfi/Renzo Piva (Eds.)

Proceedings of the Fifth GAMM-Conference on Numerical Methods in Fluid Mechanics

Rome, October 5 to 7, 1983

With 263 Figures

Friedr. Vieweg & Sohn Braunschweig/Wiesbaden

CIP-Kurztitelaufnahme der Deutschen Bibliothek

Conference on Numerical Methods in Fluid Mechanics:
Proceedings of the ... GAMM Conference on Numerical
Methods in Fluid Mechanics. – Braunschweig;
Wiesbaden: Vieweg

NE: Gesellschaft für Angewandte Mathematik und
Mechanik; HST

5. Rome, October 5 to 7, 1983. – 1984.
(Notes on numerical fluid mechanics; Vol. 7)
ISBN 3-528-08081-7

NE: GT

Prof. Karl Förster, Stuttgart, laid down the editorship for private reasons.
The Vieweg Verlag and the new editors would like to thank for his vast and
untiring efforts in establishing the series.

Produced by IVD, Industrie- und Verlagsdruck, Walluf
Printed in Germany

ISBN 3-528-08081-7

Maurizio Pandolfi
Renzo Piva (Eds.)

Proceedings of the Fifth GAMM-Conference on Numerical Methods in Fluid Mechanics

Notes on Numerical Fluid Mechanics
Volume 7

Series Editors: Ernst Heinrich Hirschel, München
Maurizio Pandolfi, Torino
Arthur Rizzi, Stockholm
Bernard Roux, Marseille

Manuscripts should have well over 100 pages. As they will be reproduced fotomechanically they should be typed with utmost care on special stationary which will be supplied on request. In print, the size will be reduced linearly to approximately 75 %. Figures and diagrams should be lettered accordingly so as to produce letters not smaller than 2 mm in print. The same is valid for handwritten formulae. Manuscripts (in English) or proposals should be sent to the general editor Prof. Dr. E. H. Hirschel, MBB-UFE 122, Postfach 80 11 60, D-8000 München 80.

FOREWORD

The GAMM-Committee for Numerical Methods in Fluid Mechanics (GAMM-Fachausschuss für Numerische Methoden in der Strömungsmechanik) organizes the GAMM-CONFERENCE ON NUMERICAL METHODS IN FLUID MECHANICS every two years.
The previous four Conferences were held at the DFVLR in Köln (1975-77-79) and at ENSTA in Paris (1981). The fifth Conference was held at the University of Rome, October 5-7, 1983.
The GAMM-Conference is intended to bring together scientists who are working on numerical methods in fluid mechanics. The main objective is to foster exchanges between the various fields of development of computational fluid mechanics such as Aerodynamics, Hydrodynamics, Propulsion, Fluidmachinery, Nuclear Reactor Technology, Meteorology, Biofluidmechanics etc.
The subjects covered in the Conference are mainly related to theoretical aspects of numerical methods in fluid mechanics (finite difference methods, finite element methods, spectral methods, etc.) or to particular applications to fluid problems which may enhance the novelties of the methods themselves.
Moreover reports are presented on the GAMM-WORKSHOPS promoted by the Committee where very definite subjects have been investigated by scientists working in those particular fields.
The 1983 Conference was attended by more than 100 scientists from 16 different countries. There were 48 contributed papers and the activity on 4 GAMM-Workshops have been reported. The contributions are here presented in alphabetical order according to the first author.
The editors, who have also been the chairmen of this Conference, would like to acknowledge the support from the Faculty of Engineering of the University of Rome and the Italian National Research Council (C.N.R.) and to express their gratitude to all colleagues and personnel of the University of Rome and the Politechnic Institute of Turin for the cooperation in organizing the Conference.

December 5, 1983

Maurizio PANDOLFI

Renzo PIVA

85605/10

CONTENTS

Page

SHORT REPORTS ON GAMM WORKSHOPS

ON THE NUMERICAL SOLUTION OF THE NAVIER STOKES EQUATIONS FOR INTERNAL INCOMPRESSIBLE FLOWS IN THE PRESENCE OF FILTRATING WALLS

R. Albanese, F. Grasso, and C. Meola
Istituto di Gasdinamica
P.le Tecchio 80, Napoli, Italy, 80125

SUMMARY

A numerical algorithm has been developed to model a non standard boundary value Navier Stokes problem. The method is a variation of a scheme developed by the authors, and successfully applied to the steady state problem of two-dimensional incompressible laminar flow confined by permeable walls. Such a method saves the implicit character of the pressure/velocity correlation on the permeable boundary, thus yielding an accurate description of the transient evolution of the phenomena. Moreover it reduces the stiffness of the pressure matrix. The latter property suggests that the model can be applied as a regularization process for non permeable walls (provided that the permeability constant approaches zero), leading to the concept of "artificial permeability".

