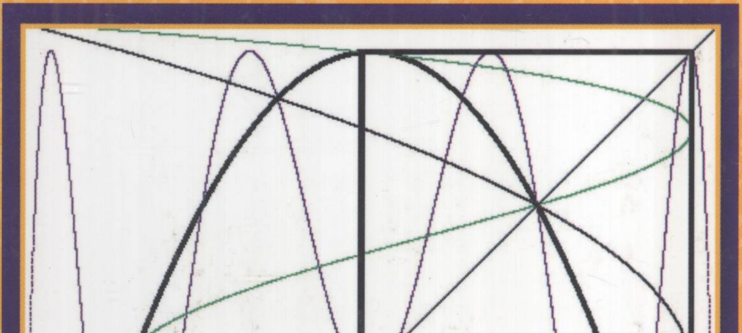
CHAOS APPLICATIONS IN TELECOMMUNICATIONS



edited by

Peter Stavroulakis

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This book is dedicated to my wife Nina and my four sons
Peter, Steven, Bill, and Stelios, whose patience and love gave me
courage when the work seemed insurmountable

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Preface

Chaos, up to a few years ago, was just an interesting field to researchers who studied the behavior of nonlinear dynamical systems of certain mathematical structure.

Chaotic systems are frequently viewed in phase space, which is a region of space where the current state of the system perpetually exists. Regions of space without the perpetual existence of chaotic dynamics are uninteresting because the points in these areas move off to infinity and do not contribute to the continuation of the chaotic process. The chaotic equations thus derived operate on coordinates in space and each iteration of the equations signifies the passage of the next time increment. This type of modeling (phase space coordinates) facilitates not only the presentation of trajectories or orbits that the chaotic system follows in the temporal evolution but also easily clarifies the concept of synchronization, which plays a central role in the communication properties of chaos. By synchronization of two chaotic systems, we define the situation by which a chaotic system is driven (coupled) by a phase signal of another chaotic system in such a way that these two systems eventually synchronize. In other words, the other phase variables of the two systems converge to the same value one to one. The concept of transmission of information from one system to the other has thus been developed due to this convergence. For example having constructed two chaotic systems that can be synchronized, we somehow modulate on one phase signal of one system the information signal to be transmitted, which drives the other system, and after synchronization, we subtract (demodulate) the information from the corresponding phase signal of the other coupled chaotic system. This technique is demonstrated in this book in various applications of communication systems. A great deal of introductory details are given in Chapter One. Chapter Two shows how chaotic signals are generated and transmitted. The design of chaotic transmitters and receivers is shown in Chapter Three whereas chaos-based modulation and demodulation techniques are presented in Chapter Four. In Chapter Five a chaos-based spreading sequence is shown to outperform classical pseudorandom sequences in two important cases such as selective and nonselective channels. In Chapter Six, several channel equalization techniques specifically designed for chaotic communications systems are developed by using knowledge of the systems dynamics, linear time-invariant representations of chaotic systems, and symbolic dynamics representations of chaotic systems. Finally a specific application is presented in Chapter Seven for optical communications. The fundamental concepts of chaos and its modeling are presented in Appendix A and Appendix B.

It is believed and hoped that this book will provide the essential reading material for those who want to have an integrated view of how an old concept such as chaos can open new roads in the communications field.

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Introduction

Peter Stavroulakis

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1.1 Chaos Modeling

In the beginning, according to ancient Greek theology and philosophy, there was chaos.[1] The Greek philosophers believed that our ordered universe, the cosmos, was formed out of this chaos. Ancient Greeks did not give a specific definition to chaos even though it was related to infinity, disorder, and unpredictability. The long-term behavior of chaos, however, was well understood because, as Heraclitus said, "**Τα παντα ρει**," which means all things flow or all is flux, and the only interpretation of that is that the cosmos was formed out of this chaos. In other words, ancient Greeks believed what modern science discovered centuries later, that disorder can result in order under certain conditions and thus the Greeks taught us of the existence of attractors or limit cycles, as explained mathematically in Appendix A. This type of evolution has been proved implicitly by science because all natural processes have one direction of evolution, which is the increase of entropy or reaching the state of minimum energy. If we extrapolate this further, it is not impossible to believe that the present-day universe is an attractor of the Big Bang. Because order cannot be generated from nothing, it is necessary to believe that chaos does not mean absence of order but that there exists order in chaos. The order in chaos is not obvious, nor are the initial conditions that are necessary to arrive at some kind of order. This subtle conclusion is what has triggered the interest of scientists from ancient times to the present. This is exactly what scientists are using, the evolution of chaos, to study many phenomena and processes, including telecommunications, as we shall see in subsequent chapters.

