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Leo A. Goodman
William H. Kruskal

Measures of Association for Cross Classifications

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Foreword

In 1954, prior to the era of modern high speed computers, Leo A. Goodman and William H. Kruskal published the first of a series of four landmark papers on measures of association for cross classifications. By describing each of several cross classifications using one or more *interpretable* measures, they aimed to guide other investigators in the use of sensible data summaries. Because of their clarity of exposition, and their thoughtful statistical approach to such a complex problem, the guidance in this paper is as useful and important today as it was on its publication 25 years ago.

Summarizing association in a cross-classification by a single number inevitably loses information. Only by the thoughtful choice of a measure of association can one hope to lose only the less important information and thus arrive at a satisfactory data summary. The series of four papers reprinted here serve as an outstanding guide to the choice of such measures and their use.

Many users view measures of association as they do correlations, applicable to essentially all data sets. To their credit, Goodman and Kruskal argue that ideally each research problem should have one or possibly several measures of association, with operational meaning, developed for its unique needs. Because the Goodman-Kruskal papers provide what amounts to a comprehensive catalogue of existing measures (several of which they themselves created), analysts may begin by examining and attempting to choose wisely from those measures currently available. If none are satisfactory, and new ones are created, the Goodman-Kruskal papers will be helpful as models and guides.

This series of papers evolved over a twenty year period. The first and core paper appeared in 1954. It suggests criteria for judging measures of association and introduces several new measures with specific contextual meanings. Examples and illustrations abound. The 1959 paper serves as a supplement to the inital one and provides additional historical and bibliographic material. The 1963 paper

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derives large-sample standard errors for the sample analogues of population measures of association and presents some numerical results about the adequacy of large-sample normal approximations. The 1972 paper presents a new look at the asymptotics, and provides a more unified way to derive large-sample variances for those measures of association that can be expressed as ratios of functions of the cell probabilities. Thus the techniques can be used for tried and true measures, and also for ones not yet invented. Only by rereading these papers many times can one appreciate the perspicacity that the authors have brought to this perplexing problem.

As a colleague of Leo and Bill at The University of Chicago, I was privileged to witness the care and scholarly attention they gave to the last of the measures of association papers. It was truly a labor of love. Thus I am delighted both personally and as a member of the Editorial Advisory Board for the Springer Statistical Series that Springer-Verlag has been able to bring together these four papers in a single volume, so that they can be shared with a new generation of statisticians and scientists.

August, 1979 STEPHEN E. FIENBERG

Preface*

In the early 1950s, as young faculty members at the University of Chicago, we had separate conversations with senior colleagues there about statistical treatment of data that were naturally arranged as cross classifications of counts. One of us talked to Bernard Berelson (then Dean of the Graduate Library School and later the President of the Population Council), who was at that time dealing with extensive cross classifications related to voting behavior. For example, he might have a number of cross classifications of intended vote against educational level for different sections of a city.

The other conversations were with the late Louis Thurstone (a major figure in the field of psychometrics, and in particular in the development of factor analysis), who also was dealing with multiple cross classifications in the context of the relationships between various personal characteristics (e.g., leadership ability) and results from various psychological tests.

In both cases the investigator had substantial numbers of cross classifications and needed a sensible way to reduce the data to try to make it coherent. One promising approach was felt to be replacement of each cross classification by a number (or numbers) that measured in a reasonable way the degree of association between the characteristics corresponding to the rows and columns of the tabulated cross classification.

Thus, the two of us were independently thinking about the same question. We discovered our mutual interest during a conversation at a party—we think that it was a New Year's Eve party at the Quadrangle (Faculty) Club—and the paper grew out of that interaction.

We knew something of the existing literature on measures of association for

*Most of this preface appeared in ''This Week's Citation Classic'', *Current Contents*, Social and Behavioral Sciences, No. 26, 25 June 1979, page 14.

cross classifications, and as we studied it further we recognized that most suggested measures of association were formal and arbitrary, without relevant interpretations—or without interpretations at all. Our contribution was to suggest a number of association measures that have interesting interpretations and to provide a simple taxonomy for cross classifications. As an example of the latter, we emphasized the importance of knowing whether or not the categories of a classification have not a natural ordering.

Since cross classifications occur throughout science, since our emphasis on interpretation was perhaps novel, and since our work was quickly incorporated into textbook expositions, citations to the paper became numerous. We continued work on the topic, digging more deeply into its history and fields of application, and treating at length the relevant approximate sampling theory in an effort to contribute some new approaches and to effect some changes in statistical thinking and practice.

One of us also developed an interest in ordinal measures of association beyond cross classifications as such.[1] The other was led to extensive research in the analysis of multi-way cross classifications, leading to what have come to be known as log-linear model theory and methodology.[2] Another outgrowth, we dare to hope, of our paper has been fresh general concern with descriptive statistics from the viewpoint of finding usefully interpretable characteristics of populations and samples.