INTRODUCTION

The numerical solution of viscous incompressible laminar flows, confined by permeable walls, was recently studied in a primitive variable formulation by the present authors [1]. The particular boundary conditions imposed along such walls (normal suction/injection velocity assumed to be proportional to the pressure jump across the permeable boundary) introduced a strong coupling between velocity and pressure fields. The implicit character of the problem was effectively bypassed by assuming a sort of delay time between pressure jump and velocity without affecting the steady state solution. However the above approach with the assumed explicit pressure/velocity correlation is not adequate to study the transient of the flow evolution and is not suitable for an implicit numerical solution of the equations.

In the present work a modified algorithm has been developed by implicitly treating the coupling between pressure and velocity along the permeable walls, so as to satisfy the implicit character of the particular boundary value problem. The proposed algorithm yields meaningful detailed informations during the transient of the phenomena.

A careful analysis of the physical and mathematical correlations between boundary conditions, continuity properties of the solution, and the proposed numerical discretization, has shown that the present treatment of this non standard boundary value Navier Stokes (BVNS) problem can also be exploited for non permeable walls. This seems to lead to the concept of "artificial permeability", in analogy with other regularization and/or optimization techniques as the "artificial compressibility" of Chorin, the "artificial viscosity" of V.Neuman etc.

The method has been tested comparing the results with the ones obtained by the approach of Ref. [1] . In the present work the effects of the gravity forces and the exit velocity and pressure boundary conditions on the flow field have also been studied. Finally the applicability of the "artificial permeability" model has been tested for a driven cavity flow where the velocity boundary conditions are exactly known.

THE MODEL

The model equations are:

$$\left.\begin{aligned} \underline{v}_t &= \underline{R} - \underline{\nabla} p + \underline{f} \\ \underline{\nabla}\cdot\underline{v} &= 0 \end{aligned}\right\} \quad \text{in } \Omega \qquad (1)$$

where $\underline{f}$ represents the mass forces, and $\underline{R} = -\underline{v}\cdot\underline{\nabla}\,\underline{v} + \nabla^2\underline{v}\,/\mathrm{Re}$.

Boundary conditions are:

$$\underline{v} = \underline{v}_1 \qquad \text{on } B\Omega_1 \qquad (2)$$

$$\left.\begin{aligned} \underline{s}\cdot\underline{v} &= v_{s2} \\ p &= p_2 \end{aligned}\right\} \quad \text{on } B\Omega_2 \qquad (3)$$

$$\left.\begin{aligned} \underline{n}\cdot\underline{v} &= kp \\ \underline{s}\cdot\underline{v} &= 0 \end{aligned}\right\} \quad \text{on } B\Omega_3 \qquad (4)$$

The strong coupling between injection/suction velocity and pressure, introduced by Eqn. (4) shows that the pressure field cannot be determined only by a constant. Moreover if the effects of conservative external forces are included in the pressure potential, then Eqns. (3)-(4) must be consequently modified.

NUMERICAL SOLUTION

According to the conclusions of Ref. [1] , the discretized governing equations have been obtained following a finite volume approach. For rectangular geometries the grid points have been evenly spaced in x and y with mesh size $\Delta x = \Delta y$. The velocity components have been defined at the grid nodes, the pressure at the center of the geometric cell.

For internal momentum cell (i,j) (centered around a velocity node) the equations are:

$$u^{np}_{ij} = u^{n}_{ij} - \alpha\left((u^{n}_{ipj}+u^{n}_{ij})^{2} - (u^{n}_{ij}+u^{n}_{imj})^{2} + (u^{n}_{ijp}+u^{n}_{ij})(v^{n}_{ijp}+v^{n}_{ij}) - (u^{n}_{ij}+u^{n}_{ijm})(v^{n}_{ij}+v^{n}_{ijm})\right) +$$

$$+\beta(u^{n}_{ipj}+u^{n}_{imj}-4u^{n}_{ij}+u^{n}_{ijp}+u^{n}_{ijm}) - 2\alpha(p^{np}_{ij}+p^{np}_{ijm}-p^{np}_{imj}-p^{np}_{imjm}) \qquad (5)$$