Chaos is an aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions.[1]

The three components of the definition are clarified as follows:

1. "Aperiodic long-term behavior" means that the system's trajectory in phase space does not settle down to any fixed points (steady state), periodic orbits, or quasi-periodic solutions as time tends to infinity. This part of the definition differentiates aperiodicity due to chaotic dynamics from the transient aperiodicity of, for example, a periodically oscillating system that has been momentarily perturbed.
2. "Deterministic" systems can have no stochastic (meaning probabilistic) parameters. It is a common misconception that chaotic systems are noisy systems driven by random processes. The irregular behavior of chaotic systems arises from intrinsic nonlinearities rather than noise.
3. "Sensitive dependence on initial conditions" requires that trajectories originating from very nearly identical initial conditions will diverge exponentially quickly. The meaning of this will be made clear in the following discussion.

The mathematical model developed, now called the Lorenz system, has been used as a paradigm for chaotic systems that satisfy the above definition. The Lorenz system consists of just three coupled first-order ordinary differential equations:

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= -x_1 x_3 + r x_1 - x_2 \\ \dot{x}_3 &= x_1 x_2 - b x_3 \end{aligned} \tag{1.1}$$

Lorenz chose parameter values $\sigma = 10$, $b = 8/3$, and $r = 28$. With these choices for the parameters, the Lorenz system is chaotic, exhibiting the traits described in the definition given for chaos.

The second component of the definition is clearly satisfied by the Lorenz system because none of the parameters is stochastic. To demonstrate the aperiodicity of the system, a numerical simulation of the Lorenz system can be performed.[1] A time-series plot of the x_1 variable is shown in Figure 1.1. The initial conditions can be chosen arbitrarily.

From direct observation of the time series in Figure 1.1a, it seems reasonable to say that the x_1 variable is aperiodic. To be certain that this system is

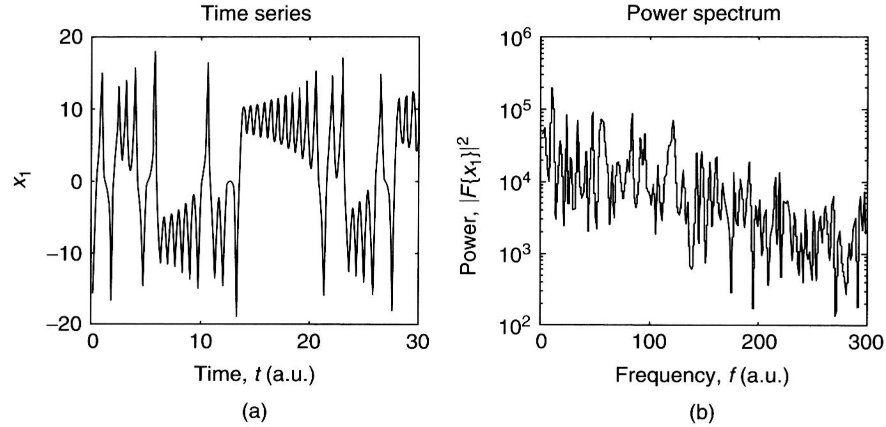

Figure 1.1 Both panels were generated using the Lorenz equations and the parameter values $\sigma = 10$, $b = 8/3$, and $r = 28$. (a) A representative time series for the x_1 Lorenz variable. (b) A typical power spectrum for the x_1 variable.

not just quasi-periodic, the power spectrum for the x_1 variable is shown in Figure 1.1b. The symbol F in the label of Figure 1.1b represents the Fourier transform operator. The power spectrum is very broad with no particularly conspicuous frequencies apparent. The irregular behavior shown for x_1 also occurs for the other variables as well, and does not diminish with increasing time.

The third component of the definition for chaos requires sensitive dependence on initial conditions. Again, the numerical simulation demonstrates that the Lorenz system satisfies this condition. The simulation can be run for two identical systems, x and y, starting from very nearly identical initial conditions. The only difference in their initial conditions was between the two variables, x_2 and y_2. Specifically, $y_2(t=0) = x_2(t=0) = 10^{-6}$. It is shown in Reference 1, Appendix A, and Appendix B that the magnitude of the difference between the two variables is a function of time, t. The linear t, even though started from nearly identical initial conditions, grows exponentially.

Another concept from chaos theory can be demonstrated using this numerical model by plotting x_2 against x_1. We obtain the well-known strange attractors as explained in Appendix A and Appendix B and in subsequent chapters as well.

1.2 Synchronization

It can be shown[1] that two identical chaotic systems can be synchronized if they are coupled together in an appropriate way.