In this reprinting, notes appear in the margin at a few points to indicate errors that were corrected in later papers of the sequence. One additional trivial error has been directly corrected. Otherwise the papers appear just as they originally appeared.

We end this preface with a statement of thanks to W. Allan Wallis, first Chairman of the Department of Statistics at the University of Chicago. There are many reasons for us to thank him, but the relevant one now is that he introduced us to Berelson and to Thurstone, and from those introductions our thinking on measures of association arose. Wallis, in fact, did far more than perform introduction: he discussed our nascent work with us, and suggested an important approach with which his name is associated in our first paper.

Chicago, Illinois
September, 1979

Leo A. Goodman
William H. Kruskal

[1]Kruskal, W. H. Ordinal measures of association. *J. Amer. Statist. Assoc.* 53:814–61, 1958.

[2]Goodman, L. A. The multivariate analysis of qualitative data: interactions among multiple classifications. *J. Amer. Statist. Assoc.* 65:226–56, 1970.

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MEASURES OF ASSOCIATION FOR CROSS CLASSIFICATIONS*

Leo A. Goodman and William H. Kruskal
University of Chicago

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When populations are cross-classified with respect to two or more classifications or polytomies, questions often arise about the degree of association existing between the several polytomies. Most of the traditional measures or indices of association are based upon the standard chi-square statistic or on an assumption of underlying joint normality. In this paper a number of alternative measures are considered, almost all based upon a probabilistic model for activity to which the cross-classification may typically lead. Only the case in which the population is completely known is considered, so no question of sampling or measurement error appears. We hope, however, to publish before long some approximate distributions for sample estimators of the measures we propose, and approximate tests of hypotheses. Our major theme is that the measures of association used by an empirical investigator should not be blindly chosen because of tradition and convention only, although these factors may properly be given some weight, but should be constructed in a manner having operational meaning within the context of the particular problem.

1. INTRODUCTION

MANY studies, particularly in the social sciences, deal with populations of individuals which are thought of as cross-classified by two or more polytomies. For example, the adult individuals living in New York City may be classified as to

Borough:	5 classes
Newspaper most often read:	perhaps 6 classes
Television set in home or not:	2 classes
Level of formal education:	perhaps 5 classes
Age:	perhaps 10 classes

For simplicity we deal largely with the case of two polytomies, although many of our remarks may be extended to a greater number. The double polytomy is the most common, no doubt because of the ease with which it can be tabulated and displayed on the printed page. Most of our remarks suppose the population completely known in regard to the classifications, and indeed this seems to be the way to begin in the construction of rational measures of association. After agreement has been reached on the utility of a measure for a known population, then

(London School of Economics and Political Science), Frederick Mosteller (Harvard University), I. Richard Savage (National Bureau of Standards), Alan Stuart (London School of Economics and Political Science), Louis L. Thurstone (University of North Carolina), John W. Tukey (Princeton (University), W. Allen Wallis (University of Chicago), and E. J. Williams (Commonwealth Scientific and Industrial Research Organisation, Australia). Part of Mr. Goodman's work on this paper was carried out at the Statistical Laboratory of the University of Cambridge under a Fulbright Award and a Social Science Research Council Fellowship. The authors were led to work on the problems of this paper as a result of conversations with Louis L. Thurstone and Bernard R. Berelson.

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one should consider the sampling problems associated with estimation and tests about this population parameter.

A double polytomy may be represented by a table of the following kind:[1]

A	B				Total
	B_1	B_2	$\cdots$	B_β	Total
A_1	ρ_{11}	ρ_{12}	$\cdots$	$\rho_{1\beta}$	$\rho_{1\cdot}$
A_2	ρ_{21}	ρ_{22}	$\cdots$	$\rho_{2\beta}$	$\rho_{2\cdot}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_α	$\rho_{\alpha 1}$	$\rho_{\alpha 2}$	$\cdots$	$\rho_{\alpha\beta}$	$\rho_{\alpha\cdot}$
Total	$\rho_{\cdot 1}$	$\rho_{\cdot 2}$	$\cdots$	$\rho_{\cdot\beta}$	1

where

Classification A divides the population into the α classes $A_1, A_2, \cdots, A_\alpha$.

Classification B divides the population into the β classes $B_1, B_2, \cdots, B_\beta$.

The proportion of the population that is classified as both A_a and B_b is ρ_{ab}.