$$v^{np}_{ij} = v^{n}_{ij} - \alpha\left((u^{n}_{ipj}+u^{n}_{ij})(v^{n}_{ipj}+v^{n}_{ij}) - (u^{n}_{ij}+u^{n}_{imj})(v^{n}_{ij}+v^{n}_{imj}) + (v^{n}_{ijp}+v^{n}_{ij})^{2} - (v^{n}_{ij}+v^{n}_{ijm})^{2}\right) +$$

$$+\beta(v^{n}_{ipj}+v^{n}_{imj}-4v^{n}_{ij}+v^{n}_{ijp}+v^{n}_{ijm}) - 2\alpha(p^{np}_{ij}+p^{np}_{imj}-p^{np}_{ijm}-p^{np}_{imjm}) \qquad (6)$$

For every mass cell but the ones along the permeable walls, the discretized conservation equation is:

$$u^{np}_{ipj}+u^{np}_{ipjp}-u^{np}_{ij}-u^{np}_{ijp}+v^{np}_{ijp}+v^{np}_{ipjp}-v^{np}_{ij}-v^{np}_{ipj} = 0 \qquad (7)$$

On cells adjacent to the permeable walls Eqn. (7) becomes:

$$u^{np}_{ipj}+u^{np}_{ipjp}-u^{np}_{ij}-u^{np}_{ijp}+2k_{i}\,p^{np}_{ij}-v^{np}_{ipj}-v^{np}_{ij} = 0 \qquad (8)$$

Boundary conditions are:

$$\underline{v}^{np}_{ij} = \underline{v}^{np}_{1ij} \qquad \text{on } B\Omega_1 \qquad (9)$$

$$\left.\begin{aligned} \underline{s}\cdot\underline{v}^{np}_{ij} &= vs^{np}_{2ij} \\ p^{np}_{ij} &= p^{np}_{2ij} \end{aligned}\right\} \qquad \text{on } B\Omega_2 \qquad (10)$$

$$\left.\begin{aligned} u^{np}_{ij} &= 0 \\ v^{np}_{ij} &= (k_{im}\,p^{np}_{imj}+k_{i}\,p^{np}_{ij})/2 \end{aligned}\right\} \qquad \text{on } B\Omega_3 \qquad (11)$$

From Eqns. (7)-(8) observe the different discretization of the divergence operator along the permeable walls (consistent with a different definition of the mass flux through such boundaries). Furthermore note the implicit treatment of the filtrating boundary conditions.

In a quasi matrix form the governing equations are:

$$\begin{pmatrix} I & -\Delta t B^{T} & 0 \\ B & B_{pm} & 0 \\ 0 & M_{p} & I \end{pmatrix} \cdot \begin{pmatrix} \underline{v} \\ P \\ \underline{v}_{m} \end{pmatrix}^{np} = \begin{pmatrix} I+\Delta t A & 0 & A \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \underline{v} \\ P \\ \underline{v}_{m} \end{pmatrix}^{n} + \begin{pmatrix} \underline{c}_{1} \\ c_{2} \\ 0 \end{pmatrix} \qquad (12)$$

where $\underline{v}_m$ is the injection/suction velocity, and $\underline{c}_1$, c_2 account for boundary conditions and external forces. A, B and $-B^T$ represent respectively the discretized $\underline{R}$, divergence and gradient operators (note that the adjoint character of B and B^T is maintained for the assumed discretization). The definitions of B_{pm} and M_p ("membrane flux" and "membrane permeability" matrices) follow from Eqns. (8),(11).

The solution of the system (12) requires the simultaneous solution of pressure and velocity. Premultiplying Eqn. (12) by the non singular matrix T, defined as:

$$T = \begin{pmatrix} I & 0 & 0 \\ -B & I & 0 \\ 0 & 0 & I \end{pmatrix}$$

the following equation for p is obtained:

$$(\Delta t B \cdot B^T + B_{pm}) \cdot p^{np} \equiv M \cdot p^{np} = -B \cdot (I + \Delta t A) \cdot \underline{v}^n + c_2' \equiv Q \qquad (13)$$

From the definitions of B, B^T and B_{pm}, M is shown to satisfy the following properties [2] : i) it is symmetric and positive definite; ii) with an appropriate reordering (i+j even/odd) it can be reduced to a two block diagonal matrix; iii) property A; iv) weak diagonal dominance (with strong character for the rows along the permeable boundary).