Before beginning a discussion of the techniques through which systems can be synchronized, some powerful conceptual tools for understanding synchronization are introduced here. To illustrate the concept of synchronization, consider two independent Lorenz systems, x and y. Synchronization of the two systems occurs if $[x(t)-y(t)]$ goes to zero as t approaches infinity. The end result of this process is actual equality of the variables, x_1, x_2, and x_3, and y_1, y_2, and y_3, respectively, as they evolve in time.

Thinking geometrically, the dynamics of the composite system, comprising the two Lorenz systems, initially occurs in a six-dimensional phase space, with three dimensions associated with each individual system. As the two systems become synchronized, however, the trajectory of the composite system moves onto a three-dimensional subspace (or hyperplane) of the original six-dimensional phase space. This subspace is often called the synchronization manifold. It contains all of the points where $x_1(t)-y_1(t)=x_2(t)-y_2(t)=x_3(t)-y_3(t)=0$.

Consider an n-dimensional chaotic system

$$\dot{u}=f(u) \tag{1.2}$$

Pecora and Caroll[3] proposed decomposing such a system into two subsystems,

$$\dot{v}=g(v,w) \quad (m \text{ dimensional}) \tag{1.3}$$

$$\dot{w}=h(v,w) \quad (k \text{ dimensional}) \tag{1.4}$$

where $n=m+k$, and together these subsystems are the drive system.

The subsystem v of Equation (1.3) is used to drive a response system with the same functional form as Equation (1.4). Thus, the response system is written as

$$\dot{w}'=h(v,w') \tag{1.5}$$

The coupling between the systems occurs through the variable v from the drive system, which is substituted for the analogous v'in the response system.

To determine whether the two systems will synchronize, the evolution of the drive response composite system along directions transverse to the synchronization manifold must be analyzed. To do so, the evolution of the difference between the two systems, $w_\delta=w-w'$, is analyzed as

$$\dot{w}_\delta=h(v,w)-h(v,w') \tag{1.6}$$

$$=D_w h(v,w)w_\delta \quad \text{for small } w_\delta \tag{1.7}$$

where $D_w h=\frac{\partial h(v,w)}{\partial w}$ is the Jacobian of the w subsystem.

To clarify the concept of synchronization and illustrate the Pecora–Carroll technique, the Lorenz system is analyzed again in light of the preceding discussion. The drive system is given by the Lorenz equations,

$$\dot{x}_1 = \sigma(x_1 - x_2) \tag{1.8}$$

$$\dot{x}_2 = rx_1 - x_2 - x_1x_3 \tag{1.9}$$

$$\dot{x}_3 = x_1x_2 - bx_3 \tag{1.10}$$

where again, $\sigma = 10$, $b = 8/3$, and $r = 28$.

This system is decomposed so that the x_2 variable is coupled to the response system; it plays the same role as the v subsystem in Equation (1.3). The response system, therefore, is given by

$$\dot{y}_1 = \sigma(x_2 - y_2) \tag{1.11}$$

$$\dot{y}_3 = y_1x_2 - by_3 \tag{1.12}$$

where the variable y_2 has been completely replaced by x_2. A simple graphical illustration of this sort of decomposition is shown in Figure 1.2.

Thus, the analog to Equation (1.7) is

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} -\sigma & 0 \\ x_2 & -b \end{pmatrix} \begin{pmatrix} e_1 \\ e_3 \end{pmatrix} \tag{1.13}$$

where $e_1 = x_1 - y_1$ and $e_3 = x_3 - y_3$. In this case, the eigenvalues of the matrix fortunately do not depend on the drive variable, x_2. There is consequently no need to integrate numerically to determine the conditional Lyapunov exponents. The eigenvalues of the Jacobian, which are also the transverse Lyapunov exponents, are easily obtained; $\lambda_1 = -\sigma$ and $\lambda_2 = -b$. Both are negative at all times, and therefore, the error variables e_1 and e_3 converge to zero as $t \to \infty$. Equation (1.13) is not an approximation, and consequently, it proves that the systems will synchronize as $t \to \infty$, regardless of the initial conditions. The synchronization is quite fast relative to

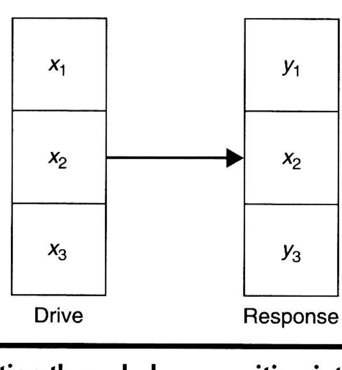

Figure 1.2 Synchronization through decomposition into drive and response systems. The variable y_2 has been completely replaced by x_2.