The marginal proportions will be denoted by

$$\rho_{a\cdot} = \text{the proportion of the population classified as } A_a.$$
$$\rho_{\cdot b} = \text{the proportion of the population classified as } B_b.$$

If the use to which a measure of association were to be put could be precisely stated, there would be little difficulty in defining an appropriate measure. For example, using the above cross-classification of the New York City population, a television service company might wish to

[1] Tables of this kind are frequently called *contingency tables*. We shall not use this term because of its connotation of a specific sampling scheme when the population is not known and one infers on the basis of a sample.

place a single newspaper advertisement which would be read by as many prospective customers as possible. Then the important information from the table of newspaper-most-often-read vs. television-set-in-home-or-not would be: which newspaper is most often read among those with television sets? And a reasonable measure of association would simply be the proportion of those with television sets who read this newspaper.

It is rarely the case, however, that the purpose of an investigation can be so specifically stated. More typically an investigation is exploratory or has a multiplicity of goals. Sometimes a measure of association is desired simply so that a large mass of data may be summarized compactly.

The basic theme of this paper is that, even though a single precise goal for an investigation cannot be specified, it is still possible and desirable to choose a measure of association which has contextual meaning, instead of using as a matter of course one of the traditional measures. In order to choose a measure of association which has meaning we propose the construction of probabilistic models of predictive activity, the particular model to be chosen in the light of the particular investigation at hand. The measure of association will then be a probability, or perhaps some simple function of probabilities, within such a model. Such is our general contention; most of the remainder of this paper is concerned with its exemplification in particular instances.

We wish to emphasize that the specific measures of association described here are *not* presented as factotum or universal measures. Rather, they are suggested as reasonable for use in appropriate circumstances only, and even in those circumstances other measures may and should be considered and investigated.

A good deal of attention has been paid in the literature to the special case of two dichotomies. We are more interested here in measures of association suitable for use with any numbers of classes in the polytomies or classifications.

2. FOUR PRELIMINARY CONSIDERATIONS

Four distinctions or cautionary remarks should be made early in any discussion of measures of association.

2.1. *Continua*

We may or may not wish to think of a polytomy as arising from an underlying continuum. For example, age may for convenience be divided into ten classifications, but it clearly does arise from an underlying continuum; however, newspaper-most-often-read would scarcely be so construed. If a polytomy does arise from an underlying continuum one may or may not wish to assume that the population has some specific kind of distribution with respect to it.

In those cases in which all the polytomies of a study arise jointly from a multivariate normal distribution on an underlying continuum, one would naturally turn to measures of association based on the correlation coefficients. These in turn might well be estimated from a sample by the tetrachoric correlation coefficient method or a generalization of it. In some cases one polytomy may arise from a continuum and the other not. An interesting discussion of this case for two dichotomies was given in 1915 by Greenwood and Yule ([3], Section 3). We do not discuss either of these cases in this paper, but restrict ourselves to situations in which there are no relevant underlying continua.

The desirability of assuming an underlying joint continuum was one of the issues of a heated debate forty years ago between Yule [15] on the one hand and K. Pearson and Heron [9] on the other. Yule's position was that very frequently it is misleading and artificial to assume underlying continua; Pearson and Heron argued that almost always such an assumption is both justified and fruitful.

2.2. *Order*

There may or may not be an underlying order between the classifications of a polytomy. For example "level of formal education" admits an obvious ordering; but borough of residence would not usually be thought of in an ordered way. If there is an ordering, it may or may not be relevant to the investigation. Sometimes an ordering may be important but not its direction. If there is an underlying one-dimensional continuum, it establishes an ordering.

When there is no natural or relevant ordering of the classes of a polytomy, one may reasonably ask that a measure of association not depend on the particular order in which the classes are tabulated.

2.3. *Symmetry*

It may or may not be that one looks at two polytomies symmetrically. When we are sure a priori that a causal relationship (if it exists) runs in one direction but not the other, then our viewpoint will be asymmetric. This will also happen if one plans to *use* the results of the experiment in one direction only. On the other hand, there is often no reason to give one polytomy precedence over another.

2.4. *Manner of Formation of the Classes*

Decisions about the definitions of the classes of a polytomy, or changes from a finer to a coarser classification (or vice-versa), can affect all the measures of association of which we know. For example, suppose we begin with the 4×4 table

0	.25	0	0
.25	0	0	0
0	0	0	.25
0	0	.25	0

and combine neighboring pairs of classes. We obtain

.5	0
0	.5

which might greatly change a measure of association. Or we might combine the three bottom rows and the three right-hand columns. This gives

0	.25
.25	.5

which presents quite a different intuitive degree of association. By other poolings one can obtain other 2×2 tables.

Although this example is extreme, similar changes can be made in the character of almost any cross-classification table. Related examples are discussed by Yule [15].

At first this consideration might seem to vitiate any reasonable discussion of measures of association. We feel, however, that it is in fact desirable that a measure of association reflect the classes *as defined for*

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