Each of the two block matrices (M', M'') satisfies properties i),iii) and iv), hence is 2-cyclic in the sense of Varga [2] .

From definition of the pressure matrix M, Eqn. (13) is shown to be formally consistent with the usual elliptic equation for p

$$\nabla^2 p = \underline{\nabla} \cdot (\underline{R} + \underline{f}) - \underline{\nabla} \cdot \underline{v}_t \qquad (14)$$

generally obtained by taking the divergence of the momentum equation [3] - [4] .

The closure of Eqn. (14) can be obtained by assuming that $\underline{v}$ satisfies some smoothness properties so that Eqn. (1) can be extended to the boundary by a limit process, thus yielding boundary conditions in terms of pressure gradient [1] , [5] , [6] . However such a naïve procedure may lead to paradoxes especially if pressure tangential derivatives are deduced [1] . Eqns. (9)-(11) seemingly allow the closure of the elliptic pressure equation yielding Neumann conditions on $B\Omega_1$, Dirichlet conditions on $B\Omega_2$, and Robin b.c. on $B\Omega_3$. Moreover the strong solution of Eqn. (14) implies $p \in C^2$, while Eqn. (1) only requires $p \in C^1$.

The solution of Eqn. (13) does not require such smoothness assumptions and it bypasses the whole closure problem for the differential BVP formulation by simply imposing:

$$\int_{\partial\Sigma} \underline{n} \cdot \underline{v} \, dS = 0 \qquad \forall \; \Sigma \subseteq \Omega$$

in a discretized form.

For an internal cell (i,j), Eqn. (13) yields:

$$p^{np}_{imjm} + p^{np}_{ipjp} - 4p^{np}_{ij} + p^{np}_{imjp} + p^{np}_{ipjm} = 2\Delta x^2 q^n_{ij} \tag{15}$$

For a cell adjacent to a non permeable boundary, Eqn. (13) gives:

$$p^{np}_{ipjm} - 2p^{np}_{ij} + p^{np}_{imjm} = \Delta x \, q^n_{ij} \tag{16}$$

For a cell along $B\Omega_3$ one has:

$$p^{np}_{ipjm} - (1 + k_i \Delta x/\Delta t) p^{np}_{ij} + p^{np}_{imjm} = \Delta x \, q^n_{ij} \tag{17}$$

Eqn. (17) shows the effect on the structure of $\mathcal{M}$ due to the implicit treatment of the b.c. and the particular definition of the numerical divergence operator (Eqns. (8),(11)). Such an equation is consistent with a Robin type b.c. for the differential equation for p; i.e. :

$$k\, p_t + p_n = \underline{n} \cdot (\underline{R} + \underline{f})$$

Such a boundary condition reduces the computational effort to obtain the pressure field with respect to the standard BVNS problem. The advantage of employing the above formulation for non permeable walls thus follows. In this case the "differential" b.c. for p would be:

$$(kp)_t + p_n = \underline{n} \cdot (\underline{R} + \underline{f})$$

with k approaching zero as t increases.

In such a case the algorithm can be interpreted as an iterative method yielding the correct non permeable steady state solution. In other words the concept of the "artificial permeability" (AP) can be viewed as an artificial compressibility limited to the cells adjacent to the solid boundary and vanishing at steady state.

RESULTS AND DISCUSSIONS

The incompressible laminar NS equations with non standard b.c. have been solved by using a finite difference algorithm that saves the implicit character of the problem (due to the incompressibility and the particular pressure/velocity correlation), guaranteeing the mass conservation for every computational cell.

For the chosen staggering (with velocity components at the geometrical nodes, and pressure unknowns at the center of the mass cells) both velocity components can be assigned at the boundaries. It can be shown that such a grid configuration corresponds to the overlapping of two grids with the classical staggering [3] , having mesh sizes $\sqrt{2}\Delta x$, and grid lines rotated of 45° with respect to x and y. Consequently the method yields two uncoupled systems of equations for the pressure by separating the p unknowns in even and odd ones. The coupling of the fluiddynamic field is obtained by an appropriate discretization of the momentum flux $\underline{R}$. Moreover proper care must be taken for the assignement of the b.c., since a well posed problem must be imposed on each of the two "overlapping" grids.