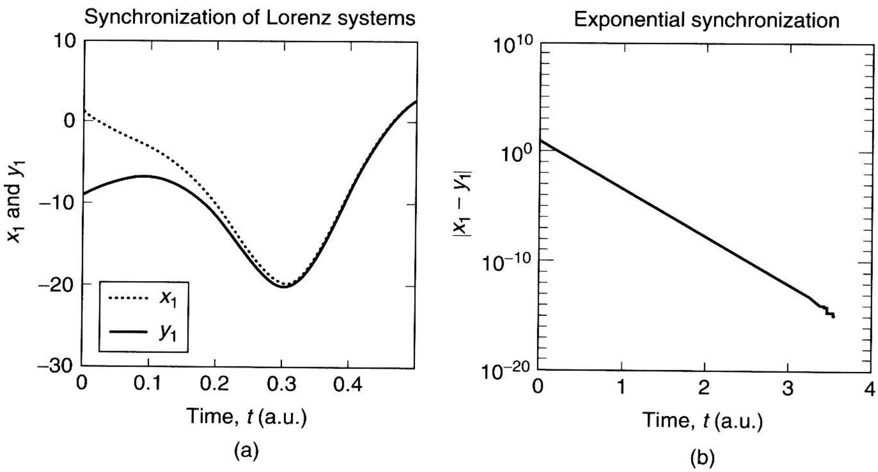

Figure 1.3 (a) Rapid synchronization of two coupled Lorenz systems even when they are started from very different initial conditions. (b) Synchronization occurs exponentially quickly. The slope can be calculated from Equation (1.13).

the speed of the oscillations of the system shown in Figure 1.1. Although Figure 1.3a indicated an exponential divergence of trajectories started at nearly identical initial conditions, Figure 1.3b shows that the trajectories between these two coupled systems converge exponentially quickly.

The slope of the line on this log-plot gives the exponent for this convergence. One could predict that this exponent ought to be the Lyapunov exponent associated with e_1, which was earlier determined to be $\lambda_1 = -\sigma = -10$. Converting σ to an exponent of 10, rather than of e, gives a value of $\sigma_{10} = -4{,}34$. This value matches the slope of the line shown in Figure 1.3b extremely well.

Other schemes for synchronization include the error feedback.[1] The simplicity of this method makes it amenable for experimental realizations of synchronization. Another of its advantages is that only small amounts of coupling between two systems are required for synchronization.

$$\begin{aligned} \dot{u} &= f(u) \\ y &= h(u) \\ y &= h(u) \end{aligned} \tag{1.14}$$

and the matching response system is written as

$$\begin{aligned} \dot{u}' &= f(u') + g(y - y') \\ y' &= h(u') \end{aligned} \tag{1.15}$$

Equation (1.15) clearly shows the manner in which the feedback is to be included. The same type of analysis leads to Equation (1.7).

Synchronization occurs for appropriate choices of feedback variable y and the coupling function g. As before, the real parts of the resulting conditional Lyapunov exponents must be negative for the systems to synchronize. The magnitude of the feedback term, $g(y-y')$, goes to zero as the systems become synchronized. Thus the error feedback technique allows chaotic systems to become predictable.[1]

The synchronization schemes that have been discussed thus far have one drive system and one response system. The error feedback method, however, permits a generalization to mutual coupling. In mutual coupling, both systems influence each other. Two systems, mutually coupled through an error-feedback signal, can be represented as follows:

$$\begin{aligned} \dot{u} &= f(u) + g(y' - y) \\ y &= h(u) \\ \dot{y}' &= f(u') + g(y - y') \\ y' &= h(u') \end{aligned} \tag{1.16}$$

For mutual coupling, each system is both a drive and response system.

1.2.1 Generalized Synchronization

The preceding discussion has focused on the concept of identical synchronization, a type of synchronization in which two identical coupled systems exhibit identical chaotic dynamics. A less restrictive manner of synchronization, called generalized synchronization (GS), has been investigated recently.[1] GS can occur between systems with mismatched parameters or even systems that are functionally dissimilar. The resulting dynamics of these coupled systems are not similar.[1]

1.3 Chaotic Communication Techniques

The relationship between synchronization and communication is not unique to chaotic communication. In AM radio communication, a message is used to modulate the amplitude of a specific frequency sine-wave carrier. A receiver tuned to that particular carrier frequency is able to recover the message. Recalling that synchronization (for nonchaotic systems) requires a common frequency for the two systems, the transmitter and receiver can be considered synchronized. In digital telecommunication systems, too, the transmitter and receiver must be synchronized. The receiver must sample bits with the proper timing to recover the message accurately. A chaotically fluctuating carrier of information represents a generalization of the more conventional sinusoidal or digital carriers. As with more traditional techniques, chaotic communication is also reliant on synchronization. The