The model has been tested to describe the flow motion in the presence of permeable walls for a variety of geometries and operating conditions. Figs. 1.a-e show the effects of different values of the exit pressure for given inlet mass flow rate, Reynolds number (Re=10), and filtrating constant. Observe that the membrane flux does vary linearly with the exit pressure. Moreover for the selected Re (Figs. 1.a-c) by simply imposing $\underline{s}\cdot\underline{v} = 0$ and p = constant at the exit, the Poiseuille flow is recovered in the outlet region. Figs. 2-3 show the computed results in forced percolators. The effects of gravity are illustrated in Figs. 3.b-c; the differences on the velocity field are due to the particular boundary conditions (Eqn. (4)). The results in a "shear filtrating pump" configuration are plotted in Fig . 4 . Finally Fig. 5 shows the application of the method to investigate the validity of the AP concept in a driven cavity flow configuration. The AP method is equivalent to a regularization one, yielding an accurate description of the steady state when k approaches zero.

In conclusions the applicability of the proposed method to calculate a variety of transient and steady flow configurations has been shown, even in the presence of external forces. The concept of artificial permeability has been introduced (with some analogies with the artificial compressibility); however the advantages of using it depend on the filtrating law k(t) and on the number of grid points.

REFERENCES

[1] Albanese, R., Grasso, F., Meola, C., "On the critical problem of f.d. pressure treatment for laminar flows confined by permeable walls", submitted for publication to Int. Jour. of Numerical Methods in Engineering, and to be presented at Third International Conference on Numerical Methods in Laminar and Turbulent Flows, Seattle, Washington, August 8-11, 1983.

[2] Young, D., "Iterative solution of large linear systems", Acad. Press, 1971.

[3] Harlow, F.H., Welch, J.A., "Numerical calculation of time dependent viscous incompressible flow of fluid with a free surface", Phys. of Fluids, 8 (1965), pp 2182-2189.

[4] Chorin, A.J., "A numerical method for solving incompressible viscous flow problems", Jour. of Comp. Phys., 2 (1967), pp 12-26.

[5] Temam, R., "Navier Stokes equations", North Holland, 1979.

[6] Moin, P., Kim, J., "On the numerical solution of time dependent viscous incompressible fluid flows involving solid boundaries", Jour. of Comp. Phys., 35 (1980), pp 381-392.

(a) $p_{ex} = 50$

(b) $p_{ex} = 45$

(c) $p_{ex} = 40$

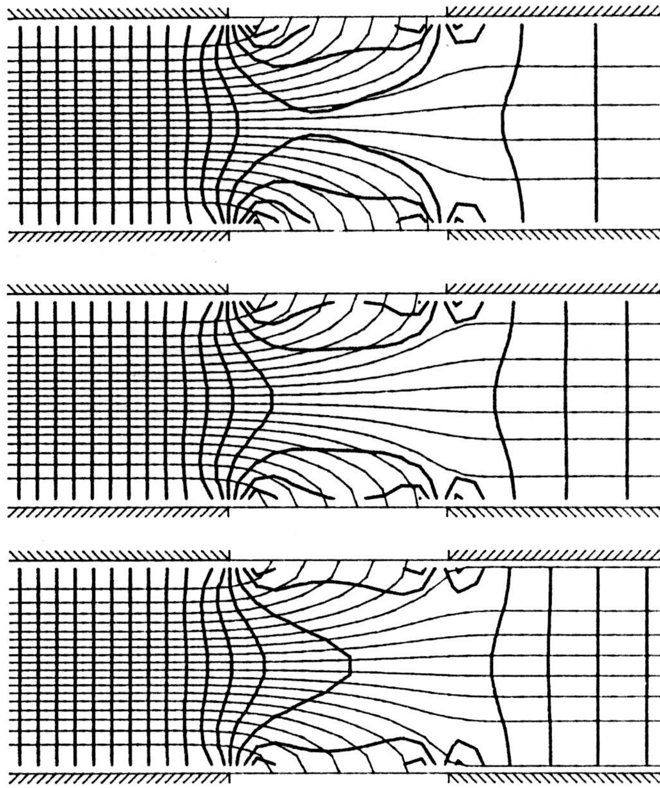

Fig. 1.a-c isobars (—), streamlines (–)
(Re=10; k=.06)

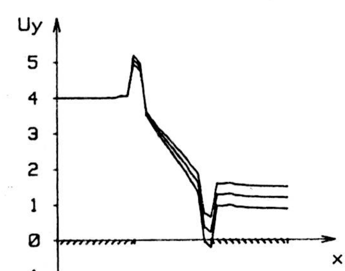

Fig. 1.d wall shear stress

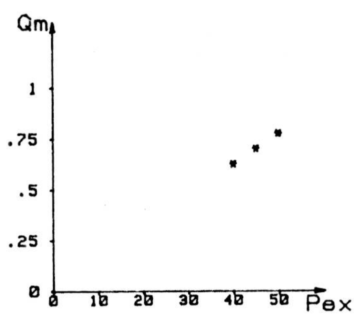

Fig. 1.e membrane mass flow rate

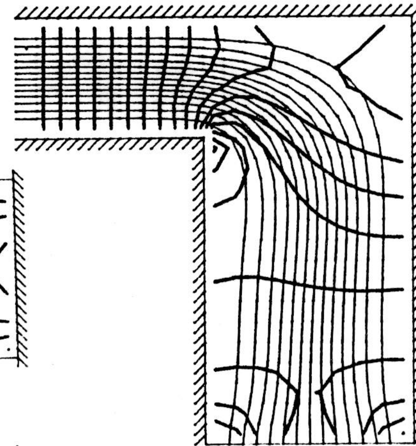

Fig. 2 (Re=10; k=.02)

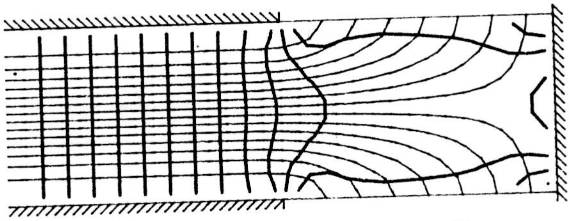

Fig. 3.a Fr=∞ (Re=10; k=.02)

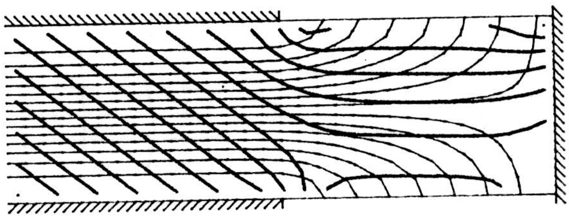

Fig. 3.b Fr=.1

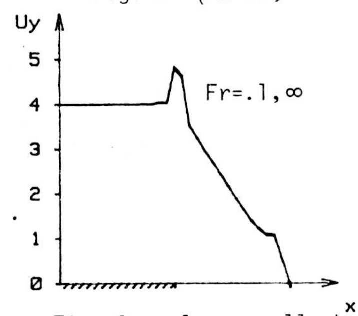

Fig. 3.c lower wall shear stress

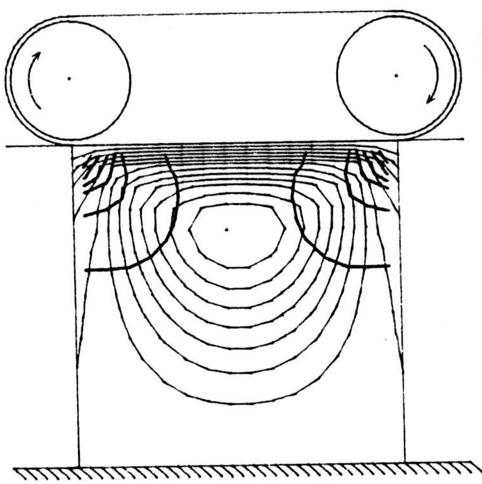

Fig. 4 Shear Filtrating Pump (Re=10; k=.02)

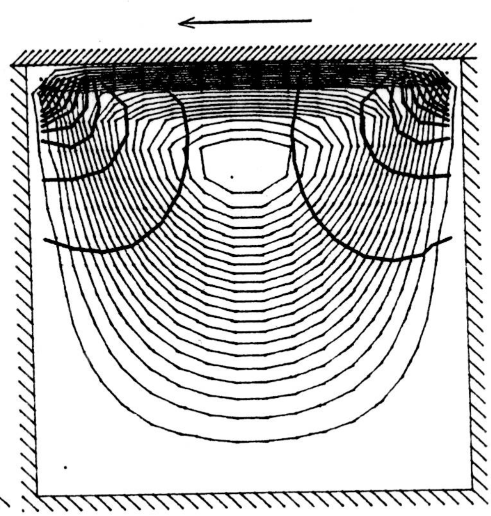

Fig. 5 Driven Cavity Flow by AP Method (Re=10; k=.02exp(-3t/1-3t))